

PROPOSAL OF UNIFIED FERMION TEXTURE*

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A unified form of mass matrix is proposed for neutrinos, charged leptons, up quarks and down quarks. Some constraints for the parameters involved are tentatively postulated. Then, the predictions are neatly consistent with available experimental data. Among the predictions are: (i) $m_\tau \simeq 1776.80$ MeV (with the inputs of m_e and m_μ), (ii) $m_{\nu_0} \ll m_{\nu_1} \sim (0.6 \text{ to } 4) \times 10^{-2}$ eV and $m_{\nu_2} \sim (0.2 \text{ to } 1) \times 10^{-1}$ eV (with the atmospheric-neutrino inputs of $|m_{\nu_2}^2 - m_{\nu_1}^2| \sim (0.0003 \text{ to } 0.01) \text{ eV}^2$ and the $\nu_\mu \rightarrow \nu_\tau$ oscillation amplitude ~ 0.8), and also (iii) $m_s \simeq 270$ MeV, $|V_{ub}/V_{cb}| \simeq 0.082$ and $\arg V_{ub} \simeq -64^\circ$ (with the inputs of $m_c = 1.3$ GeV, $m_b = 4.5$ GeV, $|V_{us}| = 0.221$ and $|V_{cb}| = 0.041$, where $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$). All elements of the Cabibbo–Kobayashi–Maskawa matrix are evaluated. All elements of its lepton counterpart are calculated up to an unknown phase (Appendix B). Some items related to dynamical aspects of the proposed fermion “texture” are briefly commented on (Appendix A). In particular, the notion of a novel dark matter, free of any Standard Model interactions (and their supersymmetric variants), appears in the case of preon option.

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1. Introduction

For the last few years we studied the “texture” of fermion mass matrices, starting from charged leptons e^- , μ^- , τ^- (the first and second Ref. [1]), then considering up and down quarks u , c , t and d , s , b (the third Ref. [1]), and finally extending the argument to neutrinos (the fourth Ref. [1]). In consequence, we came to a proposal of common structure of four mass matrices $\widehat{M}^{(\nu)}$, $\widehat{M}^{(e)}$, $\widehat{M}^{(u)}$ and $\widehat{M}^{(d)}$ in the three-dimensional family space of neutrinos (ν), charged leptons (e), up quarks (u) and down quarks (d), respectively.

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Explicitly, we proposed that

$$\widehat{M}^{(f)} = \frac{1}{29} \times \begin{pmatrix} \mu^{(f)} \varepsilon^{(f)2} & 2\alpha^{(f)} e^{i\varphi^{(f)}} & 0 \\ 2\alpha^{(f)} e^{-i\varphi^{(f)}} & \frac{4\mu^{(f)}(80+\varepsilon^{(f)2})}{9} & 8\sqrt{3}(\alpha^{(f)} - \beta^{(f)}) e^{i\varphi^{(f)}} \\ 0 & 8\sqrt{3}(\alpha^{(f)} - \beta^{(f)}) e^{-i\varphi^{(f)}} & \frac{24\mu^{(f)}(624+25C^{(f)}+\varepsilon^{(f)2})}{25} \end{pmatrix}, (1)$$

where $f = \nu, e, u, d$, while $\mu^{(f)}, \varepsilon^{(f)2}, \alpha^{(f)}, \beta^{(f)}, C^{(f)}$ and $\varphi^{(f)}$ denoted real constants to be determined from the experimental data for fermion masses and mixing parameters. The proposed form (1) followed from: (i) an idea about the origin of three fermion families, and (ii) an ansatz for the fermion mass matrix expressed in terms of the suggested family characteristics. Note that the mass matrices $\widehat{M}^{(e)}, \widehat{M}^{(u)}, \widehat{M}^{(d)}$ as given in Eq. (1) do not take the popular Georgi–Jarlskog form [2] for any choice of their parameters, if $\mu^{(f)} > 0$.

In the present paper, we do not go systematically into any motivation for the proposal (1), considering it simply as a detailed conjecture. The interested Reader may look for roots of the formula (1) in Refs [1] (note that in Refs [3] there was discussed a mass formula a bit different, especially in the quark case).

Instead, we proceed in the present paper a step further with our conjecture (1), postulating tentatively the following constraints for the parameters $\alpha^{(f)}, \beta^{(f)}$ and $C^{(f)}$ appearing there:

$$\begin{aligned} \alpha^{(\nu)} : \alpha^{(e)} &= |Q^{(\nu)}| : |Q^{(e)}|, \\ \alpha^{(u)} : \alpha^{(d)} &= |Q^{(u)}| : |Q^{(d)}|, \\ \beta^{(\nu)} &= \beta^{(e)}, \\ \beta^{(u)} &= \beta^{(d)}, \\ (\beta^{(\nu)} + \beta^{(e)}) : (\alpha^{(\nu)} + \alpha^{(e)}) &= \delta^{(l)} : (|Q^{(\nu)}| + |Q^{(e)}|), \\ (\beta^{(u)} + \beta^{(d)}) : (\alpha^{(u)} + \alpha^{(d)}) &= \left(B^{(q)} + \delta^{(q)} \frac{1}{N_C^{(q)}} \right) : (|Q^{(u)}| + |Q^{(d)}|), \\ C^{(\nu)} = C^{(e)} = 0, \quad C^{(u)} > 0, \quad C^{(d)} = 0. \end{aligned} \quad (2)$$

Here, $Q^{(\nu)} = 0, Q^{(e)} = -1, Q^{(u)} = 2/3, Q^{(d)} = -1/3, B^{(q)} = 1/3$ and $N_C^{(q)} = 3$, while $0 < \delta^{(l)} \ll 1$ and $0 < \delta^{(q)} \ll 1$ (we may introduce into Eqs (2) also $B^{(l)} = 0$ and $N_C^{(l)} = 1$, making then these relations fully symmetric under

the interchange of leptons and quarks, if $C^{(f)}$ are treated there like some charges). Thus, from Eqs (2)

$$\alpha^{(\nu)} = 0 \quad , \quad \alpha^{(u)} = 2\alpha^{(d)} > 0 \quad ,$$

$$\beta^{(\nu)} = \beta^{(e)} = \delta^{(l)}\alpha^{(e)}/2 \gtrsim 0 \quad , \quad \beta^{(u)} = \beta^{(d)} = (1 + \delta^{(q)})\alpha^{(d)}/2 \simeq \alpha^{(d)}/2 > 0$$

(we choose $\alpha^{(e)} > 0$ and $\alpha^{(d)} > 0$). We also assume that $\mu^{(\nu)} \simeq 0$ and $\varepsilon^{(\nu)2} \simeq 0$.

Then, four mass matrices (1) contain practically 14 independent parameters, say,

$$\mu^{(\nu)} \simeq 0 \quad , \quad \mu^{(e)} \quad , \quad \varepsilon^{(\nu)2} \simeq 0 \quad , \quad \varepsilon^{(e)2} \quad , \quad \beta^{(\nu)} \simeq 0 \quad , \quad \alpha^{(e)} \quad , \quad \varphi^{(\nu)} - \varphi^{(e)}$$

and

$$\mu^{(u)} \quad , \quad \mu^{(d)} \quad , \quad \varepsilon^{(u)2} \quad , \quad \varepsilon^{(d)2} \quad , \quad C^{(u)} \quad , \quad \alpha^{(d)} \quad , \quad \varphi^{(u)} - \varphi^{(d)} \quad ,$$

7 for leptons and 7 for quarks (in addition, they contain $\varphi^{(\nu)} + \varphi^{(e)}$ and $\varphi^{(u)} + \varphi^{(d)}$ that, however, will not appear in experimentally measured quantities). These 14 free parameters will describe 12 fermion masses and their 8 mixing parameters, together 20 quantities, of which 14 may be used as inputs determining consistently all parameters (except for $\varphi^{(\nu)} + \varphi^{(e)}$ and $\varphi^{(u)} + \varphi^{(d)}$ that will remain undetermined, but may be put zero as being physically irrelevant). So, we will be able to get $20 - 14 = 6$ predictions, 3 for leptons and 3 for quarks, and also an overall consistent determination of all parameters (except for the unphysical two). The agreement with available experimental data will turn out to be satisfactory.

In the framework of mass matrices (1), the observed differences between spectral properties of four types of fermions $f = \nu, e, u, d$ will follow (in a large extent) from the interplay of magnitudes of the parameters $\mu^{(f)}$ contained in the diagonal elements of $\widehat{M}^{(f)}$ and the parameters $\alpha^{(f)}$ and $\beta^{(f)}$ appearing in its off-diagonal elements. Their ratios, $\alpha^{(f)}/\mu^{(f)}$ and $\beta^{(f)}/\mu^{(f)}$, will play the role of coupling constants in our “texture dynamics” (cf. Appendix A).

2. Charged leptons

In the case of charged leptons, we will assume that the off-diagonal elements of the mass matrix $\widehat{M}^{(e)} = \left(M_{ij}^{(e)} \right) \quad (i, j = 0, 1, 2)$ given in Eq. (1) can be treated as a small perturbation of the diagonal terms. Then, in the lowest (quadratic) perturbative order we obtain

$$\begin{aligned}
m_e &= \frac{\mu^{(e)}}{29} \left[\varepsilon^{(e)2} - \frac{36}{320 - 5\varepsilon^{(e)2}} \left(\frac{\alpha^{(e)}}{\mu^{(e)}} \right)^2 \right], \\
m_\mu &= \frac{\mu^{(e)}}{29} \left[\frac{4}{9} (80 + \varepsilon^{(e)2}) + \frac{36}{320 - 5\varepsilon^{(e)2}} \left(\frac{\alpha^{(e)}}{\mu^{(e)}} \right)^2 \right. \\
&\quad \left. - \frac{10800}{31696 + 29\varepsilon^{(e)2}} \left(\frac{\alpha^{(e)} - \beta^{(e)}}{\mu^{(e)}} \right)^2 \right], \\
m_\tau &= \frac{\mu^{(e)}}{29} \left[\frac{24}{25} (624 + \varepsilon^{(e)2}) + \frac{10800}{31696 + 29\varepsilon^{(e)2}} \left(\frac{\alpha^{(e)} - \beta^{(e)}}{\mu^{(e)}} \right)^2 \right]. \quad (3)
\end{aligned}$$

These mass formulae give

$$\begin{aligned}
m_\tau &= 1776.80 \text{ MeV} \\
&\quad + \frac{216\mu^{(e)}}{3625} \left[\frac{111550}{31696 + 29\varepsilon^{(e)2}} \left(\frac{\alpha^{(e)} - \beta^{(e)}}{\mu^{(e)}} \right)^2 - \frac{487}{320 - 5\varepsilon^{(e)2}} \left(\frac{\alpha^{(e)}}{\mu^{(e)}} \right)^2 \right], \\
\varepsilon^{(e)2} &= 0.172329 + O \left[\left(\frac{\alpha^{(e)}}{\mu^{(e)}} \right)^2 \right], \\
\mu^{(e)} &= 85.9924 \text{ MeV} + O \left[\left(\frac{\alpha^{(e)}}{\mu^{(e)}} \right)^2 \right] \mu^{(e)} + O \left[\left(\frac{\alpha^{(e)} - \beta^{(e)}}{\mu^{(e)}} \right)^2 \right] \mu^{(e)}, \quad (4)
\end{aligned}$$

when the experimental values of m_e and m_μ [4] are used as inputs. Then, in the first Eq. (4) $6(351m_\mu - 136m_e)/125 = 1776.80$ MeV, in the second $320m_e/(9m_\mu - 4m_e) = 0.172329$ and in the third $29(9m_\mu - 4m_e)/320 = 85.9924$ MeV.

With $\beta^{(e)}$ neglected *versus* $\alpha^{(e)}$ due to Eq. (2), the first Eq. (4) gives

$$m_\tau = \left[1776.80 + 10.2112 \left(\frac{\alpha^{(e)}}{\mu^{(e)}} \right)^2 \right] \text{ MeV}, \quad (5)$$

what shows that

$$\left(\frac{\alpha^{(e)}}{\mu^{(e)}} \right)^2 = 0.020_{-0.020}^{+0.029}, \quad (6)$$

when the experimental value $m_\tau = 1777.00_{-0.27}^{+0.30}$ MeV [4] is used as another input. Thus, as yet, the values of $\alpha^{(f)}$ and $\beta^{(f)}$ are consistent with zero. We can see that the mass-matrix formula (1) predicts excellently the mass m_τ , even in the zero-order perturbative calculation [1].

The unitary matrix $\widehat{U}^{(e)}$, diagonalizing the mass matrix $\widehat{M}^{(e)}$ according to the equality $\widehat{U}^{(e)\dagger}\widehat{M}^{(e)}\widehat{U}^{(e)} = \text{diag}(m_e, m_\mu, m_\tau)$, gets in the lowest (linear) perturbative order the form

$$\widehat{U}^{(e)} = \widehat{1} + \frac{1}{29} \begin{pmatrix} 0 & \frac{2\alpha^{(e)}}{m_\mu} e^{i\varphi^{(e)}} & 0 \\ -2\frac{\alpha^{(e)}}{m_\mu} e^{-i\varphi^{(e)}} & 0 & 8\sqrt{3}\frac{\alpha^{(e)}-\beta^{(e)}}{m_\tau} e^{i\varphi^{(e)}} \\ 0 & -8\sqrt{3}\frac{\alpha^{(e)}-\beta^{(e)}}{m_\tau} e^{-i\varphi^{(e)}} & 0 \end{pmatrix}, \tag{7}$$

where the small $\varepsilon^{(e)2}$ is neglected. Here, due to Eq. (2), $\beta^{(e)}$ can be also neglected *versus* $\alpha^{(e)}$.

3. Neutrinos

In the case of neutrinos, the mass matrix $\widehat{M}^{(\nu)} = (M_{ij}^{(\nu)})$ ($i, j = 0, 1, 2$) as given in Eq. (1), with $\alpha^{(\nu)} = 0$ due to Eq. (2), leads exactly to the following eigenvalues interpreted as neutrino masses:

$$\begin{aligned} m_{\nu_0} &= M_{00}^{(\nu)} = \frac{\mu^{(\nu)}}{29} \varepsilon^{(\nu)2}, \\ m_{\nu_{1,2}} &= \frac{M_{11}^{(\nu)} + M_{22}^{(\nu)}}{2} \mp \left[\left(\frac{M_{11}^{(\nu)} - M_{22}^{(\nu)}}{2} \right)^2 + |M_{12}^{(\nu)}|^2 \right]^{1/2} \\ &= \left[10.9 \mp 0.478 \frac{\beta^{(\nu)}}{\mu^{(\nu)}} \sqrt{1 + \left(20.3 \frac{\mu^{(\nu)}}{\beta^{(\nu)}} \right)^2} \right] \mu^{(\nu)}, \end{aligned} \tag{8}$$

where in m_{ν_1} and m_{ν_2} the very small $\varepsilon^{(\nu)2}$ is neglected in the second step. Here, $(M_{11}^{(\nu)} - m_i)(M_{22}^{(\nu)} - m_i) = |M_{12}^{(\nu)}|^2$ ($i = 1, 2$).

The corresponding unitary matrix $\widehat{U}^{(\nu)}$ diagonalizing the mass matrix $\widehat{M}^{(\nu)}$ according to the equality $\widehat{U}^{(\nu)\dagger}\widehat{M}^{(\nu)}\widehat{U}^{(\nu)} = \text{diag}(m_{\nu_0}, m_{\nu_1}, m_{\nu_2})$, takes exactly the form

$$\widehat{U}^{(\nu)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (1 + X^2)^{-1/2} & X(1 + X^2)^{-1/2} e^{i\varphi^{(\nu)}} \\ 0 & -X(1 + X^2)^{-1/2} e^{-i\varphi^{(\nu)}} & (1 + X^2)^{-1/2} \end{pmatrix}, \tag{9}$$

where

$$\begin{aligned}
 X &= \frac{M_{11}^{(\nu)} - m_{\nu_1}}{|M_{12}^{(\nu)}|} = -\frac{M_{22}^{(\nu)} - m_{\nu_2}}{|M_{12}^{(\nu)}|} \\
 &= \frac{M_{11}^{(\nu)} - M_{22}^{(\nu)}}{2|M_{12}^{(\nu)}|} + \left[1 + \left(\frac{M_{11}^{(\nu)} - M_{22}^{(\nu)}}{2|M_{12}^{(\nu)}|} \right)^2 \right]^{1/2} \\
 &= -20.3 \frac{\mu^{(\nu)}}{\beta^{(\nu)}} + \sqrt{1 + \left(20.3 \frac{\mu^{(\nu)}}{\beta^{(\nu)}} \right)^2}. \quad (10)
 \end{aligned}$$

The experimentally observed neutrino weak-interaction states ν_e, ν_μ, ν_τ are related to their mass states $\nu_0^{(m)}, \nu_1^{(m)}, \nu_2^{(m)}$ (corresponding to the masses $m_{\nu_0}, m_{\nu_1}, m_{\nu_2}$) through the unitary transformation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \widehat{V}^\dagger \begin{pmatrix} \nu_0^{(m)} \\ \nu_1^{(m)} \\ \nu_2^{(m)} \end{pmatrix}, \quad (11)$$

where

$$\widehat{V} = \widehat{U}^{(\nu) \dagger} \widehat{U}^{(e)} \quad (12)$$

is the lepton Cabibbo–Kobayashi–Maskawa matrix. Making use of Eqs (9) and (7), we can calculate $\widehat{V} = (V_{ij})$ ($i, j = 0, 1, 2$) from the formulae $V_{ij} = \sum_k U_{ki}^{(\nu)*} U_{kj}^{(e)}$. The result, valid in the lowest (linear) perturbative order in $\alpha^{(e)}/\mu^{(e)}$ and $\beta^{(e)}/\mu^{(e)}$, reads

$$\begin{aligned}
 V_{01} &= \frac{2}{29} \frac{\alpha^{(e)}}{m_\mu} e^{i\varphi^{(e)}}, \quad V_{10} = -\frac{2}{29\sqrt{1+X^2}} \frac{\alpha^{(e)}}{m_\mu} e^{-i\varphi^{(e)}}, \\
 V_{12} &= -\frac{X}{\sqrt{1+X^2}} e^{i\varphi^{(\nu)}} + \frac{8\sqrt{3}}{29\sqrt{1+X^2}} \frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau} e^{i\varphi^{(e)}} = -V_{21}^*, \\
 V_{02} &= 0, \quad V_{20} = -\frac{2X}{29\sqrt{1+X^2}} \frac{\alpha^{(e)}}{m_\mu} e^{-i(\varphi^{(\nu)} + \varphi^{(e)})}, \\
 V_{00} &= 1, \quad V_{11} = \frac{1}{\sqrt{1+X^2}} + \frac{8\sqrt{3}X}{29\sqrt{1+X^2}} \frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau} e^{i(\varphi^{(\nu)} - \varphi^{(e)})} = V_{22}^*, \quad (13)
 \end{aligned}$$

where $\beta^{(e)}$ can be neglected *versus* $\alpha^{(e)}$ (for the numerical form of V_{ij} cf. Appendix B).

We can see from Eqs (11) and (13) that

$$\begin{aligned}
 \nu_0^{(m)} &= \nu_e + O\left(\frac{\alpha^{(e)}}{m_\mu}\right) \nu_\mu, \\
 \nu_1^{(m)} &= \left[\frac{1}{\sqrt{1+X^2}} + O\left(\frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau}\right)\right] \nu_\mu \\
 &\quad - \left[\frac{X}{\sqrt{1+X^2}} e^{i\varphi^{(\nu)}} + O\left(\frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau}\right)\right] \nu_\tau + O\left(\frac{\alpha^{(e)}}{m_\mu}\right) \nu_e, \\
 \nu_2^{(m)} &= \left[\frac{X}{\sqrt{1+X^2}} e^{-i\varphi^{(\nu)}} + O\left(\frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau}\right)\right] \nu_\mu \\
 &\quad + \left[\frac{1}{\sqrt{1+X^2}} + O\left(\frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau}\right)\right] \nu_\tau + O\left(\frac{\alpha^{(e)}}{m_\mu}\right) \nu_e. \tag{14}
 \end{aligned}$$

Thus, if X is of the order $O(1)$, in Eqs (14) there appears strong mixing between ν_μ and ν_τ beside weak mixing of ν_μ and ν_τ with ν_e . Note from Eq. (10) that $X \rightarrow 1$ in the limit of $\mu^{(\nu)}/\beta^{(\nu)} \rightarrow 0$ and, for example, $X = \sqrt{2} - 1$ or $(\sqrt{5} - 1)/2$ for $\mu^{(\nu)}/\beta^{(\nu)} = 1/20.3$ or $1/40.6$, respectively (in these cases $4X^2(1 + X^2)^{-2} \rightarrow 1$ and $4X^2(1 + X^2)^{-2} = 0.5$ or 0.8).

Once we know the elements (13) of the lepton Cabibbo–Kobayashi–Maskawa matrix \widehat{V} , we can calculate the probabilities of neutrino oscillations $\nu_i \rightarrow \nu_j$ (in the vacuum) from the familiar formulae

$$P(\nu_i \rightarrow \nu_j, t) = |\langle \nu_j | \nu_i(t) \rangle|^2 = \sum_{kl} V_{jl}^* V_{il} V_{jk} V_{ik}^* \exp\left(i \frac{m_{\nu_l}^2 - m_{\nu_k}^2}{2|\vec{p}|} t\right), \tag{15}$$

where usually $t/|\vec{p}| = L/E$ (what is equal to $4 \times 1.2663L/E$ if $m_{\nu_l}^2 - m_{\nu_k}^2$, L and E are measured in eV^2 , km and GeV, respectively). Here, L is the source-detector distance. In the notation of Eq. (15), $\nu_0 \equiv \nu_e$, $\nu_1 \equiv \nu_\mu$, $\nu_2 \equiv \nu_\tau$ are the neutrino weak-interaction states (to be distinguished from their mass states $\nu_0^{(m)}$, $\nu_1^{(m)}$, $\nu_2^{(m)}$ corresponding to m_{ν_0} , m_{ν_1} , m_{ν_2}).

After some calculations, we obtain in the lowest (linear and quadratic) perturbative order in $\alpha^{(e)}/\mu^{(e)}$ and $\beta^{(e)}/\mu^{(e)}$ the following formulae:

$$\begin{aligned}
 P(\nu_e \rightarrow \nu_\mu, t) &= \frac{16}{841(1+X^2)} \left(\frac{\alpha^{(e)}}{m_\mu}\right)^2 \sin^2\left(\frac{m_{\nu_1}^2 - m_{\nu_0}^2}{4|\vec{p}|} t\right), \\
 P(\nu_e \rightarrow \nu_\tau, t) &= \frac{16X^2}{841(1+X^2)} \left(\frac{\alpha^{(e)}}{m_\mu}\right)^2 \sin^2\left(\frac{m_{\nu_1}^2 - m_{\nu_0}^2}{4|\vec{p}|} t\right),
 \end{aligned}$$

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_\tau, t) = & \left\{ \frac{4X^2}{(1+X^2)^2} - \frac{64\sqrt{3}X(1-X^2)}{29(1+X^2)^2} \frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau} \cos(\varphi^{(\nu)} - \varphi^{(e)}) \right. \\
& + O\left[\left(\frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau}\right)^2\right] \left. \right\} \sin^2\left(\frac{m_{\nu_2}^2 - m_{\nu_1}^2}{4|\vec{p}|} t\right) \\
& + \frac{16X^2}{841(1+X^2)^2} \left(\frac{\alpha^{(e)}}{m_\mu}\right)^2 \\
& \times \left\{ \sin^2\left[\frac{m_{\nu_2}^2 - m_{\nu_0}^2}{4|\vec{p}|} t + \frac{1}{2}(\arg V_{12} - \varphi^{(\nu)} - 180^\circ + \arg V_{11})\right] \right. \\
& \left. - \sin^2\left[\frac{m_{\nu_1}^2 - m_{\nu_0}^2}{4|\vec{p}|} t + \frac{1}{2}(\arg V_{12} - \varphi^{(\nu)} - 180^\circ + \arg V_{11})\right] \right\}. \tag{16}
\end{aligned}$$

In the third Eq. (16) there appears (in the cubic perturbative order) the CP-violating phase

$$\begin{aligned}
\arg(V_{10}^* V_{21}^* V_{20} V_{11}) &= \arg V_{12} - \varphi^{(\nu)} - 180^\circ + \arg V_{11} \\
&= O\left[\frac{\alpha^{(e)} - \beta^{(e)}}{m_\tau} (\varphi^{(\nu)} - \varphi^{(e)})\right],
\end{aligned}$$

invariant under any lepton rephasing (*cf.* Appendix B). Its form presented on the rhs holds in the lepton phasing as in Eq. (13), and can be expressed through $\varphi^{(\nu)} - \varphi^{(e)}$ by means of Eqs (B) and (B.6). Then, it turns out to be vanishing if $\varphi^{(\nu)} - \varphi^{(e)} = 0$, leading in such a case to a real, CP-preserving matrix \tilde{V} in the convenient lepton phasing (B.8). Note that in the lowest (quadratic) perturbative order the mass difference $m_{\nu_2}^2 - m_{\nu_0}^2$ is not present in the second Eq. (16).

The atmospheric neutrino experiments seem to indicate that the $\nu_\mu \rightarrow \nu_\tau$ oscillation amplitude is of the order $O(1)$ [5–7]. So, let us take as an input for the leading oscillation amplitude in the third Eq. (16) the reasonable value

$$\frac{4X^2}{(1+X^2)^2} \sim 0.5 \text{ or } 0.8, \tag{17}$$

where the second figure (or one a bit larger) is more reliable. This gives $X \sim \sqrt{2} - 1 = 0.414$ or $(\sqrt{5} - 1)/2 = 0.618$, and then from Eq. (10)

$$20.3 \frac{\mu^{(\nu)}}{\beta^{(\nu)}} \sim 1 \text{ or } 0.5. \tag{18}$$

Thus, $\beta^{(\nu)}/\mu^{(\nu)} \sim 20.3$ or 40.6 and $\mu^{(\nu)}/\beta^{(\nu)} \sim 0.0493$ or 0.0246 .

As another input let us accept the recent Super-Kamiokande bound [6,7]

$$|m_{\nu_2}^2 - m_{\nu_1}^2| \sim (0.03 \text{ to } 1) \times 10^{-2} \text{ eV}^2 \quad (19)$$

with the preferable value $0.5 \times 10^{-2} \text{ eV}^2$.

Making use of Eqs (8) we get

$$\begin{aligned} m_{\nu_2}^2 - m_{\nu_1}^2 &= 2|M_{12}^{(\nu)}| \left(M_{11}^{(\nu)} + M_{22}^{(\nu)} \right) \left[1 + \left(\frac{M_{11}^{(\nu)} - M_{22}^{(\nu)}}{2|M_{12}^{(\nu)}|} \right)^2 \right]^{1/2} \\ &= 20.9 \beta^{(\nu)} \mu^{(\nu)} \sqrt{1 + \left(20.3 \frac{\mu^{(\nu)}}{\beta^{(\nu)}} \right)^2} \\ &\sim (600 \text{ or } 949) \mu^{(\nu)2}, \end{aligned} \quad (20)$$

where Eq. (18) is used. From Eqs (19) and (20) we infer that

$$\mu^{(\nu)} \sim (0.707 \text{ to } 4.08) \times 10^{-3} \text{ eV} \quad \text{or} \quad (0.562 \text{ to } 3.25) \times 10^{-3} \text{ eV}, \quad (21)$$

and then from Eq. (18)

$$\beta^{(\nu)} \sim (0.144 \text{ to } 0.828) \times 10^{-1} \text{ eV} \quad \text{or} \quad (0.228 \text{ to } 1.32) \times 10^{-1} \text{ eV}. \quad (22)$$

With the values (18) and (21), the neutrino mass formulae (8) predict

$$\begin{aligned} m_{\nu_1} &\sim -2.82\mu^{(\nu)} = -(0.199 \text{ to } 1.15) \times 10^{-2} \text{ eV} \\ &\text{or } -10.8\mu^{(\nu)} = -(0.607 \text{ to } 3.51) \times 10^{-2} \text{ eV}, \\ m_{\nu_2} &\sim 24.6\mu^{(\nu)} = (0.174 \text{ to } 1.00) \times 10^{-1} \text{ eV} \\ &\text{or } 32.6\mu^{(\nu)} = (0.183 \text{ to } 1.06) \times 10^{-1} \text{ eV} \end{aligned} \quad (23)$$

and, of course, $m_{\nu_0} \ll m_{\nu_1}$ if $\varepsilon^{(\nu)2} \simeq 0$ is small enough. The minus sign at m_{ν_1} in Eq. (23) is irrelevant in the relativistic dynamics (*cf.* the Dirac equation) and so, can be changed into the plus sign, if the mass m_{ν_1} is considered from the phenomenological point of view.

The neutrino mass m_{ν_1} as estimated in Eq. (23) leads to

$$|m_{\nu_1}^2 - m_{\nu_0}^2| \sim (0.04 \text{ to } 1) \times 10^{-4} \text{ eV}^2 \quad \text{or} \quad (0.04 \text{ to } 1) \times 10^{-3} \text{ eV}^2, \quad (24)$$

where $m_{\nu_0}^2 = \mu^{(\nu)2} \varepsilon^{(\nu)4} / 841 \simeq 0$ is neglected. Such an estimate may be used to evaluate the $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ oscillation probabilities (in the

vacuum) from the first and second Eq. (16), where the oscillation amplitudes are

$$\begin{aligned} \frac{16}{841(1+X^2)} \left(\frac{\alpha^{(e)}}{m_\mu} \right)^2 &\sim 2.2_{-2.2}^{+3.3} \times 10^{-4} \quad \text{or} \quad 1.8_{-1.8}^{+2.7} \times 10^{-4}, \\ \frac{16X^2}{841(1+X^2)} \left(\frac{\alpha^{(e)}}{m_\mu} \right)^2 &\sim 3.7_{-3.7}^{+5.5} \times 10^{-5} \quad \text{or} \quad 7.0_{-7.0}^{+10.5} \times 10^{-5}. \end{aligned} \quad (25)$$

According to recent estimations [8], the familiar two-flavor neutrino-oscillation formula (in the vacuum),

$$P = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4|\vec{p}|} t \right),$$

requires the oscillation amplitude

$$\sin^2 2\theta \sim 0.65 \quad \text{to} \quad 1$$

and the (unrealistic?) mass-squared difference

$$\Delta m^2 \sim (5 \text{ to } 8) \times 10^{-11} \text{ eV}^2$$

in order to explain the solar-neutrino deficit. Thus, our amplitudes (25) of $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ oscillations (in the vacuum) are much too small, while the related neutrino mass-squared difference (24) is much too large.

According to the recent estimations [8], the two-flavor neutrino oscillations, strengthened by the resonant MSW mechanism in the Sun matter [9], may solve the problem of solar neutrinos, if

$$(\sin^2 2\theta)_{\text{MSW}} \sim 8 \times 10^{-3}, \quad \Delta m^2 \sim 5 \times 10^{-6} \text{ eV}^2$$

(the preferred small-mixing-angle solution) or

$$(\sin^2 2\theta)_{\text{MSW}} \sim 0.6, \quad \Delta m^2 \sim 1.6 \times 10^{-5} \text{ eV}^2$$

(the alternative large-mixing-angle solution). We can see that our neutrino mass-squared difference (24) is formally not inconsistent with both MSW solutions, favouring the second.

In the present paper there is left open the actual question about interpretation of LSND events from Los Alamos [10]. They suggest the existence of $\nu_\mu \rightarrow \nu_e$ oscillations with Δm^2 of one to two orders of magnitude larger than the Super-Kamiokande Δm^2 for atmospheric neutrino events. Evidently, the LSND events are relevant for the problem of existence of only three conservative neutrinos. In fact, they seem to suggest the existence of one extra (sterile) neutrino (*cf. e.g.* Ref. [11]; for a possible origin of the hypothetical sterile neutrino *cf.* the end of Appendix A).

4. Up and down quarks

In the case of up and down quarks we will assume, similarly as for charged leptons, that the off-diagonal elements of the mass matrices $\widehat{M}^{(u,d)} = (M_{ij}^{(u,d)})$ ($i, j = 0, 1, 2$) described in Eq. (1) can be considered as small perturbations of the diagonal elements (this assumption will be verified *a posteriori*, when we estimate the coupling constants $\alpha^{(u,d)}/\mu^{(u,d)}$ and $\beta^{(u,d)}/\mu^{(u,d)}$). Then, in the lowest (quadratic) perturbative order we get

$$\begin{aligned}
 m_{u,d} &= \frac{\mu^{(u,d)}}{29} \left[\varepsilon^{(u,d)2} - \frac{36}{320 - 5\varepsilon^{(u,d)2}} \left(\frac{\alpha^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right], \\
 m_{c,s} &= \frac{\mu^{(u,d)}}{29} \left[\frac{4}{9} (80 + \varepsilon^{(u,d)2}) + \frac{36}{320 - 5\varepsilon^{(u,d)2}} \left(\frac{\alpha^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right. \\
 &\quad \left. - \frac{10800}{31696 + 1350C^{(u,d)} + 29\varepsilon^{(u,d)2}} \left(\frac{\alpha^{(u,d)} - \beta^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right], \\
 m_{t,b} &= \frac{\mu^{(u,d)}}{29} \left[\frac{24}{25} (624 + 25C^{(u,d)} + \varepsilon^{(u,d)2}) \right. \\
 &\quad \left. + \frac{10800}{31696 + 1350C^{(u,d)} + 29\varepsilon^{(u,d)2}} \left(\frac{\alpha^{(u,d)} - \beta^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right], \quad (26)
 \end{aligned}$$

where $C^{(d)} = 0$ due to Eq. (2).

These mass formulae imply the relations analogical to Eqs (4) for charged leptons, namely

$$\begin{aligned}
 m_{t,b} &= \frac{6}{125} (351m_{c,s} - 136m_{u,d}) + \frac{\mu^{(u,d)}}{29} 24C^{(u,d)} \\
 &\quad - \frac{105192}{3625} \frac{\alpha^{(u,d)2}/\mu^{(u,d)}}{320 - 5\varepsilon^{(u,d)2}} \\
 &\quad + \frac{24094800}{3625} \frac{(\alpha^{(u,d)} - \beta^{(u,d)})^2/\mu^{(u,d)}}{31696 + 1350C^{(u,d)} + 29\varepsilon^{(u,d)2}}, \\
 \varepsilon^{(u,d)2} &= \frac{320m_{u,d}}{9m_{c,s} - 4m_{u,d}} + O \left[\left(\frac{\alpha^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right] + O \left[\left(\frac{\alpha^{(u,d)} - \beta^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right], \\
 \mu^{(u,d)} &= \frac{29}{320} (9m_{c,s} - 4m_{u,d}) \\
 &\quad + O \left[\left(\frac{\alpha^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right] \mu^{(u,d)} + O \left[\left(\frac{\alpha^{(u,d)} - \beta^{(u,d)}}{\mu^{(u,d)}} \right)^2 \right] \mu^{(u,d)}. \quad (27)
 \end{aligned}$$

The mass m_s may be predicted from the first Eq. (27) in the zero perturbative order, if m_d and m_b are known. When the small ratio m_d/m_s is of the order of relative perturbative corrections [*cf.* Eq. (26)], it can be also neglected, and then

$$m_s \simeq \frac{125}{6 \cdot 351} m_b = 267 \text{ MeV} \quad (28)$$

for $m_b = 4.5 \text{ GeV}$ as an input. Similarly, the third Eq. (27) or, equivalently, the second Eq. (26) gives in the zero perturbative order

$$\mu^{(d)} \simeq \frac{29 \cdot 9}{320} m_s \simeq 218 \text{ MeV}, \quad (29)$$

while, due to the estimation (41) of $(\alpha^{(d)}/\mu^{(d)})^2$ discussed later on, the first Eq.(26) leads jointly with its perturbation to

$$\varepsilon^{(d)2} \simeq \frac{29m_d}{\mu^{(d)}} + \frac{36}{320} \left(\frac{\alpha^{(d)}}{\mu^{(d)}} \right)^2 \simeq 0.932 + 1.41 = 2.34 \quad (30)$$

for $m_d = 7 \text{ MeV}$ as another input. If, however, the first Eq. (26) gives $m_d < 0$, then with the input $|m_d| = 7 \text{ MeV}$ one obtains $\varepsilon^{(d)2} \simeq -0.932 + 1.41 = 0.478$. Here, $C^{(d)} = 0$. If $C^{(d)} > 0$, then m_s is smaller. For instance, $m_s \simeq 200 \text{ MeV}$ and $\mu^{(d)} \simeq 163 \text{ MeV}$, when $C^{(d)} \simeq 8.37$.

In an analogical way,

$$\mu^{(u)} \simeq \frac{29 \cdot 9}{320} m_c = 1060 \text{ MeV} \quad (31)$$

and

$$\varepsilon^{(u)2} \simeq \frac{29m_u}{\mu^{(u)}} + \frac{36}{320} \left(\frac{\alpha^{(u)}}{\mu^{(u)}} \right)^2 \simeq 0.109 + 0.239 = 0.348 \quad (32)$$

for $m_u = 4 \text{ MeV}$ and $m_c = 1.3 \text{ GeV}$ as inputs. However, if $m_u < 0$, then with the input $|m_u| = 4 \text{ MeV}$ one gets $\varepsilon^{(u)2} \simeq -0.109 + 0.239 = 0.130$. Further, from the first Eq. (27)

$$C^{(u)} \simeq \frac{29}{24} \frac{1}{\mu^{(u)}} \left(m_t - \frac{6 \cdot 351}{125} m_c \right) \simeq 175 \quad (33)$$

for $m_t = 175 \text{ GeV}$ as another input. If the constant $C^{(u)}$ were known from some conjecture, the value of one of the masses, m_c or m_t , could be a prediction.¹

¹ For instance, the *ad hoc* formula $C_N^{(f)} = (N_C^{(f)} - 1)(Q + B)^{(f)}(N - 1)(N - 3)(2N + 1)$ ($N = 1, 3, 5$) gives $C_N^{(\nu)} = 0$, $C_N^{(e)} = 0$, $C_N^{(u)} = 176\delta_{N5}$, $C_N^{(d)} = 0$, *i.e.*, $C^{(u)} = 176$, what predicts $m_t \simeq 176 \text{ GeV}$ with the input of $m_c = 1.3 \text{ GeV}$.

The unitary matrices $\widehat{U}^{(u,d)}$, diagonalizing the mass matrices $\widehat{M}^{(u,d)}$ through the relations $\widehat{U}^{(u,d)\dagger}\widehat{M}^{(u,d)}\widehat{U}^{(u,d)} = \text{diag}(m_{u,d}, m_{c,s}, m_{t,b})$, take in the lowest (linear or quadratic) perturbative order the form

$$\widehat{U}^{(f)} = \begin{pmatrix} A_0^{(f)} & 0 & 0 \\ 0 & A_1^{(f)} & 0 \\ 0 & 0 & A_2^{(f)} \end{pmatrix} + \frac{1}{29} \begin{pmatrix} 0 & 2\frac{\alpha^{(f)}}{m_{c,s}}e^{i\varphi^{(f)}} & \frac{16\sqrt{3}}{29}\frac{\alpha^{(f)}\gamma^{(f)}}{m_{t,b}^2}e^{2i\varphi^{(f)}} \\ -2\frac{\alpha^{(f)}}{m_{c,s}}e^{-i\varphi^{(f)}} & 0 & 8\sqrt{3}\frac{\gamma^{(f)}}{m_{t,b}}e^{i\varphi^{(f)}} \\ \frac{16\sqrt{3}}{29}\frac{\alpha^{(f)}\gamma^{(f)}}{m_{c,s}m_{t,b}}e^{-2i\varphi^{(f)}} & -8\sqrt{3}\frac{\gamma^{(f)}}{m_{t,b}}e^{-i\varphi^{(f)}} & 0 \end{pmatrix}, \tag{34}$$

where $(f) = (u, d)$ and $\gamma^{(f)} = \alpha^{(f)} - \beta^{(f)}$, while

$$\begin{aligned} A_0^{(u,d)} &= 1 - \frac{1}{2} \frac{4}{841} \left(\frac{\alpha^{(u,d)}}{m_{c,s}} \right)^2, \\ A_1^{(u,d)} &= 1 - \frac{1}{2} \frac{4}{841} \left(\frac{\alpha^{(u,d)}}{m_{c,s}} \right)^2 - \frac{1}{2} \frac{192}{841} \left(\frac{\alpha^{(u,d)} - \beta^{(u,d)}}{m_{t,b}} \right)^2, \\ A_2^{(u,d)} &= 1 - \frac{1}{2} \frac{192}{841} \left(\frac{\alpha^{(u,d)} - \beta^{(u,d)}}{m_{t,b}} \right)^2. \end{aligned} \tag{35}$$

Here, in the mass denominators, we keep only leading terms.

The down-quark weak-interaction states $d^{(w)}, s^{(w)}, b^{(w)}$ are related to their experimentally observed mass states d, s, b by the unitary transformation

$$\begin{pmatrix} d^{(w)} \\ s^{(w)} \\ b^{(w)} \end{pmatrix} = \widehat{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{36}$$

Here,

$$\widehat{V} = \widehat{U}^{(u)}\dagger\widehat{U}^{(d)} \tag{37}$$

is the familiar (quark) Cabibbo–Kobayashi–Maskawa matrix [to be distinguished from its lepton counterpart (12)]. Using both Eqs (34), we can calculate $\widehat{V} = (V_{ij})$ ($i, j = 0, 1, 2$) from the formulae $V_{ij} = \sum_k U_{ki}^{(u)*}U_{kj}^{(d)}$. In the lowest (linear or quadratic) perturbative order in $\alpha^{(u,d)}/\mu^{(u,d)}$ and $\beta^{(u,d)}/\mu^{(u,d)}$, we obtain

$$V_{us} \ (\equiv V_{01}) = -V_{cd}^* \ (\equiv -V_{10}^*) = \frac{2}{29} \left(\frac{\alpha^{(d)}}{m_s}e^{i\varphi^{(d)}} - \frac{\alpha^{(u)}}{m_c}e^{i\varphi^{(u)}} \right),$$

$$\begin{aligned}
V_{cb} \ (\equiv V_{12}) &= -V_{ts}^* \ (\equiv -V_{21}^*) \\
&= \frac{8\sqrt{3}}{29} \left(\frac{\alpha^{(d)} - \beta^{(d)}}{m_b} e^{i\varphi^{(d)}} - \frac{\alpha^{(u)} - \beta^{(u)}}{m_t} e^{i\varphi^{(u)}} \right) \\
&\simeq \frac{8\sqrt{3}}{29} \frac{\alpha^{(d)} - \beta^{(d)}}{m_b} e^{i\varphi^{(d)}}, \\
V_{ub} \ (\equiv V_{02}) &= -\frac{16\sqrt{3}}{841} \left[\frac{\alpha^{(u)}(\alpha^{(d)} - \beta^{(d)})}{m_c m_b} e^{i(\varphi^{(u)} + \varphi^{(d)})} \right. \\
&\quad \left. - \frac{\alpha^{(u)}(\alpha^{(u)} - \beta^{(u)})}{m_c m_t} e^{2i\varphi^{(u)}} \right. \\
&\quad \left. - \frac{\alpha^{(d)}(\alpha^{(d)} - \beta^{(d)})}{m_b^2} e^{2i\varphi^{(d)}} \right] \\
&\simeq -\frac{16\sqrt{3}}{841} \frac{\alpha^{(u)}(\alpha^{(d)} - \beta^{(d)})}{m_c m_b} e^{i(\varphi^{(u)} + \varphi^{(d)})}, \\
V_{td} \ (\equiv V_{20}) &= -\frac{16\sqrt{3}}{841} \left[\frac{(\alpha^{(u)} - \beta^{(u)})\alpha^{(d)}}{m_t m_s} e^{-i(\varphi^{(u)} + \varphi^{(d)})} \right. \\
&\quad \left. - \frac{\alpha^{(d)}(\alpha^{(d)} - \beta^{(d)})}{m_s m_b} e^{-2i\varphi^{(d)}} \right. \\
&\quad \left. - \frac{\alpha^{(u)}(\alpha^{(u)} - \beta^{(u)})}{m_t^2} e^{-2i\varphi^{(u)}} \right] \\
&\simeq \frac{16\sqrt{3}}{841} \frac{\alpha^{(d)}(\alpha^{(d)} - \beta^{(d)})}{m_s m_b} e^{-2i\varphi^{(d)}}, \\
V_{ud} \ (\equiv V_{00}) &\simeq |V_{ud}|, \quad V_{cs} \ (\equiv V_{11}) \simeq |V_{cs}|, \quad V_{tb} \ (\equiv V_{22}) \simeq |V_{tb}|.
\end{aligned} \tag{38}$$

The approximate equalities in Eqs (38) are due to $m_t \gg m_b$ and $m_b > m_c$, and also to $\alpha^{(u)} = 2\alpha^{(d)}$ and $\beta^{(u)} = \beta^{(d)} \simeq \alpha^{(d)}/2$ (this gives $\alpha^{(u)}m_b/\alpha^{(d)}m_c = 6.3$).

Taking the experimental value $|V_{cb}| = 0.041 \pm 0.003$ as an input, we calculate from the second Eq. (38)

$$\alpha^{(d)} - \beta^{(d)} \simeq \frac{29}{8\sqrt{3}} m_b |V_{cb}| = (386 \pm 28) \text{ MeV} \tag{39}$$

for the $m_b = 4.5$ GeV already used in Eq. (28). Hence, invoking Eq. (2), we evaluate

$$\begin{aligned}
\alpha^{(d)} &= 2(\alpha^{(d)} - \beta^{(d)}) \simeq (772 \pm 56) \text{ MeV}, \\
\alpha^{(u)} &= 2\alpha^{(d)} \simeq (1544 \pm 112) \text{ MeV}, \\
\beta^{(u)} &= \beta^{(d)} \simeq \frac{1}{2}\alpha^{(d)} \simeq (386 \pm 28) \text{ MeV}.
\end{aligned} \tag{40}$$

We can see from Eqs (30), (32) and (40) that

$$\begin{aligned} \left(\frac{\alpha^{(u)}}{\mu^{(u)}}\right)^2 &\simeq 2.12 = O(1) , & \left(\frac{\alpha^{(d)}}{\mu^{(d)}}\right)^2 &\simeq 12.5 = O(10) , \\ \left(\frac{\alpha^{(u)} - \beta^{(u)}}{\mu^{(u)}}\right)^2 &\simeq 1.19 = O(1) , & \left(\frac{\alpha^{(d)} - \beta^{(d)}}{\mu^{(d)}}\right)^2 &\simeq 3.14 = O(1) . \end{aligned} \quad (41)$$

In spite of these nonperturbative values, the small relative numerical coefficients in the formulae (26) for $m_{c,s}$ and $m_{t,b}$ cause that the effective perturbative corrections are small (maximally of 1%, in the case of m_s). But, for $m_{u,d}$ the perturbative effects are essential [as large as 220% and 150%, respectively; *cf.* Eqs (32) and (30)], what, strictly speaking, invalidates the perturbative calculation in this case (where this calculation ought to be replaced by the numerical evaluation of $m_{u,d}$).

The value of $\alpha^{(u)}$ as estimated in Eqs (40) leads through the second and third Eq. (38) to the prediction

$$\left|\frac{V_{ub}}{V_{cb}}\right| \simeq \frac{2}{29} \frac{\alpha^{(u)}}{m_c} \simeq 0.082 \pm 0.006 \quad (42)$$

for the $m_c = 1.3$ GeV already used in Eq. (32) [the corrections following from the neglected terms in V_{ub} do not change the figure (42)]. Hence, we predict equivalently $|V_{ub}| \simeq 0.0034 \pm 0.0005$. We can see that the result (42) agrees neatly with the experimental value $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ [4].

With $\alpha^{(u)}$ and $\alpha^{(d)}$ as given in Eqs (40), we find from the first Eq. (38) that

$$V_{us} \simeq \frac{2}{29} \frac{772 \pm 56}{267} \left[1 - \frac{2 \cdot 267}{1300} e^{i(\varphi^{(u)} - \varphi^{(d)})} \right] e^{i\varphi^{(d)}} , \quad (43)$$

and then

$$|V_{us}|^2 \simeq \left(\frac{2}{29} \frac{772 \pm 56}{267}\right)^2 \left[1 + \left(\frac{534}{1300}\right)^2 - \frac{2 \cdot 534}{1300} \cos(\varphi^{(u)} - \varphi^{(d)}) \right] . \quad (44)$$

Hence, taking the experimental value $|V_{us}| = 0.2205 \pm 0.0018$ as another input, we evaluate $\cos(\varphi^{(u)} - \varphi^{(d)}) \simeq -0.0658$ and

$$\varphi^{(u)} - \varphi^{(d)} \simeq -86.2^\circ + 180^\circ , \quad (45)$$

when the central values are used. Then, from Eq. (43) we calculate $\tan(\arg V_{us} - \varphi^{(d)}) \simeq -0.399$ and

$$\arg V_{us} \simeq -21.8^\circ + \varphi^{(d)} , \quad \arg V_{cd} \simeq 21.8^\circ - \varphi^{(d)} - 180^\circ . \quad (46)$$

From other Eqs (38) we can see that

$$\begin{aligned} \arg V_{cb} &\simeq \varphi^{(d)} , & \arg V_{ts} &\simeq -\varphi^{(d)} + 180^\circ , \\ \arg V_{ub} &\simeq \varphi^{(u)} + \varphi^{(d)} - 180^\circ , & \arg V_{td} &\simeq -2\varphi^{(d)} , \\ \arg V_{ud} &\simeq \arg V_{cs} \simeq \arg V_{tb} \simeq 0 . \end{aligned} \quad (47)$$

Further, making use of Eqs (46) and (47) with (45), we predict two mutually dependent CP-violating phases:

$$\arg(V_{us}^* V_{cb}^* V_{ub} V_{cs}) \simeq 21.8^\circ + \varphi^{(u)} - \varphi^{(d)} - 180^\circ \simeq -64.4^\circ$$

and

$$\arg(V_{cd}^* V_{ts}^* V_{td} V_{cs}) \simeq -21.8^\circ .$$

These, being invariant under any quark rephasing, reduce to

$$\arg V_{ub} \simeq -64.4^\circ , \quad \arg V_{td} \simeq -21.8^\circ \quad (48)$$

in the special quark phasing, where

$$\begin{aligned} \arg V_{ud} &= 0 , & \arg V_{us} &= 0 , & \arg V_{ub} &\simeq -64.4^\circ , \\ \arg V_{cd} &= 180^\circ , & \arg V_{cs} &= 0 , & \arg V_{cb} &= 0 , \\ \arg V_{td} &\simeq -21.8^\circ , & \arg V_{ts} &= 180^\circ , & \arg V_{tb} &= 0 . \end{aligned} \quad (49)$$

Finally, we can evaluate the rest of magnitudes $|V_{ij}|$ of the Cabibbo–Kobayashi–Maskawa matrix elements (of them $|V_{us}|$ and $|V_{cb}|$ are used as inputs, while $|V_{ub}|$ was predicted). In fact, we obtain from Eqs (38)

$$\begin{aligned} |V_{cd}| &= |V_{us}| = 0.221 , \\ |V_{ts}| &= |V_{cb}| = 0.041 , \\ |V_{td}| &\simeq \frac{\alpha^{(d)} m_c}{\alpha^{(u)} m_s} |V_{ub}| \simeq 0.0083 , \end{aligned} \quad (50)$$

and from the perturbative unitarity of \widehat{V}

$$\begin{aligned} |V_{ud}| &\simeq 1 - \frac{1}{2}|V_{us}|^2 = 0.976 , \\ |V_{cs}| &\simeq 1 - \frac{1}{2}|V_{us}|^2 - \frac{1}{2}|V_{cb}|^2 = 0.975 , \\ |V_{tb}| &\simeq 1 - \frac{1}{2}|V_{cb}|^2 = 0.999 . \end{aligned} \quad (51)$$

Thus, in the case of convenient phasing (49), we predict the following approximate form of Cabibbo–Kobayashi–Maskawa matrix:

$$\widehat{V} \simeq \begin{pmatrix} 0.976 & 0.221 & 0.0034 e^{-i 64^\circ} \\ -0.221 & 0.975 & 0.041 \\ 0.0083 e^{-i 22^\circ} & -0.041 & 0.999 \end{pmatrix}. \quad (52)$$

Here, $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and $\arg V_{ub}$ can be considered as independent. As inputs we used the experimental values of $|V_{us}|$ and $|V_{cb}|$ as well as m_u , m_d , m_c , m_b and m_t . We predicted $|V_{ub}|$ and $\arg V_{ub}$ as well as m_s . From these 7 inputs we were also able to determine consistently 7 of all 7 + 1 independent parameters involved in the mass matrices $\widehat{M}^{(u,d)}$ (only the unphysical phase $\varphi^{(u)} + \varphi^{(d)}$ remained undetermined).

It is interesting to compare our perturbative form (38) of \widehat{V} with its convenient Wolfenstein parametrization (*cf. e.g.* Ref. [12]),

$$\widehat{V} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A \lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A \lambda^2 \\ A \lambda^3(1 - \rho - i\eta) & -A \lambda^2 & 1 \end{pmatrix} + O(\lambda^4), \quad (53)$$

being the base for the discussion of popular unitary triangle in the complex $\rho + i\eta$ plane:

$$V_{ub}^* + V_{td} = A \lambda^3 + O(\lambda^5) \simeq A \lambda^3. \quad (54)$$

This parametrization can be considered as an expansion in λ of the standard parametrization [4], where

$$V_{us} - O(\lambda^7) = s_{12} \equiv \lambda, \quad V_{cd} - O(\lambda^8) = s_{23} \equiv A \lambda^2,$$

$$V_{ub} = s_{13} e^{-i\delta} \equiv A \lambda^3(\rho - i\eta) \quad (55)$$

and $c_{ij} = \sqrt{1 - s_{ij}^2}$ ($s_{ij} > 0$ and $c_{ij} > 0$). Note that

$$V_{ts} = -A \lambda^2 + O(\lambda^4) \quad \text{and} \quad V_{td} = A \lambda^3(1 - \rho - i\eta) + O(\lambda^5).$$

When considering Eqs (53) and (52), we get

$$\begin{aligned} \lambda &\simeq 0.221, & A &\simeq 0.839, \\ \rho &\simeq 0.164, & \eta &\simeq 0.337, \\ \delta &\equiv \arctan \frac{\eta}{\rho} \simeq 64^\circ, & \beta &\equiv \arctan \frac{\eta}{1 - \rho} \simeq 22^\circ. \end{aligned} \quad (56)$$

In the unitary triangle (54), $\arg V_{ub}^* = \delta \equiv \gamma$ and $\arg V_{td} \simeq -\beta$ if $O(\lambda^5)$ is neglected in V_{td} [i.e., on the rhs of Eq. (54)]. According to Ref. [10], the present uncertainties of γ and β are

$$41^\circ < \gamma < 134^\circ, \quad 11^\circ < \beta < 27^\circ. \quad (57)$$

Our predictions (56) are consistent with these limits (however, in the future, our γ and β may lie at the new lower and upper experimental limit, respectively).

5. Summary

We proposed here the unified form (1) of mass matrix for all fundamental fermions: neutrinos, charged leptons, up quarks and down quarks. In this framework, their spectral differences are related only to the differences in values of the parameters involved, subject to the tentative constraints (2).

With some inputs, we obtained a number of predictions neatly consistent with available experimental data.

In the case of charged leptons e^- , μ^- , τ^- , from the inputs of m_e and m_μ , we predicted $m_\tau = 1776.80 \text{ MeV} + \Delta m_\tau$ with Δm_τ denoting a perturbative correction, quadratic in coupling constants, which measured the relative strength of the off-diagonal part of mass matrix *versus* its diagonal part. If the experimental value of m_τ was also taken as an input, then 3 of all 4 independent parameters in the charged-lepton mass matrix were consistently determined (only the phase $\varphi^{(e)}$ remained undetermined). This enabled us to evaluate (up to our ignorance of the phase $\varphi^{(e)}$) the charged-lepton contribution to the lepton Cabibbo–Kobayashi–Maskawa matrix.

In the case of neutrinos ν_e , ν_μ , ν_τ , from the atmospheric-neutrino inputs of $|m_{\nu_2}^2 - m_{\nu_1}^2| \sim (0.0003 \text{ to } 0.01) \text{ eV}^2$ and the $\nu_\mu \rightarrow \nu_\tau$ oscillation amplitude ~ 0.8 , we predicted $m_{\nu_0} \ll m_{\nu_1} \sim (0.6 \text{ to } 4) \times 10^{-2} \text{ eV}$ and $m_{\nu_2} \sim (0.2 \text{ to } 1) \times 10^{-1} \text{ eV}$. We were also able to evaluate (up to our ignorance of the phase $\varphi^{(\nu)}$) the neutrino contribution to the lepton Cabibbo–Kobayashi–Maskawa matrix and, taking into account the previous charged-lepton contribution, the whole matrix (up to an unknown phase, dependent on $\varphi^{(\nu)} - \varphi^{(e)}$). Then, the neutrino oscillations $\nu_e \rightarrow \nu_\mu$ and $\nu_e \rightarrow \nu_\tau$ (in the vacuum) got amplitudes $\sim 2_{-2}^{+3} \times 10^{-4}$ and $7_{-7}^{+11} \times 10^{-5}$, respectively, and $|m_{\nu_1}^2 - m_{\nu_0}^2| \sim (0.04 \text{ to } 1) \times 10^{-3} \text{ eV}^2$. So, while not fitted at all to the (unrealistic?) vacuum solution of the solar-neutrino problem, these oscillations require to be strengthened by the resonant MSW mechanism in the Sun matter to solve this problem.

In both lepton cases, the number of inputs was 5, and it was sufficient to determine consistently 5 of all $7 + 1$ independent parameters appearing

in the charged-lepton and neutrino mass matrices (the very small $\varepsilon^{(\nu)^2} \simeq 0$, the phase $\varphi^{(\nu)} - \varphi^{(e)}$ and the unphysical phase $\varphi^{(\nu)} + \varphi^{(e)}$ remained undetermined).

In the case of up and down quarks, from the inputs of $m_u, m_d, m_c = 1.3$ GeV, $m_b = 4.5$ GeV and m_t as well as $|V_{us}|$ and $|V_{cb}|$, we predicted $m_s \simeq 270$ MeV as well as $|V_{ub}/V_{cs}| \simeq 0.082$ and $\arg V_{ub} \simeq -64^\circ$. Hence, we were able to evaluate all elements of the Cabibbo–Kobayashi–Maskawa matrix.

In both quark cases the number of inputs was 7, and it was sufficient to determine consistently 7 of all $7 + 1$ independent parameters, involved in the up-quark and down-quark mass matrices (only the unphysical phase $\varphi^{(u)} + \varphi^{(d)}$ remained undetermined).

I am much indebted to Dr. Danuta Kielczewska for her advice about recent Super-Kamiokande data.

Appendix A

Unified “texture dynamics”

Let us introduce the following 3×3 matrices in the space of three fermion families:

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}. \quad (\text{A.1})$$

With the matrix

$$\hat{n} = \hat{a}^\dagger \hat{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad (\text{A.2})$$

they satisfy the commutation relations

$$[\hat{a}, \hat{n}] = \hat{a}, \quad [\hat{a}^\dagger, \hat{n}] = -\hat{a}^\dagger \quad (\text{A.3})$$

characteristic for annihilation and creation matrices, while \hat{n} plays the role of an occupation-number matrix. However, in addition, they obey the “truncation” identities

$$\hat{a}^3 = 0, \quad \hat{a}^{\dagger 3} = 0. \quad (\text{A.4})$$

Note that due to Eqs (A.4) the bosonic canonical commutation relation $[\hat{a}, \hat{a}^\dagger] = \hat{1}$ does not hold, being replaced by the relation $[\hat{a}, \hat{a}^\dagger] = \text{diag}(1, 1, -2)$.

In consequence of Eqs (A.1), (A.2) and (A.3), we get $\hat{n}|n\rangle = n|n\rangle$ as well as $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ and $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ ($n = 0, 1, 2$), however, $\hat{a}^\dagger|2\rangle = 0$ (*i.e.*, $|3\rangle = 0$) in addition to $\hat{a}|0\rangle = 0$ (*i.e.*, $|-1\rangle = 0$). Evidently,

$n = 0, 1, 2$ may play the role of a vector index in our three-dimensional matrix calculus.

It is natural to expect that the Gell–Mann matrices (generating the horizontal $SU(3)$ algebra) can be built up from \hat{a} and \hat{a}^\dagger . In fact,

$$\begin{aligned}
 \hat{\lambda}_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2} (\hat{a}^2 \hat{a}^\dagger + \hat{a} \hat{a}^{\dagger 2}), \\
 \hat{\lambda}_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2i} (\hat{a}^2 \hat{a}^\dagger - \hat{a} \hat{a}^{\dagger 2}), \\
 \hat{\lambda}_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2} (\hat{a}^2 \hat{a}^{\dagger 2} - \hat{a} \hat{a}^{\dagger 2} \hat{a}), \\
 \hat{\lambda}_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} (\hat{a}^2 + \hat{a}^{\dagger 2}), \\
 \hat{\lambda}_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} = \frac{1}{i\sqrt{2}} (\hat{a}^2 - \hat{a}^{\dagger 2}), \\
 \hat{\lambda}_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}} (\hat{a}^\dagger \hat{a}^2 + \hat{a}^{\dagger 2} \hat{a}), \\
 \hat{\lambda}_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} = \frac{1}{i\sqrt{2}} (\hat{a}^\dagger \hat{a}^2 - \hat{a}^{\dagger 2} \hat{a}), \\
 \hat{\lambda}_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} = \frac{1}{\sqrt{3}} (\hat{a} \hat{a}^\dagger - \hat{a}^\dagger \hat{a}), \\
 \hat{1} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{2} (\hat{a}^2 \hat{a}^{\dagger 2} + \hat{a} \hat{a}^{\dagger 2} \hat{a} + \hat{a}^{\dagger 2} \hat{a}^2). \tag{A.5}
 \end{aligned}$$

Inversely, $\hat{a} = (\hat{\lambda}_1 + i\hat{\lambda}_2)/2 + \sqrt{2}(\hat{\lambda}_6 + i\hat{\lambda}_7)/2$ and $\hat{a}^\dagger = (\hat{\lambda}_1 - i\hat{\lambda}_2)/2 + \sqrt{2}(\hat{\lambda}_6 - i\hat{\lambda}_7)/2$. A message we get from these relationships is that a horizontal field formalism, always simple (linear) in terms of $\hat{\lambda}_A$ ($A = 1, 2, \dots, 8$) and $\hat{1}$, is generally not simple in terms of \hat{a} and \hat{a}^\dagger . In particular, a nontrivial $SU(3)$ -symmetric horizontal formalism is not simple in \hat{a} and \hat{a}^\dagger . Inversely, a nontrivial horizontal field formalism, if simple (linear and/or quadratic and/or cubic) in terms of \hat{a} and \hat{a}^\dagger , cannot be $SU(3)$ -symmetric.

Now, let us consider the following ansatz [1]:

$$\widehat{M}^{(f)} = \widehat{\rho}^{1/2} \widehat{h}^{(f)} \widehat{\rho}^{1/2} \quad (f = \nu, e, u, d), \tag{A.6}$$

where

$$\widehat{\rho}^{1/2} = \frac{1}{\sqrt{29}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{24} \end{pmatrix} \tag{A.7}$$

and

$$\begin{aligned} \widehat{h}^{(f)} &= \mu^{(f)} \left[(1 + 2\widehat{n})^2 + (\varepsilon^{(f)})^2 - 1 \right] (1 + 2\widehat{n})^{-2} + \widehat{C}^{(f)} \\ &+ \left(\alpha^{(f)} \widehat{1} - \beta^{(f)} \widehat{n} \right) \widehat{a} e^{i\varphi^{(f)}} + \widehat{a}^\dagger \left(\alpha^{(f)} \widehat{1} - \beta^{(f)} \widehat{n} \right) e^{-i\varphi^{(f)}} \end{aligned} \tag{A.8}$$

with $\widehat{n} = \widehat{a}^\dagger \widehat{a}$ and

$$\widehat{1} + 2\widehat{n} = \widehat{N} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}, \quad \widehat{C}^{(f)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C^{(f)} \end{pmatrix}. \tag{A.9}$$

It is the matter of an easy calculation to show that the matrices (A.6) get explicitly the form (1).

In a more detailed construction following from our idea about the origin of three fermion families [1], each eigenvalue $N = 1, 3, 5$ of the matrix \widehat{N} corresponds (for any $f = \nu, e, u, d$) to a wave function carrying $N = 1, 3, 5$ Dirac bispinor indices: $\alpha_1, \alpha_2, \dots, \alpha_N$ of which one, say α_1 , is coupled to the external Standard Model gauge fields, while the remaining $N - 1 = 0, 2, 4$: $\alpha_2, \dots, \alpha_N$ are fully antisymmetric under permutations. So, the latter obey Fermi statistics along with the Pauli principle implying that really $N - 1 \leq 4$, because each $\alpha_i = 1, 2, 3, 4$. Then, the three wave functions corresponding to $N = 1, 3, 5$ can be reduced to three other wave functions carrying only one Dirac bispinor index α_1 (and so, spin 1/2),

$$\begin{aligned} \psi_{1\alpha_1}^{(f)} &\equiv \psi_{\alpha_1}^{(f)}, \\ \psi_{3\alpha_1}^{(f)} &\equiv \frac{1}{4} (C^{-1} \gamma^5)_{\alpha_2 \alpha_3} \psi_{\alpha_1 \alpha_2 \alpha_3}^{(f)} = \psi_{\alpha_1 12}^{(f)} = \psi_{\alpha_1 34}^{(f)}, \\ \psi_{5\alpha_1}^{(f)} &\equiv \frac{1}{24} \varepsilon_{\alpha_2 \alpha_3 \alpha_4 \alpha_5} \psi_{\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5}^{(f)} = \psi_{\alpha_1 1234}^{(f)}, \end{aligned}$$

and appearing with the multiplicities 1, 4, 24, respectively (the chiral representation is used here). In this argument, the requirement of relativistic covariance of the wave function (and the related probability current) is applied explicitly [1]. The weighting matrix $\widehat{\rho}^{1/2}$ as given in Eq. (A.7) gets

as its elements the square roots of these multiplicities, normalized in such a way that $\text{tr } \widehat{\rho} = 1$.

It is interesting to observe that in the case when *also* the index α_1 is not coupled to the Standard Model gauge fields, and so *all* $\alpha_1, \alpha_2, \dots, \alpha_N$ are fully antisymmetric under permutations, then (due to the Pauli principle) only the $N = 1$ bispinor of the three $N = 1, 3, 5$ bispinors may exist. It might correspond to a *sterile neutrino* of Dirac type, if it could participate in some oscillations joint with ν_e, ν_μ, ν_τ (*cf.* also the end of this Appendix for the preon option).

Note that all four matrices $\widehat{M}^{(f)}$ ($f = \nu, e, u, d$) defined by Eqs (A.6)–(A.9) and (A.1) have a common structure, differing from each other only by the values of their parameters $\mu^{(f)}, \varepsilon^{(f)2}, \alpha^{(f)}, \beta^{(f)}, C^{(f)}$ and $\varphi^{(f)}$. We propose the fermion mass matrices to be of this unified form. Then, Eqs (A.6) and (A.8) define a quantum mechanical model for the “texture” of mass matrices $\widehat{M}^{(f)}$ ($f = \nu, e, u, d$). Such an approach may be called “texture dynamics”.

The fermion mass matrix $\widehat{M}^{(f)}$, containing the kernel $\widehat{h}^{(f)}$ given in Eq. (A.8), consists of a diagonal part proportional to $\mu^{(f)}$, and of an off-diagonal part involving linearly $\alpha^{(f)}$ and $\beta^{(f)}$. The off-diagonal part of $\widehat{h}^{(f)}$ describes the mixing of three eigenvalues

$$\mu^{(f)} \left[N^2 + \left(\varepsilon^{(f)2} - 1 \right) N^{-2} + \delta_{N5} C^{(f)} \right] \quad (N = 1, 3, 5) \quad (\text{A.10})$$

of its diagonal part. Beside the term $\mu^{(f)} C^{(f)}$ that appears only for $N = 5$, each of these eigenvalues is the sum of two terms containing N^2 . They are: (i) a term $\mu^{(f)} N^2$ that may be interpreted as an “interaction” of N elements (“intrinsic partons”) treated on the same footing, and (ii) another term

$$\mu^{(f)} \left(\varepsilon^{(f)2} - 1 \right) P_N^2 \quad \text{with } P_N = N^{-1} = [N!/(N-1)!]^{-1}$$

that may describe an additional “interaction” with itself of one element arbitrarily chosen among N elements of which the remaining $N - 1$ are undistinguishable. Therefore, the total “interaction” with itself of this (arbitrarily) distinguished “parton” is $\mu^{(f)} [1 + (\varepsilon^{(f)2} - 1) N^{-2}]$, so it becomes $\mu^{(f)} \varepsilon^{(f)2}$ in the first fermion family.

It seems natural to conjecture that each “intrinsic parton” carries a Dirac bispinor index. In fact, such a possibility, as already described in the context of the weighting matrix (A.7), follows from our idea about the origin of three fermion families [1]. Then, for the (arbitrarily) distinguished “parton”, this index, considered in the framework of a fermion wave equation, is coupled to the external gauge fields of the Standard Model. Thus, this “parton” carries a set of Standard Model charges corresponding to $f = \nu, e, u, d$. For the

$N - 1$ undistinguishable “partons”, obeying Fermi statistics along with the Pauli principle, their Dirac bispinor indices are mutually coupled, resulting into Lorentz scalars, while their number $N - 1 = 0, 2, 4$ differentiates between three fermion families (for each $f = \nu, e, u, d$). These “partons” are free of Standard Model charges.

Evidently, the intriguing question arises, how to interpret two possible boson families corresponding to the number $N - 1 = 1, 3$ of undistinguishable “partons” [13]. In the present paper this problem is not discussed. Here, we would like only to point out that three fermion families $N = 1, 3, 5$ differ from these two hypothetic boson families $N = 2, 4$ by the full pairing of their $N - 1 = 0, 2, 4$ undistinguishable “partons”. So, the boson families, containing an odd number $N - 1 = 1, 3$ of such “partons”, might be considerably heavier.

A priori, the “intrinsic partons” may be either strictly *algebraic* objects providing fundamental fermions (leptons and quarks) with new family degrees of freedom, or may give us a signal of a new *spatial* substructure of fundamental fermions (built up of spatial “intrinsic partons” = preons). Our idea about the origin of three fermion families [1] chooses the first option. The difficult problem of new non-Standard-Model forces, responsible for the binding of N preons within fundamental fermions, does not arise in this option.

However, if the second option is true, then this irksome (though certainly profound) problem does arise and must be solved. It seems that in this case the most natural preon dynamics may be based (at the naïve phenomenological level) on a very strong and very shortrange effective attraction $\sum_{ij} V_{ij}^{(N)}$ ($i, j = 1, 2, \dots, N$) binding N spin-1/2 preons in some S-wave *ground* states (and *only* in such states). Among these preons, one is (arbitrarily) distinguished by carrying a set $f = \nu, e, u, d$ of Standard Model charges, while the remaining $N - 1$ are undistinguishable and obey Fermi statistics along with the Pauli principle. This implies (much as in the case of the first option) that $N - 1 \leq 4$ and so, $N - 1 = 0, 2, 4$ for the halfinteger total spin (that is then $1/2$), what is in consistency with the phenomenon of three fermion families. In particular, the fundamental fermion of the family $N = 1$ (*i.e.*, the lepton ν_e or e^- or quark u or d) is essentially nothing else as the (arbitrarily) distinguished preon, but dressed by the Standard Model radiative effects.

In view of Eq. (A.10), this attraction, jointly with the Standard Model radiative effects, should give

$$\begin{aligned}
 Z^{(f)} \langle Nm^{(N)} + \text{internal kinetic contribution} + \sum_{ij} V_{ij}^{(N)} \rangle^{(f)} \\
 = \mu^{(f)} \left[N^2 + \left(\varepsilon^{(f)2} - 1 \right) N^{-2} + \delta_{N5} C^{(f)} \right] > 0 \quad (\text{A.11})
 \end{aligned}$$

for the fermion $f = \nu, e, u, d$ from any family $N = 1, 3, 5$. Hopefully, the values $\mu^{(\nu)} \sim (0.6 \text{ to } 3) \times 10^{-3} \text{ eV}$, $\mu^{(e)} = 85.9924 \text{ MeV}$, $\mu^{(u)} \simeq 1060 \text{ MeV}$ and $\mu^{(d)} \simeq 218 \text{ MeV}$ as well as $C^{(u)} \simeq 175$ should be reasonably reproduced in terms of Standard Model characteristics carried by the (arbitrarily) distinguished “parton” within the fundamental fermion f (in any family N), as well as in terms of the preon (effective) mass $m^{(N)}$ and a few new parameters introduced through the function $V_{ij}^{(N)}$.

Whatever might be the origin of such a phenomenological shortrange attraction, this would be certainly an exciting physical problem, related or not to the (future) quantum gravitation.

Those of the preons that are free of Standard Model charges (and play within fundamental fermions the role of undistinguishable preons) may form a *novel* dark matter, transparent for *any* Standard Model interactions (and their supersymmetric variants). It is so, if they can appear also as *free* particles and/or as *sole* constituents of some new bound states. Evidently, such a Standard Model-dark matter is able to interact gravitationally.

A characteristic feature of the undistinguishable preons within fundamental fermions is that, though they carry no Standard Model charges, their configurations $N - 1 = 0, 2, 4$ corresponding to the families $N = 1, 3, 5$ can mix in charge-changing weak interactions. In the case of neutrinos, this implies neutrino oscillations (while for charged leptons the mass and weak-interaction states are identical).

One might go a (bold) step further and ask, if a free preon of those carrying no Standard Model charges could participate in some oscillations together with the *conventional neutrinos* ν_e, ν_μ, ν_τ (which contain one Standard-Model-active preon $f = \nu$ and $N - 1 = 0, 2, 4$ Standard-Model-sterile preons of the same sort as the considered free preon). If it could, it would be nothing else as a massive *sterile neutrino* of Dirac or Majorana type. Note that here the conventional neutrinos ν_e, ν_μ, ν_τ are of Dirac type and so, though they are no mass states, include both the (active) left-handed and (sterile) right-handed components, the latter coupled only through mass terms (*via* the righthanded components of mass states). Thus, in this (intriguing) case, the novel dark matter would consist just of sterile neutrinos (and/or their bound states), although such a sterile neutrino would be an additional one, different from the sterile ν_{eR} or, more correctly, sterile $\nu_{eR} + (\nu_{eR})^c$ (to speak of possible mass states, of Majorana type in this example). Here, $(\nu_{eR})^c = (\nu_e^c)_L \neq \nu_{eL}$. Then, when bound within fundamental fermions $f = \nu, e, u, d$ from families $N = 1, 3, 5$, such sterile neutrinos would play also the role of $N - 1 = 0, 2, 4$ undistinguishable preons.

Appendix B

Lepton Cabibbo–Kobayashi–Maskawa matrix

Let us consider the lepton Cabibbo–Kobayashi–Maskawa matrix $\widehat{V} = (V_{ij})$ ($i, j = 0, 1, 2$) as given in Eqs (13) involving four parameters X , $\alpha^{(e)}$, $\varphi^{(\nu)}$ and $\varphi^{(e)}$ (if $\beta^{(e)}$ is neglected *versus* $\alpha^{(e)}$). We are able to ascribe some values only to two of these parameters: $X \sim (\sqrt{5} - 1)/2 = 0.618$ and $(\alpha^{(e)}/\mu^{(e)})^2 = 0.020_{-0.020}^{+0.029}$ with $\mu^{(e)} = 85.9924$ MeV, what gives the central value $\alpha^{(e)} = 12$ MeV [*cf.* Eqs (17) and (6)]. Then,

$$\begin{aligned}
 |V_{01}| &= \frac{2}{29} \frac{\alpha^{(e)}}{m_\mu} = 0.0079, \\
 |V_{10}| &= \frac{2}{29\sqrt{1+X^2}} \frac{\alpha^{(e)}}{m_\mu} = 0.0057, \\
 |V_{12}| = |V_{21}| &= \left[\frac{X^2}{1+X^2} + \frac{192}{871(1+X^2)} \left(\frac{\alpha^{(e)}}{m_\tau} \right)^2 \right. \\
 &\quad \left. - \frac{16\sqrt{3}X}{29\sqrt{1+X^2}} \frac{\alpha^{(e)}}{m_\tau} \cos(\varphi^{(\nu)} - \varphi^{(e)}) \right]^{1/2} = 0.53, \\
 |V_{02}| &= 0, \\
 |V_{20}| &= \frac{2X}{29\sqrt{1+X^2}} \frac{\alpha^{(e)}}{m_\mu} = 0.0035, \\
 |V_{00}| &= 1, \\
 |V_{11}| = |V_{22}| &= \left[\frac{1}{1+X^2} + \frac{192X^2}{871(1+X^2)} \left(\frac{\alpha^{(e)}}{m_\tau} \right)^2 \right. \\
 &\quad \left. - \frac{16\sqrt{3}X}{29\sqrt{1+X^2}} \frac{\alpha^{(e)}}{m_\tau} \cos(\varphi^{(\nu)} - \varphi^{(e)}) \right]^{1/2} = 0.85, \tag{B.1}
 \end{aligned}$$

where the numbers correspond to the central value of $\alpha^{(e)}$. Note that in the third and seventh Eq. (B.1)

$$\frac{16\sqrt{3}X}{29\sqrt{1+X^2}} \frac{\alpha^{(e)}}{m_\tau} \cos(\varphi^{(\nu)} - \varphi^{(e)}) = 0.0034 \cos(\varphi^{(\nu)} - \varphi^{(e)}) \tag{B.2}$$

and so, it is negligible, giving practically no chance for determining $\varphi^{(\nu)} - \varphi^{(e)}$ from a (future) experimental value of $|V_{12}|$. If $\alpha^{(e)} = 0$, then only the elements

$$\begin{aligned} |V_{12}| &= |V_{21}| = \frac{X}{\sqrt{1+X^2}} = 0.53, \\ |V_{00}| &= 1, \\ |V_{11}| &= |V_{22}| = \frac{1}{\sqrt{1+X^2}} = 0.85 \end{aligned} \quad (\text{B.3})$$

remain different from zero.

In terms of an unknown phase difference $\varphi^{(\nu)} - \varphi^{(e)}$, we can evaluate from Eq. (13) the following CP-violating phase:

$$\arg(V_{10}^* V_{21}^* V_{20} V_{11}) = (\arg V_{12} - \varphi^{(\nu)} - 180^\circ) + \arg V_{11}, \quad (\text{B.4})$$

where

$$\tan(\arg V_{12} - \varphi^{(\nu)} - 180^\circ) = \frac{8\sqrt{3}(\alpha^{(e)} - \beta^{(e)}) \sin(\varphi^{(\nu)} - \varphi^{(e)})}{29m_\tau X - 8\sqrt{3}(\alpha^{(e)} - \beta^{(e)}) \cos(\varphi^{(\nu)} - \varphi^{(e)})} \quad (\text{B.5})$$

and

$$\tan(\arg V_{11}) = \frac{8\sqrt{3}(\alpha^{(e)} - \beta^{(e)}) X \sin(\varphi^{(\nu)} - \varphi^{(e)})}{29m_\tau + 8\sqrt{3}(\alpha^{(e)} - \beta^{(e)}) X \cos(\varphi^{(\nu)} - \varphi^{(e)})} \quad (\text{B.6})$$

depend on $\varphi^{(\nu)} - \varphi^{(e)}$. Since the lhs of (B.4) is invariant under any lepton rephasing, it reduces to

$$\arg V_{20}^{\text{new}} = \arg V_{12} - \varphi^{(\nu)} - 180^\circ \quad (\text{B.7})$$

in the special lepton phasing, where

$$\begin{aligned} \arg V_{00}^{\text{new}} &= 0, & \arg V_{01}^{\text{new}} &= 0, \\ \arg V_{10}^{\text{new}} &= 180^\circ, & \arg V_{11}^{\text{new}} &= \arg V_{11}, & \arg V_{12}^{\text{new}} &= 0, \\ & & \arg V_{21}^{\text{new}} &= 180^\circ, & \arg V_{22}^{\text{new}} &= -\arg V_{11}. \end{aligned} \quad (\text{B.8})$$

Here, $\arg V_{02}^{\text{new}}$ is irrelevant because of $|V_{02}^{\text{new}}| = |V_{02}| = 0$, while $\arg V_{20}^{\text{new}}$ is given as in Eq. (B.7).

Thus, we predict the following approximate form of lepton Cabibbo–Kobayashi–Maskawa matrix:

$$\widehat{V} \simeq \begin{pmatrix} 1 & 0.0079 & 0 \\ -0.0057 & 0.85 \exp(i \arg V_{11}^{\text{new}}) & 0.53 \\ 0.0035 \exp(i \arg V_{20}^{\text{new}}) & -0.53 & 0.85 \exp(-i \arg V_{11}^{\text{new}}) \end{pmatrix} \quad (\text{B.9})$$

with $\arg V_{20}^{\text{new}} = \arg V_{12} - \varphi^{(\nu)} - 180^\circ$ and $\arg V_{11}^{\text{new}} = \arg V_{11}$ expressed in terms of $\varphi^{(\nu)} - \varphi^{(e)}$ as in Eqs (B.5) and (B.6). If $\varphi^{(\nu)} - \varphi^{(e)} = 0$, then in Eq. (B.9) the CP-violating phases vanish, $\arg V_{20}^{\text{new}} = 0$ and $\arg V_{11}^{\text{new}} = 0$, what leads to a real matrix \widehat{V} . In general, a nonreal matrix \widehat{V} could violate CP-parity, but only in processes, where neutrino mass states (instead of their weak-interaction states) might be detected experimentally. The expected neutrino oscillations are processes, where (nondegenerate) neutrino mass states may be detected indirectly. There, CP-parity is generally violated [*cf.* Eq. (16)]. If $\alpha^{(e)} = 0$, then

$$\widehat{V} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.85 & 0.53 \\ 0 & -0.53 & 0.85 \end{pmatrix} \quad (\text{B.10})$$

in the lepton phasing (B.8). With the transformation $\nu_i^{(\text{m})} = \sum_j V_{ij} \nu_j$, we can express explicitly the neutrino mass states through their weak-interaction states [*cf.* Eq. (14)].

We can see that the lepton matrix \widehat{V} has physically a different structure than the quark matrix \widehat{V} evaluated in Eq. (52): the former gives strong mixing of leptons ν_μ and ν_τ from the second and third family, while for the latter rather the quarks d and s from the first and second family are strongly mixed. This difference, however, follows from only quantitative difference in lepton and quark couplings [*cf.* Eqs (2)] in the unified fermion mass matrix (1).

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