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SHADOWING OF VIRTUAL PHOTONS IN NUCLEI AT SMALL $x_{\rm Bj}$ IN THE QCD DIPOLE PICTURE

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Compact and well defined formulae for the shadow of the virtual photon interacting with a large nucleus at small $x_{\rm Bj}$ are given in the QCD dipole picture. Two classes of contributions are considered: (a) quasi-elastic interaction of the $q\bar{q}$ dipole and (b) multi-pomeron coupling.

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1. Introduction

We have now at our disposal the QCD dipole picture of virtual photon interactions [1–3] with hadrons [4,5] which is able to account for existing data on proton structure function F_2 (*i.e.* on total virtual photon–proton cross-section) at small $x_{\rm Bj}$. It is therefore interesting to investigate whether the model can be also applied to nuclear targets, in particular if it can describe the shadowing of the virtual photons. This becomes especially interesting in view of a possible extention of the HERA experimental program to nuclear targets [6].

In our recent paper [7] we have calculated in this model the double scattering contribution to the virtual photon shadowing[1]. The multiple scattering corrections, which are more difficult to handle, were merely estimated, however. As the resulting shadowing at small $x_{\rm Bj}$ turned out to be rather substantial and not too far from experimental data, it seems worthwhile to pursue the problem further and to evaluate more precisely the multiple scattering terms. This is the purpose of the present paper.

[1] The subject of photon shadowing in nuclei has a vast literature. For recent reviews, see *e.g.* [6] and [8].

As in [7] (*c.f.* also [9,10]) we are dealing with two types of contributions:

(a) The "quasi-elastic" interaction of the $q\bar{q}$ dipole (first introduced in the dipole picture in [10]) contributing mostly at small mass excitations of the virtual photon.

(b) The "direct" interaction of the dipole cascade (corresponding to the so-called multiple pomeron coupling in the Regge language) which dominates the high-mass excitations.

The amplitudes describing these processes were derived and discussed in [1–3,9–11]. When applied to the interaction with nuclear targets, however, they must be modified in two important points:

(i) a phase difference related to the change in the longitudinal momentum of system travelling through the nucleus must be taken into account. Consequently, at some point it is nessesary to transform the amplitudes into the momentum representation. This makes the calculation substantially more complicated.

(ii) Since the transverse size of the gluon cascade is much smaller than that of a large nucleus, we can use the standard approximation and integrate over the impact-parameter dependence of the elementary amplitudes, *i.e.* we need to deal with forward scattering amplitudes only. Indeed, the convolution of the onium and the nucleus transverse profiles is approximately

$$\int T_A\,(\vec{s})\,T(\vec{b}-\vec{s})\,d^2s \approx T_A\,(b)\int T\,(b')\,d^2b' = \tilde{T}(k=0)\,T_A\,(b)\,, \qquad (1)$$

where $\tilde{T}(k=0)$ is the forward scattering amplitude and $T_A(b)$ is the nucleus profile.

This simplifies the calculation in an essential way.

Our ultimate goal is to give an explicit expression for the shadow, defined as

$$C(A, x_{\rm Bj}, Q^2) = \frac{\sigma_{\rm tot}(A)}{A\sigma_{\rm tot}(n)} - 1, \qquad (2)$$

where $\sigma_{\rm tot}(A)$ is the total photon-nucleus cross-section and $\sigma_{\rm tot}(n)$ is the total photon-nucleon cross-section. $\sigma_{\rm tot}(n)$ has been measured [12] and successfully computed in the dipole picture [4]. This will be our input.

To compute $\sigma_{\rm tot}(A)$ we have to know the forward amplitudes for (a) quasi-elastic scattering of the dipole and (b) direct scattering of the dipole cascade from a given number, N, of nucleons. We then dress them up with the light-cone photon wave functions [13] and with single nucleon densities characterizing the nuclear shape. Finally we add them up for $2 \leq N \leq A$. This calculation is described in the next two sections.

In Sec. 4 we discuss the obtained results and formulate our conclusions.

2. Forward γ^*-nucleus amplitude from the direct interaction of the dipole cascade

We start by giving the final result for the forward amplitude for scattering on N nucleons of the dipole cascade emerging from an onium. It reads

$$F_N^{\rm dir}(r,r_0;Y,y) = (2N-3)!!\pi\alpha^2(\alpha^3N_c)^{N-1}r_0^{2N}n_{\rm eff}^N {\rm e}^{i(z_1-z_N)mx_P}$$
$$\times\prod_{k=1}^{N}\left(\int\limits_{c_k-i\infty}^{c_k+i\infty}\frac{d\lambda_k}{2\pi i}h(\lambda_k){\rm e}^{\varDelta(\lambda_k)y}G(\lambda_k)\right){\rm e}^{\varDelta(\gamma)(Y-y)}\left(\frac{r}{r_0}\right)^{\gamma}\frac{1}{G(\gamma)}, \quad (3)$$

where

$$\varDelta(\lambda) = \frac{\alpha N_c}{\pi}\chi(\lambda),\ \chi(\lambda) = 2\psi(1)-\psi\left(1-\tfrac{\lambda}{2}\right)-\psi(\tfrac{\lambda}{2}), \quad (4)$$

α is the strong coupling constant, $N_{\rm c}$ is the number of colours, r_0 is a parameter of the order of the effective size of the nucleon, r is the transverse size of the incident onium. The remaining symbols denote:

$$h(\lambda) = \frac{4}{\lambda^2(2-\lambda)^2}\,,\quad G(\lambda) = \frac{\varGamma(\frac{\lambda}{2})}{\varGamma(1-\frac{\lambda}{2})}\,,\quad \gamma = \sum_{k=1}^{N}\lambda_k\,,$$
$$Y = -\log\tfrac{x_{\rm Bj}}{c}\,,\quad y = -\log\tfrac{x_P}{c}\,,\quad x_P = \tfrac{x_{\rm Bj}}{\beta}\,,\quad \beta = \tfrac{Q^2}{Q^2+M^2}\,, \quad (5)$$

and M is the mass of the dipole cascade; c is a constant giving the energy scale, $n_{\rm eff}$ is an effective number of onia in one nucleon (estimated in [4], *c.f.* also [7]).

The origin of the phase factor ${\rm e}^{i(z_1-z_N)mx_P} = {\rm e}^{i(z_1-z_N)\frac{Q^2+M^2}{2\nu}}$ where z_1 and z_N are the longitunal positions of the first and the last scattering (m is nucleon mass and ν is the energy of the photon) was discussed in detail already by Gottfried and Yennie [14][2].

The starting point in the derivation of Eq. (3) is the generalization of the Mueller–Patel formula [2] for triple-pomeron coupling. In the case of N nucleons it reads

$$F_N^{\rm dir}(r,r_1,\dots r_N;Y,y)$$
$$= {\rm e}^{i(z_1-z_N)mx_P}\int\left(\prod_{k=1}^{N}\frac{dx_k}{x_k}\frac{dx'_k}{x'_k}\tau(x_k,x'_k)n_1(r_k,x_k,y^*)\right)$$
$$\times n_N(r,x'_1,\dots,x'_N;Y-y^*,y-y^*). \quad (6)$$

[2] When only one mass is excited, as it is in the present case, the phase factor depends only on the first and last points of the interaction.

Here

$$n_1(r_k, x_k, y^*) = \int\limits_{c-i\infty}^{c+i\infty} \frac{d\lambda}{2\pi i} \mathrm{e}^{\Delta(\lambda)y^*} \left(\frac{r_k}{x_k}\right)^{\lambda} \tag{7}$$

are the densities of the dipoles of the transverse size x_k *integrated over the trasverse positions of the dipoles* within an onium of size r_k moving with rapidity y^*. $n_N(r, x_1, \ldots, x_n; Y-y^*, y-y^*)$ is the density of N dipoles of the transverse sizes $x_1, \ldots x_N$ again *integrated over the transverse positions of the dipoles* within the cascade of the transverse size r evolved from the incident onium. $\tau(x, x')$ is the forward scattering amplitude of two dipoles of transverse sizes x and x' (in the two-gluon exchange approximation it is energy independent [2]). The arguments $r_1, \ldots, r_N$ are the transverse sizes of the onia representing the target nucleons. Y, y are defined above in Eq. (5). In the terminology employing the pomeron concept, y is the rapidity which divides the process into the region of one pomeron (rapidities between Y and y) and N pomerons (rapidities smaller than y) in the $N+1$ pomeron coupling.

To obtain a formula for $n_N(r, x_1, \ldots, x_n; Y-y^*, y-y^*)$ one has to solve an integro-diferential equation which one obtains by N-fold differentiation of the generating functional given in [3]. We do this by the method described in [10] for double-dipole scattering. An extension of this procedure to arbitrary N is described in Appendix A. At this point it should be emphasized that a solution in compact form, necessary to derive the result of Eq. (3), can be obtained only for distributions integrated over the transverse position of the dipoles with respect to the original onium. The solution for arbitrary dipole position is much more involved. Fortunately, as already mentioned in the Introduction, for large nuclei the transverse size of the incident onium is much smaller than the nuclear diameter and thus we only need the distributions integrated over the transverse positions inside the onium.

In order to obtain the contribution to the shadow (Eq. (2)) one has to add the contributions from $N = 2, 3, \ldots A$ nucleons and average them over the positions of the nucleon inside the nucleus and over the virtual photon wave functions. Finally we have to integrate over the excited mass M (*i.e.* the rapidity y). Using (2) and

$$\sigma_{\mathrm{tot}} = 2\mathrm{Re}F \tag{8}$$

one obtains

$$C^{\mathrm{dir}}(A, x_{\mathrm{Bj}}, Q^2) = \frac{-2}{A\sigma_{\mathrm{tot}}(n)} \times \mathrm{Re}\left(\int\limits_0^Y dy < \sum_{N=2}^{A} (-1)^N \frac{A!}{(A-N)!} n_{\mathrm{eff}}^N \langle\langle \psi_Q \mid F_N^{\mathrm{dir}} \mid \psi_Q \rangle\rangle \right). \tag{9}$$

The average $\langle\psi_Q \mid F_N^{\rm dir} \mid \psi_Q\rangle$ is the integral of $F_N^{\rm dir}$ multiplied by light-cone photon densities [13] given by

$$\begin{aligned}
\mid \Psi_Q(r,\eta) \mid^2 &= \frac{N_c\alpha_{\rm em}e_{\rm f}^2}{\pi^2}W(r,\eta,Q), &(10)\\
W^{\rm T}(r,\eta,Q) &= \tfrac{1}{2}[\eta^2+(1-\eta)^2]\hat{Q}^2K_1^2(\hat{Q}r), &(11)\\
W^{\rm L}(r,\eta,Q) &= 2\eta(1-\eta)\hat{Q}^2K_0^2(\hat{Q}r), &(12)
\end{aligned}$$

where $\hat{Q}=[\eta(1-\eta)]^{\frac{1}{2}}Q, \alpha_{\rm em}=\frac{1}{137}, e_{\rm f}^2$ is the sum of the squares of the quark charges. η is the light-cone momentum fraction of one of the quarks in the photon.

The relevant integrals $\int d^2rd\eta\ldots$ can be found in [16] and they result in the following prescription: r^γ in (3) should be replaced by the expressions

$$r^\gamma \to \frac{N_c\alpha_{\rm em}e_{\rm f}^2}{\pi}\frac{\Gamma^2(2-\frac{\gamma}{2})\Gamma^4(1+\frac{\gamma}{2})}{\Gamma(4-\gamma)\Gamma(2+\gamma)}\left(\frac{Q}{2}\right)^{-\gamma}I^{\rm T,L}(\gamma), \tag{13}$$

where

$$I^{\rm T}(\gamma)=\frac{(2-\frac{\gamma}{2})(1+\frac{\gamma}{2})}{(\frac{\gamma}{2})(1-\frac{\gamma}{2})}, \tag{14}$$

$$I^{\rm L}(\gamma)=2\ . \tag{15}$$

Averaging over the nucleon positions reduces to multiplying $\langle\psi_Q \mid F_N^{\rm dir} \mid \psi_Q\rangle$ by N single particle nucleon densities $\rho(b,z_k)$ and integrating over z_k's kept ordered:

$$\begin{aligned}
&\langle\langle\psi_Q \mid F_N^{\rm dir} \mid \psi_Q\rangle\rangle = \langle\psi_Q \mid F_N^{\rm dir} \mid \psi_Q\rangle\\
&\times\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{z_1}^{+\infty}dz_N{\rm e}^{i(z_1-z_N)mx_P}\rho(b,z_1)\rho(b,z_N)\\
&\times\int_{z_1}^{z_N}dz_2\rho(b,z_2)\ldots\int_{z_{N-2}}^{z_N}dz_{N-1}\rho(b,z_{N-1})\\
&=\langle\psi_Q \mid F_N^{\rm dir} \mid \psi_Q\rangle\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{z_1}^{+\infty}dz_N{\rm e}^{i(z_1-z_N)mx_P}\rho(b,z_1)\rho(b,z_N)\\
&\times\frac{1}{(N-2)!}\left(\int_{z_1}^{z_N}dz\rho(b,z)\right)^{N-2}\ . \qquad (16)
\end{aligned}$$

Inserting (16) into (9) we obtain the required formula for the shadow.

For the sake of completeness let us also give the explicit formula for $\sigma_{\rm tot}(n)$:

$$\sigma_{\rm tot}(n) = \sigma(r_0, Y) = 2\pi\alpha^2 r_0^2 n_{\rm eff} \int \frac{d\gamma}{2\pi i} \left(\frac{r}{r_0}\right)^{\gamma} {\rm e}^{\Delta(\gamma) y} h(\gamma)\,, \tag{17}$$

where r^γ is given by Eq. (13).

3. Forward γ^*-nucleus amplitude from the quasi-elastic scattering of a QCD dipole in the nucleus

As is well-known since the seminal paper of Stodolsky [15] and of Gottfried and Yennie [14], the multiple elastic scattering of a photon fluctuation inside a nucleus contributes in an essential way to the nuclear shadowing. In the QCD dipole picture this corresponds to quasi-elastic scattering of an onium whose transverse size is distributed according to the light-cone wave functions of the photon given in [13]. In the present section we derive the formula for this contribution to the shadow.

The N-fold scattering contribution to the forward γ^*-nucleus amplitude is built as a product of N onium-nucleon amplitudes (including the eikonal phases) sandwiched between the initial and final state of the virtual photon and summed over all intermediate states of the onium as follows

$$\begin{aligned}
&\langle \psi_Q \mid F_N^{\rm qel} \mid \psi_Q\rangle = n_{\rm eff}^N \langle Q \mid T(z_N) \mid k_{N-1}\rangle \\
&\times \frac{d^2 k_{N-1}}{4\pi^2} \langle k_{N-1} \mid T(z_{N-1}) \mid k_{N-2} > \frac{d^2 k_{N-2}}{4\pi^2} \\
&\times \langle k_{N-2} \mid T(z_{N-2} \mid k_{N-3}\rangle \ldots \frac{d^2 k_3}{4\pi^2} \langle k_3 \mid T(z_3) \mid k_2\rangle \frac{d^2 k_2}{4\pi^2} \\
&\times \langle k_2 \mid T(z_2) \mid k_1\rangle \frac{d^2 k_1}{4\pi^2} \langle k_1 \mid T(z_1) \mid Q\rangle.
\end{aligned} \tag{18}$$

Here $k_1, \ldots k_{N-1}$ are the relative transverse momenta of the quark and antiquark forming the onium. To shorten the expression, we skip the integral symbols (and we shall continue to so doing).

At this point it is important to realize that the amplitudes entering (18) depend on the longitudinal position, acquiring the eikonal phase. So we can write

$$\langle k' \mid T(z) \mid k\rangle = {\rm e}^{izk'_{\rm L}} \langle k' \mid T \mid k\rangle {\rm e}^{-izk_{\rm L}}, \tag{19}$$

where $k_{\rm L}$ and $k'_{\rm L}$ are the longitudinal momenta of the onium before and after scattering.

The first step is to transform the transverse momentum amplitudes $\langle k' \mid T \mid k\rangle$ into transverse position amplitudes, so that we can use the explicit expression for the onium-onium forward elastic amplitude, $T(r)$, derived in [2, 3]. We shall use here the path integral representation, given by:

$$T(r,r_0,Y) = \pi\alpha^2 r r_0 \int\limits_{c-i\infty}^{c+i\infty} \frac{d\lambda}{2\pi i}\left(\frac{r}{r_0}\right)^{\lambda-1} \mathrm{e}^{\Delta(\lambda)Y} h(\lambda), \tag{20}$$

where r and r_0 are the transverse sizes of the colliding onia. In terms of $T(r,r_0,Y) \equiv T(r)$ we thus obtain

$$\langle k' \mid T(z) \mid k\rangle = \mathrm{e}^{iz(k^2-k'^2)\xi} d^2\rho \mathrm{e}^{i(k'-k)\rho} T(\rho) \tag{21}$$

with

$$\xi = \frac{1}{2\nu\eta(1-\eta)}. \tag{22}$$

In arriving at (22) we have used the high energy approximation for k_{L}, *viz.*

$$k_{\mathrm{L}} = \nu - \frac{M^2}{2\nu} = \nu - \frac{k^2}{2\nu\eta(1-\eta)}. \tag{23}$$

Similarly we obtain

$$\langle k \mid T(z) \mid Q\rangle = \mathrm{e}^{iz(x_{\mathrm{Bj}}m - \xi k^2)} d^2\rho \mathrm{e}^{ik\rho}\langle \rho \mid T \mid Q\rangle, \tag{24}$$

where $\langle \rho \mid T \mid Q\rangle$ includes the photon wave function:

$$\langle \rho \mid T \mid Q\rangle = T(\rho)\Psi_Q(\rho). \tag{25}$$

Putting this in (18) and rescaling the variables

$$k = Q\kappa, \quad \rho = \frac{r}{Q}, \quad \zeta = \frac{mz}{\eta(1-\eta)} \tag{26}$$

we obtain

$$\begin{aligned}
&\langle \psi_Q \mid F_N^{\mathrm{qel}} \mid \psi_Q\rangle = \tfrac{1}{Q^2}\mathrm{e}^{i(z_1-z_N)mx_{\mathrm{Bj}})} n_{\mathrm{eff}}^N \langle Q \mid T \mid r_N\rangle d^2 r_N \\
&\times \Phi[r_{N-1} - r_N, x_{\mathrm{Bj}}(\zeta_N - \zeta_{N-1})] T(r_{N-1}) d^2 r_{N-1} \\
&\times \Phi[r_{N-2} - r_{N-1}, x_{\mathrm{Bj}}(\zeta_{N-1} - \zeta_{N-2})] T(r_{N-2}) \dots \Phi[r_2 - r_3, x_{\mathrm{Bj}}(\zeta_3 - \zeta_2)] \\
&\times T(r_2) d^2 r_2 \Phi[r_1 - r_2, x_{\mathrm{Bj}}(\zeta_2 - \zeta_1)] T(r_1) d^2 r_1 \langle r_1 \mid T \mid Q\rangle,
\end{aligned} \tag{27}$$

with

$$\Phi(\Delta, a) \equiv \int \frac{d^2\kappa}{(2\pi)^2} \mathrm{e}^{i\kappa\Delta} \mathrm{e}^{ia\kappa^2} = \frac{i}{4\pi a} \mathrm{e}^{-i\frac{\Delta^2}{4a}}, \tag{28}$$

where we have assumed that $a = \mathrm{Re}(a) + i\varepsilon$ to give a definite meaning to the integral.

Further calculations are continued in the limit $x_{\mathrm{Bj}} \ll 1$. In this limit the following formula can be derived by the saddle point method for $a, a' \ll 1$

$$\int d^2r \Phi(s-r,a) F(r) \Phi(r-s',a') = \Phi(s-s', a+a') F(\bar{s}) \exp\left[i\bar{a} \left(\frac{\nabla F(\bar{s})}{F(\bar{s})} \right)^2 \right], \tag{29}$$

where

$$\bar{s} = \frac{a's + as'}{a+a'}, \tag{30}$$

and

$$\bar{a} = \frac{aa'}{a+a'}. \tag{31}$$

By repeated application of this formula we finally arrive at the following result

$$\langle \psi_Q \mid F_N^{\mathrm{qel}} \mid \psi_Q \rangle = \int_0^1 d\eta \int d^2\rho \mid \Psi_Q(\rho,\eta) \mid^2 \left[n_{\mathrm{eff}} T(\rho, r_0, x_{\mathrm{Bj}}) \right]^N \times \mathrm{e}^{-\frac{i}{2\nu} \sum_{j=1}^{N} z_j \left(M_j^2 - M_{j+1}^2 \right)}, \tag{32}$$

where T is given by (20) and

$$M_1^2 = M_{N+1}^2 = -Q^2, \quad M_j^2 = \left(\frac{N-j+1}{(\eta(1-\eta))^{1/2}} \frac{\vec{\rho}}{\rho^2} + \vec{\nabla} \log[\Psi(\rho)] \right)^2, \ j = 2, \ldots, N. \tag{33}$$

Some details of these calculations are given in the Appendix B.

The shadow for the quasi-elastic process is now

$$C^{\mathrm{qel}}(A, x_{\mathrm{Bj}}, Q^2) = \frac{-2}{A\sigma_t(n)} \mathrm{Re} \left(\sum_{N=2}^{A} (-1)^N \frac{A!}{(A-N)!} \langle\langle \psi_Q \mid F_N^{\mathrm{qel}} \mid \psi_Q \rangle\rangle \right) \tag{34}$$

with the following averaging over the nuclear densities

$$\langle\langle \psi_Q \mid F_N^{\mathrm{qel}} \mid \psi_Q \rangle\rangle = \int_0^1 \int d\eta d^2\rho \mid \Psi_Q(\rho,\eta) \mid^2 \left[n_{\mathrm{eff}} T(\rho, r_0, x_{\mathrm{Bj}}) \right]^N \times \int d^2b \int_{z_N \geq z_{N-1} \geq \cdots \geq z_1} dz_N dz_{N-1} \ldots . dz_1 \rho(b, z_N) \mathrm{e}^{-\frac{i}{2\nu} z_N \left(M_N^2 - M_{N+1}^2 \right)}$$

$$\times \rho(b, z_{N-1}) \mathrm{e}^{-\frac{i}{2\nu} z_{N-1}(M_{N-1}^2 - M_N^2)} \ldots$$
$$\times \ldots \rho(b, z_2) \mathrm{e}^{-\frac{i}{2\nu} z_2(M_2^2 - M_3^2)} \rho(b, z_1) \mathrm{e}^{-\frac{i}{2\nu} z_1(M_1^2 - M_2^2)} . \qquad (35)$$

4. Conclusions and outlook

We have shown that the QCD dipole picture can provide a well defined and compact formulae for nuclear shadowing of the virtual photons. The formulae are well suited for numerical evaluation. They contain the following parameters:

(i) The pomeron intercept Δ_P;

(ii) The nucleon size parameter r_0;

(iii) The effective number of dipoles in the proton n_{eff};

(iv) The effective number of flavours (reflecting on e_{f}^2);

(v) The scales in the elastic and in the diffractive γ^*-proton scattering.

All these parameters can, in principle, be determined from the fit of the dipole picture to the proton data. Such a fit has been completed for the total γ^*-proton cross-section [4]. That gave $\Delta_P = .285$, $Q_0 \equiv \frac{2}{r_0} = .622$ GeV; $n_{\mathrm{eff}} e_{\mathrm{f}}^2 = 3.8$; $Y = \log \frac{1.65}{x_{\mathrm{Bj}}}$. So, as long as the fits to the diffractive production are not available, we are left with two not fully determined parameters: the effective number of flavours and the scale in the diffractive dissociation.

It is important to remember that the nuclear shadowing effects are determined by the *forward* diffractive amplitudes. Our formulae can thus be employed to obtain the cross section for the forward diffractive dissociation of the virtual photon on one nucleon. Indeed, such cross sections on *one nucleon* can be obtained from the formulae of Sections 2 and 3 for $F_{N=2}^{\mathrm{dir}}$ and $F_{N=2}^{\mathrm{qel}}$:

$$\frac{d\sigma^{\mathrm{dir}}}{dy d^2 p_t}|_{p_t=0} = \frac{1}{(2\pi)^2} \langle \Psi_Q | F_{N=2}^{\mathrm{dir}}(z_1 = z_2) | \Psi_Q \rangle \,, \qquad (36)$$

and

$$\frac{d\sigma^{\mathrm{qel}}}{d^2 p_t}|_{p_t=0} = \frac{1}{(2\pi)^2} \langle \Psi_Q | F_{N=2}^{\mathrm{qel}}(z_1 = z_2) | \Psi_Q \rangle \,, \qquad (37)$$

where p_t is the transverse momentum of the final proton, and y defined in Eq. (5) contains M, the diffractively produced mass. Therefore (36) is, in fact, a differential cross section for production from the incident virtual photon of an object of mass M. On the other hand (37) is the total diffractive dissociation cross section *integrated over diffractively excited masses.* One can resolve (37) into the differential cross sections $\frac{d\sigma^{\mathrm{qel}}}{dM^2 d^2_{p_t}}\,|_{p_t=0}$ applying a straightforward procedure. This will be discussed elsewhere [19].

Our formulae, given in Sections 2 and 3 refer to two mechanisms described in the Introduction: (a) the quasi-elastic interaction of the $q\bar{q}$ dipole and (b) the multiple-pomeron coupling. One must remember, however, that these two mechanisms are not mutually exclusive: they mix in the multiple scattering terms, *i.e.* there are collisions in which they both take place. Such mixed amplitudes are also readily calculable by the methods developped in the present paper.

Any further discussion strongly depends on the outcome of the numerical estimates. Before this is available let us simply list the problems which, in our opinion, seem interesting.

(a) The relative importance of the multi-pomeron and the quasi-elastic interactions. Apart from its primary interest, it is essential for an estimation of the importance of the mixing terms.

(b) An extrapolation for very small x_{Bj} where the unitarity is expected to break down. It is likely that for nuclear targets this effect will occur much earlier (*i.e.* for larger x_{Bj}) than estimated in [3, 17].

(c) The dependence of the kind $x_{\mathrm{Bj}}^{-\Delta_P}$ which appear in BFKL forward amplitudes imply the existence of a large ratio of the real to imaginary part of the forward amplitudes. We have shown already [7,18] that this influences significantly the predicted amount of shadowing. It will thus be interesting to discuss this problem again with the present, more precise, formulation.

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Appendix A

In this Appendix we indicate some details of the derivation of the formula for n_N, Eq. (6).

We start with n_2.

Using the generating function given in [3] and assuming that evolution of the two exchanged pomerons goes until the rapidity y is reached, we obtain the following equation for n_2 (*c.f.* also Eq. (52) of [2]):

$$\begin{aligned}
&\frac{dn_2(x_{01},Y,y,x,x')}{dY} \\
&= \frac{\alpha N_c}{\pi^2}\int\limits_R \frac{x_{01}^2 d^2x_2}{x_{02}^2 x_{12}^2} n_1(x_{12},y,x)n_1(x_{02},y,x')\delta(Y-y) \\
&+\frac{2\alpha N_c}{\pi}\int dx_{12}K(x_{01},x_{12})n_2(x_{12},Y,y,x,x'), \qquad (38)
\end{aligned}$$

where n_1 is the single dipole density and

$$K(x_{01},x_{12}) = \frac{1}{2\pi}\int\limits_R \frac{x_{01}^2 d^2x_2}{x_{02}^2 x_{12}^2} - \delta(x_{01}-x_{12})\log(x_{01}/\rho) \qquad (39)$$

is the Lipatov kernel (*c.f.* [1]) which satisfies the eigenvalue equation:

$$\int K(x_{01},x_{12})x_{12}^{\lambda}dx_{12} = x_{01}^{\lambda}\chi(\lambda) \qquad (40)$$

with $\chi(\lambda)$ given by (4). We now introduce Mellin transforms to take advantage of (40):

$$\begin{aligned}
n_2(x_{01},Y,y,x,x') &= \int\limits_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i}\tilde{n}_2(\gamma,Y,y,x,x')x_{01}^{\gamma} \qquad (41)\\
\tilde{n}_2(\gamma,Y,y,x,x') &= \int\limits_0^{\infty} dx_{01}x_{01}^{-1-\gamma}n_2(x_{01},Y,y,x,x'). \qquad (42)
\end{aligned}$$

Substituting (41) into (38) and using (40) we obtain a differential equation for $\tilde{n}_2$ which can be easily solved giving

$$\tilde{n}_2(\gamma,Y,y,x,x') = \mathrm{e}^{\Delta(\gamma)(Y-y)}g_2(\gamma,y,x,x'), \qquad (43)$$

where $\Delta(\gamma)$ is given by (4) and

$$g_2(\gamma,y,x,x') = \frac{\alpha N_c}{\pi^2}\int\limits_0^{\infty} dx_{01}x_{01}^{-1-\gamma}\int\limits_R \frac{x_{01}^2 d^2x_2}{x_{02}^2 x_{12}^2} n_1(x_{12},y,x)n_1(x_{02},y,x'). \qquad (44)$$

Substituting (44) and (43) into (41) we finally have

$$\begin{aligned}
n_2(r,Y,y,x,x') &= \frac{\alpha N_c}{\pi^2}\int\limits_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} r^{\gamma}\mathrm{e}^{\Delta(\gamma)(Y-y)} \\
&\times \int\limits_0^{\infty} dx_{01}x_{01}^{-1-\gamma}\int\limits_R \frac{x_{01}^2 d^2x_2}{x_{02}^2 x_{12}^2} n_1(x_{12},y,x)n_1(x_{02},y,x'). \qquad (45)
\end{aligned}$$

We now turn to n_3.

Using the generating function given in [3] and assuming that evolution of the three exchanged pomerons goes until the rapidity y is reached, we obtain the following equation for n_3:

$$\begin{aligned}
&\frac{dn_3(x_{01},Y,y,x,x',x'')}{dY}\\
&=\frac{\alpha N_c}{\pi^2}\int\limits_R \frac{x_{01}^2 d^2x_2}{x_{02}^2x_{12}^2}[n_1(x_{12},y,x)n_2(x_{02},Y,y,x',x'')]_{\rm sym}\delta(Y-y)\\
&+\frac{2\alpha N_c}{\pi}\int dx_{12}K(x_{01},x_{12})n_3(x_{12},Y,y,x,x',x''),
\end{aligned} \tag{46}$$

where $n_2(x_{02},Y,y,x',x'')$ is given by (45). The symbol $_{\rm sym}$ denotes a sum of three terms, symmetrized with respect to x,x',x''.

Introducing Mellin transforms, as in (41), (42), substituting them into (38) and using (40) we obtain a differential equation for $\tilde{n}_3$. The solution is found again in the form

$$\tilde{n}_3(\gamma,Y,y,x,x',x'')={\rm e}^{\Delta(\gamma)(Y-y)}g_3(\gamma,y,x,x',x''), \tag{47}$$

where

$$\begin{aligned}
g_3(\gamma,y,x,x',x'')&=\frac{\alpha N_c}{\pi^2}\int\limits_0^\infty dx_{01}x_{01}^{-1-\gamma}\\
&\times\int\limits_R \frac{x_{01}^2 d^2x_2}{x_{02}^2x_{12}^2}[n_1(x_{12},y,x)n_2(x_{02},y,y,x',x'')]_{\rm sym},
\end{aligned} \tag{48}$$

and $n_2(x_{02},y,y,x',x'')$ is given by (45).

The formulae for $n_4,n_5,\ldots$ are derived in a similar way.

The integrals over $x_1,\ldots x_N;x'_1,\ldots,x'_N$ in (6) are performed using the formula for the forward onium-onium amplitude $T(r,r',y)$

$$\begin{aligned}
T(r,r',y)&=\int\frac{dx}{x}\frac{dx'}{x'}\tau(x,x')n_1(r,y*,x)n_1(r',y-y*,x')\\
&=\pi\alpha^2rr'\int\limits_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}{\rm e}^{\Delta(\gamma)y}\left(\frac{r}{r'}\right)^{1-\gamma}h(\gamma)
\end{aligned} \tag{49}$$

which was first derived in [2] and [4, 5] in a sligthly more simplified form.

Using (49) and (45), (6) gives

$$F_2^{\rm dir}(r,r_1,r_2;Y,y)={\rm e}^{i\,(z_1-z_2\,)mx_p}\frac{\alpha N_c}{\pi^2}r_1r_2\int\limits_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}r^\gamma{\rm e}^{\Delta(\gamma)(Y-y)}$$

$$\times\int\limits_{c-i\infty}^{c+i\infty}\frac{d\lambda_1}{2\pi i}r_1^{\lambda_1-1}{\rm e}^{\Delta(\lambda_1)y}h(\lambda_1)$$

$$\times\int\limits_{c-i\infty}^{c+i\infty}\frac{d\lambda_2}{2\pi i}r_2^{\lambda_2-1}{\rm e}^{\Delta(\lambda_2)y}h(\lambda_2)\Omega(\gamma,\lambda_1,\lambda_2)\,, \tag{50}$$

where

$$\Omega(\gamma,\lambda_1,\lambda_2)=\int\limits_0^\infty dx_{01}x_{01}^{-1-\gamma}\int\limits_R\frac{x_{01}^2d^2x_2}{x_{02}^2x_{12}^2}x_{02}^{2-\lambda_1}x_{12}^{2-\lambda_2}\,, \tag{51}$$

$\Omega(\gamma,\lambda_1,\lambda_2)$ can be calculated using [1]

$$\frac{x_{01}^2d^2x_2}{x_{02}^2x_{12}^2}=2\pi x_{01}^2\frac{dx_{02}}{x_{02}}\frac{dx_{12}}{x_{12}}\int\limits_0^\infty kdkJ_0(kx_{01})J_0(kx_{02})J_0(kx_{12}), \tag{52}$$

from which the following useful identity can be obtained:

$$\int\limits_0^\infty dx_{01}x_{01}^{-1-\gamma}\int\limits_R\frac{x_{01}^2d^2x_2}{x_{02}^2x_{12}^2}x_{02}^{\lambda}x_{12}^{\lambda'}=\pi\frac{G(\lambda)G(\lambda')}{G(\gamma)}2\pi i\delta(\gamma-\lambda-\lambda'). \tag{53}$$

Using (53) in (50) and setting $r_1=r_2=r_0$ we obtain the formula (3) for $F_2^{\rm dir}$. With the same technique one can obtain (3) for more than two collisions. We indicate below the derivation for three collisions.

Substituting (45) into (48) and using (47) we obtain

$$n_3(x_{01},Y,y,x,x',x'')=\left(\frac{\alpha N_c}{\pi^2}\right)^2\int\limits_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}x_{01}^\gamma{\rm e}^{\Delta(\gamma)(Y-y)}\int\limits_0^\infty dx_{01}x_{01}^{-1-\gamma}$$

$$\times\int\limits_R\frac{x_{01}^2d^2x_2}{x_{02}^2x_{12}^2}[n_1(x_{12},y,x)\int\limits_{c-i\infty}^{c+i\infty}\frac{d\gamma'}{2\pi i}x_{02}^{\gamma'}$$

$$\times\int\limits_0^\infty dx'_{01}(x'_{01})^{-1-\gamma'}\int\limits_R\frac{(x'_{01})^2d^2x'_2}{(x'_{02})^2(x'_{12})^2}n_1(x'_{12},y,x')n_1(x'_{02},y,x'')]_{\rm sym}. \tag{54}$$

When (54) is substituted into (6) with $r_1 = r_2 = r_3 = r_0$ we obtain

$$
\begin{aligned}
& F_3^{\mathrm{dir}}(r, r_0; Y, y) = \mathrm{e}^{i(z_1 - z_N)mx_P} \\
& \times 3\left(\frac{\alpha N_c}{\pi^2}\right)^2 \int\limits_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} x_{01}^{\gamma} \mathrm{e}^{\Delta(\gamma)(Y-y)} \int\limits_0^{\infty} dx_{01} x_{01}^{-1-\gamma} \\
& \times \int\limits_R \frac{x_{01}^2 d^2 x_2}{x_{02}^2 x_{12}^2} T(x_{12}, r_0, y) \int\limits_{c-i\infty}^{c+i\infty} \frac{d\gamma'}{2\pi i} x_{02}^{\gamma'} \\
& \times \int\limits_0^{\infty} dx'_{01} (x'_{01})^{-1-\gamma'} \int\limits_R \frac{(x'_{01})^2 d^2 x'_2}{(x'_{02})^2 (x'_{12})^2} T(x'_{12}, r_0, y) T(x'_{02}, r_0, y)\,, \qquad (55)
\end{aligned}
$$

where $T(r, r', y)$ is the dipole–dipole forward scattering amplitude given by (49).

The integrals over dx_{01},d^2x_2 and dx'_{01} $d^2x'_2$ can be performed using again the identity (53) and taking into account the fact that the dependence of $T(r, r', y)$ on r and r' is in the form of a power law. The result of these operations is the Eq. (3) for $N = 3$.

Appendix B

In this Appendix we give a few intermediate steps of the calculations of Section 3 which lead to Eq. (32).

First, by repeated application of (29) we obtain

$$
\begin{aligned}
& \langle \psi_Q \mid F_N^{\mathrm{qel}} \mid \psi_Q \rangle = n_{\mathrm{eff}}^N Q^{-2} \mathrm{e}^{i(z_1 - z_N)mx_{\mathrm{Bj}})} \langle Q \mid T \mid r_N \rangle d^2 r_N \\
& \times \Phi(r_1 - r_N, x_{\mathrm{Bj}}(\zeta_N - \zeta_{N-1})) S(r_N, r_1; \zeta_N, \ldots, \zeta_1) d^2 r_1 \langle r_1 \mid T \mid Q \rangle, \qquad (56)
\end{aligned}
$$

where

$$
\begin{aligned}
S(r_N, r_1; \zeta_N, \ldots, \zeta_1) = & \prod_{j=2}^{N-1} \frac{d\lambda_j}{2\pi i} \Lambda(\lambda_j, r_0, Y) r_j^{\lambda_j} \\
& \times \exp\left(i \frac{a_j \lambda_j}{r_j^2} + i2 \sum_{j,k=2}^{N-1} \lambda_j \lambda_k \frac{\vec{r}_j \vec{r}_k}{r_j^2 r_k^2} a_{jk} \right) \qquad (57)
\end{aligned}
$$

with

$$
\vec{r}_j \;=\; \frac{(z_N - z_j)\vec{r}_1 + (z_j - z_1)\vec{r}_N}{z_n - z_1}, \qquad (58)
$$

$$a_j = \frac{(z_N - z_j)(z_j - z_1)}{z_n - z_1}, \tag{59}$$

$$a_{jk} = \frac{(z_N - z_j)(z_k - z_1)}{z_n - z_1}. \tag{60}$$

$$\Lambda(\lambda, r_0, Y) = n_{\text{eff}} \pi \alpha^2 r_0^{2-\lambda} \mathrm{e}^{\Delta(\lambda)Y} h(\lambda) \tag{61}$$

is taken from the integrand of Eq. (20) and n_{eff} added for the reasons explained in the main text.

We finally perform the integral over $d^2 r_N$, again in the saddle point approximation which is valid for small x_{Bj}. Then $\Phi(r_1 - r_N, x_{\text{Bj}}(\zeta_N - \zeta_1))$ is very strongly peaked around $r_n \approx r_1$, so that in leading order all r_j's become r_1. This approximation puts a new kind of factor, $\nabla\psi_Q/\psi_Q$, into the over all phase of the expression and we obtain (returning to z_j's and ρ_j's)

$$\langle \psi_Q \mid F_N^{\text{qel}} \mid \psi_Q \rangle = n_{\text{eff}}^N \mathrm{e}^{ix_{\text{Bj}}(z_1 - z_N)} \int_0^1 d\eta \int d^2\rho \mid \psi_Q(\rho, \eta) \mid^2$$
$$\times \exp\left[\frac{i}{2\nu}\left[\frac{1}{\eta(1-\eta)\rho^2} \sum_{j=2}^N [2(N-j)+1](z_j - z_1)\right.\right.$$
$$\left.\left. + 2\sum_{j=2}^N (z_j - z_1) \frac{\vec{\rho} \cdot \vec{\nabla}\psi_Q(\rho)}{[(\eta(1-\eta)]^{1/2} \rho^2 \psi_Q(\rho)} + (z_n - z_1) \frac{[\vec{\nabla}\psi_Q(\rho)]^2}{\psi_Q(\rho)^2}\right]\right]$$
$$\times \prod_{j=1}^N \int \frac{d\lambda_j}{2\pi i} \Lambda(\lambda_j, r_0, Y) \rho^{\lambda_j}\,. \tag{62}$$

One can check by a direct substitution of M_j^2 given by (33) into (32) that it reduces to (62). The version of $\langle \psi_Q \mid F_N^{\text{qel}} \mid \psi_Q \rangle$ given in (33) has, however, a clearer physical interpretation.

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