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PHASE TRANSITIONS AT HIGH LEVEL DENSITY AND GIANT RESONANCES IN HOT NUCLEI*

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The properties of highly excited states embedded in the continuum of decay channels are investigated. First some general properties of open quantum systems are discussed. Especially, the restructuring taking place at high level density is considered and shown to be, in many cases, a second-order phase transition. The characteristic features of the phase transition are formulated. Then, the formalism is applied to giant resonances. The interferences of the resonance states via the continuum lead to a redistribution of the transition strength under critical conditions. Calculations for the ISMGR in ^{208}Pb are performed in the continuum-RPA. Narrow resonances at about 30 MeV excitation energy are shown to arise from resonance trapping. These resonance states have an almost pure 1p-1h structure.

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1. Introduction

Recent experimental results on giant resonances in hot nuclei showed that collisional damping is nearly the same at the temperature $T \approx 2$MeV as at the temperature $T \approx 0$MeV [1]. This result is a challenge for the theory. It is necessary to investigate in detail the properties of nuclear states under the conditions of extremely high excitation energy. In doing this, one has to study open quantum systems, since all the nuclear states at high excitation energy are embedded in the continuum of decay channels.

The properties of open quantum systems are discussed in several theoretical papers, see *e.g.* [2–11] and further references therein. At high level density, the states of the system interfere. As a result, collective short-lived

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states appear together with trapped long-lived states. The collectivity of the short-lived states originates from their coupling to the continuum. This so-called external collectivity appears additionally to the collectivity of intrinsic nature which is well known from the properties of giant resonances at low excitation energy.

The collective states can hardly be observed experimentally due to their short lifetime. Of special interest are therefore the long-lived states which may exist still at high excitation energy. A study of their spectroscopic properties allows us to gain more insight into the properties of open quantum systems.

In Section 2, some characteristic properties of open quantum systems are discussed. The nature of the restructuring taking place in an open quantum system under critical conditions is considered in Section 3. The interplay of internal and external collectivity in giant resonances is investigated in Section 4 while in Section 5, the results of calculations in the continuum-RPA for ^{208}Pb are given. Some conclusions are drawn in the last Section.

2. Open quantum systems

An open quantum system is described usually by a Hamiltonian of the type

$$\begin{aligned} \mathcal{H} &= H_0 + V + W \equiv H + W \\ &\approx H - ie^{i\theta} V V^+ , \end{aligned} \tag{1}$$

where $H = H_0 + V$ is the Hamiltonian of the closed quantum system whose discrete states are coupled to the continuum of decay channels by means of the coupling matrix $W \approx -ie^{i\theta} V V^+$. Due to this coupling, the states of the open system get a finite lifetime. The angle θ is nonzero, generally. The Hamiltonian $\mathcal{H}$ consists of the two parts H and W which both have, as a rule, a different rank. The rank of H is equal to the number N of states while that of W is equal to the number K of open decay channels. Of special interest is the case $K < N$ which is realized in many physical systems. For details see *e.g.* [2, 7].

The states of the open system are mixed due to the residual interaction V (internal or configurational mixing) in the same manner as the states of the closed system. Additionally they are mixed via the continuum if the non-diagonal matrix elements of W,

$$W^{\rm ext}_{RR'} = \langle \Phi^{SM}_R | V G^{(+)}_P V | \Phi^{SM}_{R'} \rangle \, , \tag{2}$$

are non-vanishing. Here $G^{(+)}_P = P(E^{(+)} - PHP)^{-1}P$ is the Green function in the function space $\mathcal{P}$ of the decay channels, P is the projector onto $\mathcal{P}$ and

Φ_R^{SM} are the eigenfunctions of H. Since H is Hermitian, the Φ_R^{SM} are real and the matrix elements $W_{RR'}^{\rm ext}$ are symmetrical in relation to an exchange of R and R'. The Hamiltonian $\mathcal{H}$ of the open system is non-Hermitian. Its eigenfunctions Φ_R are complex. Due to the symmetry of the $W_{RR'}^{\rm ext}$, it holds $\mathcal{H}\,|\Phi_R\rangle = \mathcal{E}_R\,|\Phi_R\rangle$ and $\langle\Phi_R^*|\,\mathcal{H} = \mathcal{E}_R\,\langle\Phi_R^*|$ and therefore

$$\langle\Phi_R^*|\Phi_{R'}\rangle = \delta_{RR'}\ . \tag{3}$$

From Eq. (3), it follows

$$\langle\Phi_R|\Phi_R\rangle = b_R\ \geq\ 1\ ; \qquad \langle\Phi_R|\Phi_{R'}\rangle \quad \text{complex for}\ \ R \neq R'\ . \tag{4}$$

The real value b characterizes the biorthogonality (3) of the eigenfunctions of $\mathcal{H}$. It is $b > 1$ in the neighbourhood of the crossing of every two resonances R and R'. Here, the wavefunctions of the states become mixed with each other.

Let us illustrate the differences between open and closed quantum systems by means of the simple case of two states coupled to one common open decay channel. The Hamiltonian is

$$\mathcal{H}^{(2)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - 2i\alpha e^{-i\theta} \begin{pmatrix} \cos^2\varphi & \cos\varphi\,\sin\varphi \\ \cos\varphi\,\sin\varphi & \sin^2\varphi \end{pmatrix}\ . \tag{5}$$

The angle θ determines the ratio between the real and the imaginary part of the coupling term. The symmetry of the problem (for $\theta = 0$) is determined by the coupling vector $V = \sqrt{\frac{2\alpha}{\pi}}\ (\cos\varphi, \sin\varphi)$, i.e. by the angle φ. The coupling strength between the discrete states at $E_\pm = \pm 1$ and the continuum can varied by means of the parameter α.

The eigenvalues of $\mathcal{H}^{(2)}$ are

$$\mathcal{E}_\pm = -i\alpha e^{-i\theta} \pm \sqrt{1 - 2i\alpha e^{-i\theta}\ \cos 2\varphi - \alpha^2 e^{-2i\theta}}\ . \tag{6}$$

Analytical and numerical studies show the existence of a critical value of the parameter α at which the two states have a minimum distance in the complex plane [5]:

$\alpha < \alpha_{\rm crit}$:
the energies of both states are almost independent of α (for $\theta = 0$);
the widths of both states increase with increasing α; $b \approx 1$

$\alpha \to \alpha_{\rm crit}$:
the levels attract each other in energy; the widths bifurcate; $b > 1$
(one of the states aligns with the channel)

$\alpha > \alpha_{\rm crit}$:
the energies are almost independent of α (for $\theta = 0$); resonance trapping occurs: the width of the state aligned with the channel, increases further with increasing α, while the width of the other (trapped) state decreases; $b \approx 1$

At the critical value $\alpha_{\rm crit}$, the two resonances repulse each other in the complex plane. Their wavefunctions become mixed. At $\alpha > \alpha_{\rm crit}$, the differences in the lifetimes of the two states increase: one of the states is aligned with the decay channel and becomes short-lived while the other one is trapped and becomes long-lived.

In an open quantum system with N states coupled to K common decay channels, the situation is similar: K states align with the K open channels and become short-lived. The remaining $N - K$ states ($N > K$) become trapped and long-lived. Mathematically, this is achieved by the different rank of the two parts H and W of the Hamiltonian $\mathcal{H}$. At $\alpha \ll \alpha_{\rm crit}$, the rank of $\mathcal{H}$ is determined by that of H while at $\alpha \gg \alpha_{\rm crit}$, it becomes equal to that of W. The restructuring in an open quantum system takes place if the nondiagonal matrix elements in Eq. (2) cannot neglected.

In an open quantum system, it holds

$$\mathcal{E}_R \cdot b_R = \langle \Phi_R | \mathcal{H} | \Phi_R \rangle \,, \tag{7}$$

where $\mathcal{E}_R = E_R - \frac{i}{2}\Gamma_R$ are the complex eigenvalues of $\mathcal{H}$. Thus, the value b_R appears also in this basis equation.

3. Phase transitions and time scales in quantum systems

In [8], the question is investigated whether the restructuring taking place in an open quantum system, can be considered as a phase transition. To this purpose, analytical and numerical studies are performed for the picket fence and for some other model systems coupled to one open decay channel. The Hamiltonian is

$$\mathcal{H}' = H - i\,\alpha\, e^{i\theta}\, VV^{+} \tag{8}$$

in analogy to (1). The parameter α plays the role of a control parameter.

In many cases, the restructuring taking place in the system is a second-order phase transition. The conditions are formulated analytically in the limit $N \to \infty$. The order parameter is Γ_0/N where Γ_0 is the width of the state which aligns with the channel. The numerical results for finite N show that the characteristic properties of the phase transition appear already at $N < 100$.

In [8], the following two values are introduced: first

$$B = \frac{1}{N} \sum_{R=1}^{N} b_R \geq 1 \tag{9}$$

is defined in analogy to b_R given by Eq. (4) and secondly

$$N_R^p \; = \; \frac{1}{N \; \sum_{R'} |\hat{\beta}_{RR'}|^4} \; ; \qquad 1/N \;\; \leq \;\; N_R^p \;\; \leq 1 \tag{10}$$

being the number of principal components. Here, the representation $\Phi_R = \sum_{R'} \; \beta_{RR'} \; \Phi_{R'}^{SM}$ with the normalization $|\hat{\beta}_{RR'}|^2 = \frac{|\beta_{RR'}|^2}{\sum_{R'} |\beta_{RR'}|^2}$ is used. It is $N_R^p = 1/N$ for a pure state (in relation to the eigenfunctions $\{\Phi_R^{SM}\}$ of H) but $N_R^p = 1$ for a state maximal mixed in the $\{\Phi_R^{SM}\}$. Such a state is a collective state.

Considering these values and the lifetimes of the states, the phase transition at $\alpha \to \alpha_{\rm crit}$ is charaterized in the following manner.

— It is $B \gg 1$. This means, all resonance states cross at (almost) the same value $\alpha = \alpha_{\rm crit}$ and the system changes its properties suddenly as a function of the parameter α.

— The state $R = 0$ aligns with the channel: the value $N_{R=0}^p$ changes suddenly from $1/N$ to 1. The state $R = 0$ becomes a collective one. Its wavefunction is coherently mixed in relation to the eigenfunctions $\{\Phi_R^{SM}\}$ of H while the wavefunctions of the other states $R \neq 0$ remain almost pure.

— A separation of time scales (bifurcation of the widths) takes place. For $K = 1$ and $\alpha > \alpha_{\rm crit}$, the average width $\bar{\Gamma}^{\rm \; long} = \frac{1}{N-1} \sum_{R'=1}^{N-1} \Gamma_{R'}$ of the long-lived trapped states may differ considerably from the width $\bar{\Gamma}^{\rm short} = \Gamma_{R=0}$ of the short-lived aligned state. (At $\alpha = \alpha_{\rm crit}$, $\Gamma_{R=0}$ is much smaller than the length of the spectrum).

The relation between the phase transition and the appearance of a collective state at different values of θ is investigated in the one-channel case in [8, 9]. The restriction to the one-channel case is justified also at high excitation energy since the system is coupled with a different strength to the different channels. A detailed study of this question can be found in [7].

Collectivity arising at $\theta = 0$ is called external collectivity [6] in order to distinguish it from the well-known internal collectivity. The latter one appears at $\theta = \pi/2$. In this case, the Hamiltonian $\mathcal{H}$, Eq. (1), is Hermitian and the collectivity is of internal origin. Both cases are border-line cases.

At high excitation energy, the collective states can hardly be identified due to their large widths. More interesting is therefore to study the long-lived states which accompany the formation of the collective states at high excitation energy. Their existence is not only a hint to resonance trapping in real systems but shows also that an equilibration of the states in relation to their decay widths does not occur in open quantum systems.

4. Interplay of internal and external collectivity in giant resonances

Describing the interplay between internal and external collectivity in the case of a giant resonance at high excitation energy we follow the two papers [6]. The Hamilton operator is non-hermitean,

$$\tilde{\mathcal{H}} = \tilde{H}_0 + \tilde{\mathcal{H}}^{\rm int} \equiv \tilde{H} - \frac{i}{2} A A^T \equiv \tilde{H}' + \boldsymbol{D}\boldsymbol{D}^T \ . \tag{11}$$

The residual interaction $\tilde{\mathcal{H}}^{\rm int} \equiv \boldsymbol{D}\boldsymbol{D}^T - \frac{i}{2} A A^T$ contains both the internal and external interaction of the states: $\boldsymbol{D}\boldsymbol{D}^T$ is the dipole-dipole residual interaction and the elements of the coupling matrix AA^T are determined by the sums $\sum_c A^c_m A^c_n$ (c denotes a channel and m and n the discrete states). The strength of the internal and external mixing, respectively, is governed by the parameters

$$\kappa = \frac{\Delta_e}{\boldsymbol{D}^2} \quad ; \quad \kappa' = \frac{\Delta_e}{\langle (\boldsymbol{A}^c)^2 \rangle} \, , \tag{12}$$

where Δ_e is the variance of the distribution of the parental (unmixed) levels and $\langle (\boldsymbol{A}^c)^2 \rangle$ is the mean coupling matrix element, $(\boldsymbol{A}^c)^2 = \sum_n (A^c_n)^2$, of the parental states to the continuum.

In the case $\kappa \ll 1$ (and $\kappa' \gg 1$), the internal collectivity is concentrated on one of the levels (giant resonance) which is shifted in energy by $\boldsymbol{D}^2$ from the remaining $N-1$ states. In the opposite case, $\kappa' \ll 1$ (and $\kappa \gg 1$), we have K broad states with large external collectivity, which are aligned with the K channels. The widths of the remaining $N-K$ states approach zero with $\kappa' \to 0$, i.e. they are (almost) decoupled from the continuum. The K broad states are shifted in energy to the centre of the position of the parental levels.

The interference between the two types of collective states depends mainly on the ratio

$$\lambda = \frac{\langle (\boldsymbol{A}^c)^2 \rangle}{\boldsymbol{D}^2} \tag{13}$$

of the strengths of external and internal interaction and on the angle Θ, $0 \leq \Theta \leq \pi/2$, between the dipole vector $\boldsymbol{D}$ and the K-dimensional subspace spanned by the decay vectors $\boldsymbol{A}^c$.

The arising picture [6] is similar to that obtained for the two-resonance case discussed in Section 2:

> $\lambda < \lambda_{\rm crit}$:
> both κ and κ' are so large that there exist $1 + K$ collective states (together with $N - K - 1$ trapped states); the energies and dipole strengths of the states are almost independent of λ; the widths increase with increasing λ
>
> $\lambda \to \lambda_{\rm crit}$:
> the $K + 1$ collective states attract each other in energy; a large part of the dipole strength is transferred to externally collective states; the widths bifurcate
>
> $\lambda > \lambda_{\rm crit}$:
> the energies and dipole strengths are almost independent of λ; one of the externally collective states is trapped: the widths of K states increase with further increasing λ, while the width of the externally collective state with the smallest width and largest dipole strength decreases

It is interesting to see the following: The dipole strengths belong to the real part of the Hamiltonian (11) and show, under critical conditions, the same dependence on λ as the energies (which are also related to the real part of the Hamiltonian).

Thus, we have the following picture. As long as λ is small, we see radiation only from the giant resonance. If, however, all the resonances are mixed via the continuum of decay channels ($\lambda \to \lambda_{\rm crit}$), the interference leads to a strong redistribution of the dipole strengths as well as of the escape widths between the states. The redistribution is accompanied by energy shifts. When λ exceeds the critical value we see the resonance trapping: the escape width of one of the states with large external collectivity starts to decrease while the escape widths of the other states increase further as a function of increasing λ. That means, some part of external collectivity is transferred from one state to some others. This transfer is, as the results show, accompanied by the transfer of some part of internal collectivity in the opposite direction. As a result, the internal collectivity (dipole strength) is no longer concentrated on one resonance state, but is distributed over more resonances. All resonances lie at an energy which is lower than the original energy of the dipole resonance.

Another consequence of resonance trapping is that states with pure wavefunctions (in relation to the eigenfunctions $\{\Phi_R^{SM}\}$ of H) exist still at high

excitation energy. It is shown in [12] that collisional damping is determined not by the absolute value of the internal interaction but by its value relative to the coupling strength of the system to the continuum. The latter value increases with the excitation energy surely stronger than the former. The sharp transition at $\lambda = \lambda_{\rm crit}$ will therefore not be washed out due to spreading. As a consequence, the results [6] obtained by us point to a new mechanism which could possibly shed an additional light on the quenching of the multiplicity and the existence of a limiting excitation energy for the observation of giant dipole resonances through its γ-emission [13].

5. Calculations in the continuum-RPA for ^{208}Pb

Within the random phase approximation in the continuun (continuum-RPA), calculations are performed for the $4\hbar\omega$ isoscalar monopole giant resonance (ISMGR) in ^{208}Pb. It lies at about 30 MeV excitation energy and consists of $N = 34$ basis states of $4\hbar\omega$ particle-hole type in which the particle is in a stationary or quasistationary state. These basis states are coupled to $K = 27$ open decay channels. For details see [10].

In the calculated strength function, there are one broad resonance and seven narrow resonances lying near to the maximum of the broad resonance. The widths of the $N - K = 7$ narrow states are small due to resonance trapping. The results of further calculations show that the seven narrow resonance states are almost pure 1p-1h states (for the parameters of the narrow resonances in the energy dependence of the strength function see Table I taken from [10]). This means that the interaction of the 1p-1h states is effectively reduced due to their strong coupling to the continuum.

TABLE I

Parameters of the narrow resonances (from [10])

n	ω_n (MeV)	Γ_n (keV)	s_n (%)	basis state	ω_n^{ph} (MeV)	s_n^{ph} (%)
1	29.8	26	0.64	$2s^n_{1/2} - 4s^n_{1/2}$	29.9	0.49
2	30.8	39	0.71	$1p^p_{3/2} - 3p^p_{3/2}$	30.9	0.55
3	31.3	66	0.96	$1d^n_{5/2} - 3d^n_{5/2}$	31.4	0.91
4	31.5	12	0.04	$1p^p_{1/2} - 3p^p_{1/2}$	31.6	0.16
5	31.9	11	0.40	$1d^n_{3/2} - 3d^n_{3/2}$	31.9	0.49
6	33.6	73	0.77	$1d^p_{5/2} - 3d^p_{5/2}$	33.8	0.95
7	34.3	146	0.39	$1d^p_{3/2} - 3d^p_{3/2}$	34.4	0.45

These results show that the continuum influences strongly not only the widths but also the energies and strengths of the individual components of giant resonances. In calculations without the continuum the role of the residual (internal) interaction is overestimated.

The spreading width of the $4\hbar\omega$ ISMGR is expected to be not larger than 5 to 6 MeV. We expect therefore that the $4\hbar\omega$ ISMGR in ^{208}Pb can be observed experimentally *e.g.* by means of (α, α') reactions at large momentum transfer. The result would prove the trapping mechanism in open quantum systems.

6. Summary

The experimental and theoretical studies of highly excited states in nuclei raised some interesting questions. Both, the theoretical as well as the experimental investigations, prove the existence of long-lived states with almost pure nuclear structure (in relation to the wavefunctions of the closed system). This result is unexpected from the point of view of the statistical theory of nuclear reactions. It means that an equilibration of the states in relation to their decay widths does not occur in highly excited quantum systems.

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REFERENCES

[1] A. Bracco *et al.*, *Phys. Rev. Lett.* **74**, 3748 (1995); Nuovo Cimento (Proceedings SNEC98); M. Mattiuzzi *et al.*, Phys. Lett. B364, 13 (1995); *Nucl. Phys.* **A612**, 262 (1997).

[2] I. Rotter, *Rep. Prog. Phys.* **54**, 635 (1991).

[3] V.V. Sokolov, V.G. Zelevinsky, *Ann. Phys.* (N.Y.) **216**, 323 (1992).

[4] M. Desouter-Lecomte, J. Liévin, V. Brems, *J. Chem. Phys.* **103**, 15 (1995).

[5] M. Müller, F.M. Dittes, W. Iskra, I. Rotter, *Phys. Rev.* **E52**, 5961 (1995).

[6] V.V. Sokolov, I. Rotter, D.V. Savin, M. Müller, *Phys. Rev.* **C56**, 1031 and 1044 (1997).

[7] E. Persson, T. Gorin, I. Rotter, *Phys. Rev.* **E** (1998), in press.

[8] C. Jung, M. Müller, I. Rotter, *quant-ph/9804020.*

[9] W.D. Heiss, M. Müller, I. Rotter, *quant-ph/9805038.*

[10] S.E. Muraviev, I. Rotter, S. Shlomo, M.H. Urin, *mpi-pks/9804006.*

[11] further references in I. Rotter, *J. Chem. Phys.* **106**, 4810 (1997).

[12] E. Persson, I. Rotter, to be published.

[13] M. Thoennessen (ed.), Proceedings of the Gull Lake Nuclear Physics Conference on Giant Resonances, *Nucl. Phys.* **A569** (1994); T. Suomijärvi *et al.*, *Phys. Rev.* **C53**, 2258 (1996).