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RELATING MICROSCOPIC RESULTS TO SEMICLASSICAL MODELS OF COLLECTIVE EXCITATIONS*

Dietrich Zawischa

Institut für Theoretische Physik, Universität Hannover
D-30167 Hannover, Germany

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The interpretation of the low energy orbital magnetic dipole excitations of nonspherical nuclei, commonly called scissors modes, is examined. It is demonstrated that the "fluid dynamical model" which evolves from the Bohr–Mottelson collective model when the nuclear fluid is treated in Thomas–Fermi approximation, agrees much closer with microscopic calculations than other semiclassical models. This allows to interpret the microscopic results in more detail than usually done, and sheds light on the origin of the apparent discrepancies between semiclassical theory and experiment.

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The Random Phase Approximation (RPA) is appropriate for the calculation of low energy small amplitude collective excitations and has been widely used. Starting from a well adjusted shell model potential and employing a realistic effective interaction, good agreement with experimental data can be reached. Therefore an analysis of the RPA state vectors should supply their correct physical interpretation. In most cases, *e.g.* in collective electric excitations of nonspherical nuclei, the nature of the excitations is clearly seen in the transition densities [1].

However, in the case of the low energy isovector magnetic dipole excitations of deformed nuclei which are called scissors modes, the discussion about their true nature was not ended when good microscopic results were available.

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One quantity which has been widely used in favour of a “scissors” interpretation is the squared overlap with the so-called synthetic or pure scissors state which has been obtained from the RPA state vectors. But the squared overlaps are of the order of 20% or less which makes it difficult to draw definite conclusions.

Let us look at the competing models for isovector rotational vibrations. Different current distributions producing a change in the orientation of the nuclear surface or, in the case of isovector modes, the orientation of the proton and neutron bodies, are shown in Fig. 1. Rigid rotation (Fig. 1 b) has been assumed in early predictions of scissors modes in the Two Rigid Rotors Model (TRRM) [2, 3], irrotational quadrupole flow (Fig. 1 c) in generalizations of the collective model to isovector oscillations (Extended Collective Model (ECM) [4] or Neutron-Proton-Deformation model (NPD) [5]). Both the TRRM and the ECM (or NPD) lead to the following expressions for the frequency and magnetic dipole transition strength:

$$\omega^2 = \frac{\delta^2 C}{I_1} \qquad B(\mathrm{M1})\uparrow = \frac{3\mu_{\mathrm{N}}^2}{16\pi\hbar^2}\,\hbar\omega I_1\,. \tag{1}$$

Here δ is the deformation parameter, C the restoring force constant, and I_1 the moment of inertia around the x-axis, rigid or irrotational, respectively.

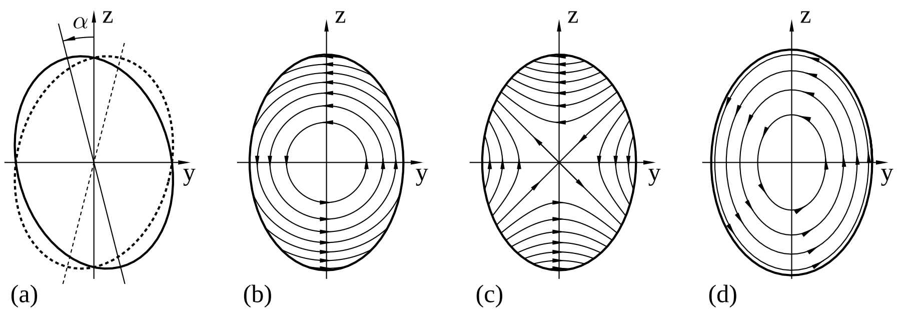

Fig. 1. (a) — Scissors-like isovector rotational oscillation of protons (solid line) against neutrons (broken line) at the classical turning point. It can originate from rigid rotation (b) — from irrotational quadrupole flow (c) — or from a superposition of both. There is a special superposition of both modes (d) — which does not lead to a change of the surfaces’ orientation, thus satisfying boundary conditions of Steinwedel–Jensen type. In the drawings (b) to (d), only the proton current is shown, the neutron current going in the opposite direction.

As both models with parameters C and I_1 derived from the liquid drop model badly fail to reproduce the experimental data (see Table I), the parameters have been adjusted to the data. Then, of course, the results of both

TABLE I

Comparison of semiclassical models for ^{156}Gd with parameters as derived from the liquid drop model. The experimental data are from [6, 7].

	E_{x} (MeV)	B(M1) (μ_{N}^2)	B(E2) ($e^2\mathrm{fm}^4$)
Rigid Rotors	6.18	21.31	404
Irrotational Flow	20.5	6.43	1338
Fixed Surface	3.90	10.34	0
Experiment	3.07	1.30(20)	40(6)

models coincide, the models have been considered to be equivalent, though in their basic assumptions they are contradictory.

A still different semiclassical model has been proposed by Lipparini and Stringari [8]: They assume Steinwedel-Jensen boundary conditions (i.e. a fixed surface) which force the flow in the interior to be a superposition of irrotational and rotational flow. This mode (depicted in Fig. 1 d) is very interesting: for a classical nonviscous fluid there would be no restoring force at all, it thus would have zero frequency. However, if there is some nuclear elasticity as postulated by Bertsch [9], due to the deformation of the volume elements a restoring force arises.

Now there are already three different flow patterns which may correspond to the low energy M1 states. Having an RPA state vector, can we decide which pattern comes closest?

Irrotational flow can be ruled out by the form factor [10]. It occurs (to a good approximation) in the $|K| = 1$ component of the giant quadrupole resonance but not in a low energy state. It has turned out that the form factors are not sensitive to differentiate between the models of Fig. 1 (b) and (d) [10]. Let us therefore look at the transition current densities from which the form factors are calculated. Considering the linear combination of $K = 1$ and $K = -1$ modes which corresponds to a rotation around the x-axis, it is not too difficult to visualize the transition current distribution: Consider a point labelled by the cylindrical coordinates ρ, ϕ, and z, and decompose the vector of the local current density $\vec{j}$ in its components j_ρ, j_ϕ, and j_z. The dependence of these quantities on the azimuthal angle ϕ is very simple:

$$
\begin{gathered}
j_\rho(\rho,\phi,z) = j_\rho(\rho,\tfrac{\pi}{2},z)\,\sin(\phi) \qquad j_z(\rho,\phi,z) = j_z(\rho,\tfrac{\pi}{2},z)\,\sin(\phi) \\
j_\phi(\rho,\phi,z) = j_\phi(\rho,0,z)\,\cos(\phi)\,.
\end{gathered}
\tag{2}
$$

We need to know j_ϕ only in the x-z-plane and j_ρ, j_z in the y-z-plane to know $\vec{j}$ everywhere. Moreover, the pattern in the x-z-plane is symmetric with respect to the z- and antisymmetric with respect to the x-axis, while in the y-z-plane j_ρ and j_z are antisymmetric with respect to both axes. Therefore we show only one quadrant of both planes.

Fig. 2 shows the result for the most prominent low energy state of ^{164}Dy with theoretical $B^{\rm orb}({\rm M1})\!\uparrow = 1.13\,\mu_{\rm N}^2$. It is clearly seen that there is a certain amount of isovector rotational motion present, but there is additional small-scale vorticity superimposed which makes it difficult to decide whether it is more similar to Fig. 1 (b) or (d). Only when a larger number of scissors states of different nuclei is examined, one realizes that there is the general tendency of the flow to be more or less parallel to the surface in the outer region, thus favouring the "fixed surface" model (Fig. 1 d).

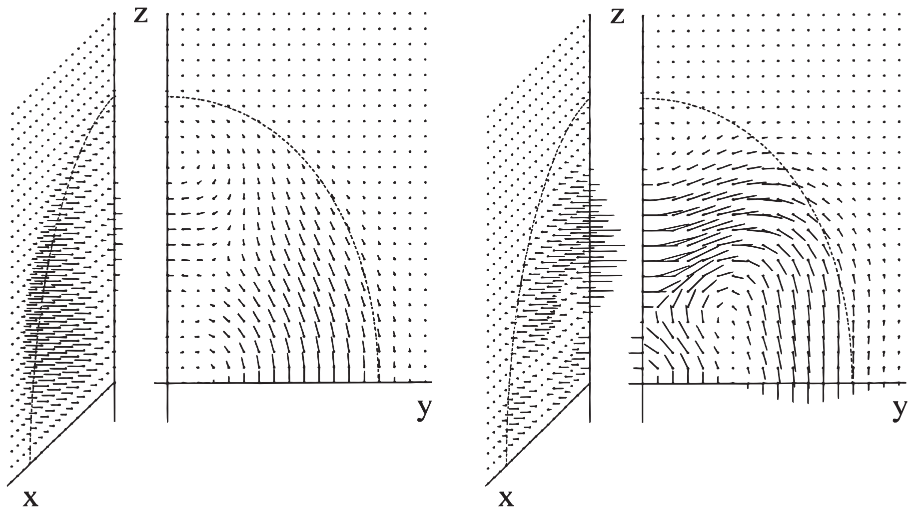

Fig. 2. Flow pattern of the M1 excitation at 3.2 MeV of ^{164}Dy (in arbitrary units). Left hand side: protons, right hand side: neutrons.

The merit of the TRRM has been the introduction of the rotational degree of freedom, and the microscopic transition currents confirm that this degree of freedom does exist in nuclei. We also see in other states (Fig. 3), that (almost) irrotational quadrupole flow is possible, too. This is the kind of flow assumed in the collective model and its extensions. If both modes are treated simultaneously assuming a classical fluid, the rotational degree of freedom is absorbed in a zero energy mode and only irrotational flow survives. This should not be taken as an argument in favour of the ECM. Actually, with parameters extracted from the liquid drop model, the ECM describes the quadrupole giant resonance quite well, but not the low energy states, as Table I shows.

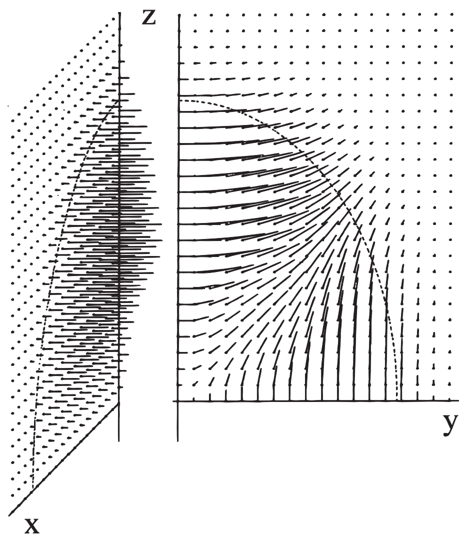

Fig. 3. Neutron transition current distribution of the $|K| = 1$ component of the giant quadrupole resonance of ^{165}Gd, an example of a highly collective state.

In Table I also quadrupole transition strengths are shown. The quadrupole strength comes out to be proportional to ω^{-1}, which means that changing parameters to lower the energy to 3 MeV would increase the quadrupole strength dramatically. This would never escape experimental detection.

It is, therefore, clear that in the isovector rotational degree of freedom the situation is similar to the isoscalar case in that the motion is neither rigid rotation nor irrotational. By adjusting parameters to the experimental data one implicitely admits more complex flow patterns than initially assumed. Thus from the TRRM one goes to the Two Rotor Model, (TRM). One should be aware of the fact that, when changing the restoring force parameter and the moment of inertia, also the variables may change their meaning [11].

This is easily demonstrated comparing the different flow patterns shown in Fig. 1. They all are special cases of the TRM, when the condition of rigidity is relaxed. (Restricting to small oscillation amplitudes, we need not bother about the problems connected with multiple-valuedness of the angle.) In the TRRM we have rigid rotation, and the variable canonically conjugate to the angular momentum of one of the bodies is just the geometrical angle of rotation, visible as rotation of the shape. In the fixed surface model, there is also rotational motion and therefore an angular momentum, but the variable conjugate to angular momentum certainly is not the geometrical angle of rotation of the shape which remains zero.

To improve the semiclassical description all relevant degrees of freedom and all possible sources of restoring force should be included from the very beginning. One then arrives at nuclear fluid dynamics (NFD) as proposed by Holzwarth and Eckart [12, 13], which incorporates nuclear elasticity [9] and has no free parameter.

Details of the calculation have already been published [10, 11], here only the results are presented: Considering two degrees of freedom (rotational and $|K| = 1$ quadrupole) we get two eigenmodes of the system. While the higher energy mode looks very similar to the quadrupole mode itself (Fig. 1 c), the low energy mode is clearly a superposition of rotational and quadrupole motion. It is shown in a drawing to scale in Fig. 4, which illustrates the situation at the classical turning point. It is not exactly equal to the result obtained with Steinwedel-Jensen boundary conditions, but very similar. Certainly, this also is one of the possible modes encompassed by the TRM.

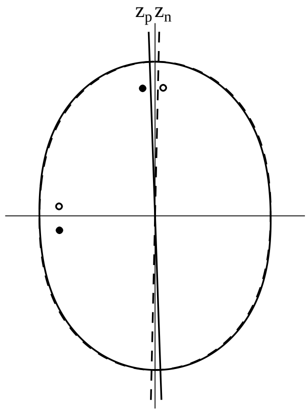

Fig. 4. The low-energy isovector rotational vibration at the classical turning point. The surface and symmetry axis of the proton fluid is drawn as full line, those of the neutron fluid are dashed. Two volume elements of the protons (neutrons) are marked by full circles (open circles). At rest, the full and open circles would coincide. It is seen that the angles of rotation of the volume elements are larger than those of the symmetry axes. The drawing is to scale, with parameters corresponding to ^{164}Dy.

It is instructive to compare the semiclassical NFD results with microscopic RPA. The RPA does not yield a single collective low energy orbital M1 state but rather a group of states which, in the rare earths, ranges up to ≈ 10 MeV (see [11] and references cited there). An other concentration of orbital magnetic dipole strength is found above 20 MeV. As is seen from Table II, there is very good agreement between the summed RPA and semiclassical results (and both, in fact, are very close to the sum-rule estimates of Lipparini and Stringari 1983 [14]). The only slight discrepancy, namely

TABLE II

Comparison of RPA with Migdal force (average excitation energy and summed strengths), NFD and rigid rotors model for the example ^{164}Dy. The table is taken from [11].

	$E_{\rm x}^{\rm ave}$ MeV	$\sum B({\rm M1})^{\rm orb}$ $\mu_{\rm N}^2$	$\sum E_{\rm x} B({\rm M1})^{\rm orb}$ $\mu_{\rm N}^2$MeV	$\sum B({\rm E2})$ $e^2{\rm fm}^4$
RPA 0–10 MeV	5.6	7.28	40.6	103
NFD low energy	3.26	11.6	37.8	71.6
TRRM	6.15	22.9	141	439
RPA high energy	21.2	3.2	67.6	1148
NFD high energy	23.8	4.36	103.8	1234

the somewhat low exitation energy of the semiclassical model, is explained by the fact that pairing has not been included, which would shift the energy up while conserving the energy weighted sum rule.

From the agreement of the RPA with experimental data on one hand, with the semiclassical Fermi-fluid model on the other hand, we conclude that the experimentally detected low energy M1-strength corresponds to the low energy wing of the collective mode which, due to quantum mechanical effects, is spread in the range up to $\approx$ 10 MeV in the rare earths [11], and that the closest possible semiclassical visualization of the mode is given by the Fermi-fluid (or NFD) model.

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