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EFFECTS OF THE PION WAVE DISTORTION ON THE ABSORPTION/EMISSION MECHANISM OF THE DCX REACTION ON ^{56}Fe

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We have studied the effects of pion wave-function distortion on the absorption/emission mechanism cross-section of the pionic double charge exchange reaction on ^{56}Fe. We are using pion-nucleus optical potential and quasiparticle proton–neutron random phase approximation formalisms. We confirm the resonant behaviour of the foward cross-section at around 50 MeV, opposite to the plane wave-function results.

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1. Introduction

Recently the observation of a resonance-like behaviour of the forward DCX (double charge exchange) cross-section around the pion kinetic energy $T_\pi = 50\,\mathrm{MeV}$ raised the question of possible existence of a d' dibaryon [1], proposed to explain this phenomenon. We have already shown [2] that the use of *conventional* two-nucleon mechanism with distorted pion wave-function could do the job as well.

We use the framework of Quasiparticle Random Phase Approximation, very successful in describing nucleon–nucleon correlations — the phenomenon that plays crucial role in the DCX process.

We perform the calculations on ^{56}Fe for the reasons that it is heavy enough to expect that the QRPA gives reliable results and recent experimental data are available. [3, 4]. Last but not least we want to confirm our results obtained on the tellurium ^{128,130}Te nuclei [2].

In the next section we recapitulate shortly the formalism, point out new elements regarding distortion (Section 2), present details and results of the calculations together with their discussion (Section 3) and finally draw some conclusions (Section 4).

2. Formalism

2.1. Klein–Gordon equation and optical potential

We follow closely the formalism developed and presented in a series of previous papers [5, 6], taking QRPA to describe the DCX process and extending it to account for distortion effects [2]. The wave-function of the incoming and outgoing pions ϕ are solutions of Klein–Gordon equation [7] with pion–nucleus optical potential consisting of nuclear (U_N) and Coulomb (U_C) parts. Making the partial-wave expansion of these functions:

$$\phi(\boldsymbol{k},\boldsymbol{r}) = 4\pi \sum_l \mathrm{i}^l \frac{u_l(r)}{r} \sum_m Y_{lm}(\vartheta_k,\varphi_k) Y_{lm}(\vartheta,\varphi), \tag{1}$$

one can find the following equation for each partial wave u_l:

$$\left[-\frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{r^2}\right] u_l(r) = \left[k^2 - 2\xi(U_\mathrm{C} + U_\mathrm{N}) + U_\mathrm{C}^2\right] u_l(r). \tag{2}$$

From the pion–nucleon scattering amplitude one derives the first-order pion–nucleus optical potential in the so-called Kisslinger form. Taking into account effects of Pauli blocking, polarization of the nucleonic medium by pions (Lorentz–Lorenz—Ericson–Ericson correction) and true pion absorption on a pair of nucleons the complete second-order optical potential emerges [8–10] and equation (2) is to be solved numericaly.

2.2. Distorted wave DCX amplitudes

The DCX reaction takes place generally on two correlated nucleons. Because of the weakness of this reaction we assume that all other nucleons play an indirect role through wave distortion and other medium effects. In recent work we considered only the contribution of pion absorption/emission (so-called p-wave mechanism) on the correlated nucleon pair (Fig. 1). As shown by Koltun and Singham [11] the absorption on $T = 1$ pairs contributes substantially to DCX in the low energy region, especially at resonance. Therefore, we limit our attention to influence of the pion distortion on this stronger mechanism only.

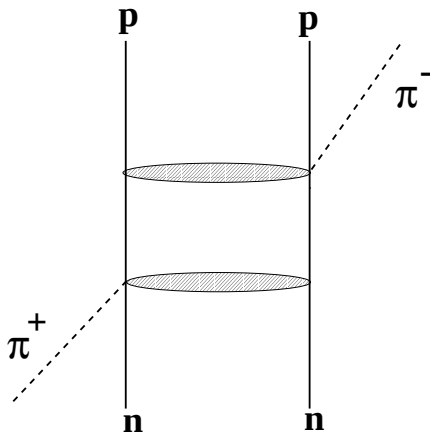

Fig. 1. Conventional two-nucleon absorption/emission mechanism of the DCX reaction.

The differential cross-section of the DCX reaction is expressed by the total amplitude:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\vartheta_{kk'}) = \left| \frac{1}{4\pi} \sum_{J^\pi} F^p_{J^\pi}(\boldsymbol{k}, \boldsymbol{k}') \right|^2 , \tag{3}$$

where $\boldsymbol{k}$ and $\boldsymbol{k}'$ are momenta of the incoming and outgoing pions, respectively. The partial amplitude $F^p_{J^\pi}(\boldsymbol{k}, \boldsymbol{k}')$ is connected with p-wave pion–nucleon effective Hamiltonian of the general form:

$$h_p(\boldsymbol{k}) = -\sqrt{2\pi}\mathrm{i}\frac{f}{m_\pi} \sum_{pn} \left[\int \mathrm{d}\boldsymbol{x} \Psi^\star_p(\boldsymbol{x}) \vec{\sigma} \cdot \boldsymbol{\nabla} \phi_\pi(\boldsymbol{k}, \boldsymbol{x}) \Psi_n(\boldsymbol{x}) \right] \hat{c}^\dagger_p \hat{c}_n, \tag{4}$$

describing the elementary process of absorption/emission of a pion with a momentum $\boldsymbol{k}$. Here $\Psi_{p(n)}(\boldsymbol{x})$ is the solution of Schrödinger equation for the chosen average nuclear potential (harmonic oscillator or Woods–Saxon). Taking into account the partial wave expansion of the pion wave–function (1) we obtain for (4) the expression of the form:

$$h_p(\boldsymbol{k}) = -\sqrt{48\pi}\mathrm{i}\frac{f}{m_\pi} \sum_{pn} \sum_{JM} (-1)^{\frac{l_n + l_p + J}{2}} Y^*_{JM}(\vartheta_k, \varphi_k) G^J_{pn} \mathcal{R}^{JM}_{pn}, \tag{5}$$

where $\mathcal{R}^{JM}_{pn} = [\hat{c}^\dagger_p \hat{c}_n]_{JM}$ is the transition density operator and f is the constant determined to reproduce the experimental data for pion–nucleon elastic scattering [12].

The nuclear form-factor is modified with respect to [5] to take into account the distortion of the pion wave-function [6]. Its explicit form can be found in [2, 5].

The DCX transition amplitude can be expressed as follows [5, 6]

$$F^p_{J^\pi}(\boldsymbol{k},\boldsymbol{k}') = \sum_{M,mm'} \langle 0^+_{\rm f};\pi^-(\boldsymbol{k}')|h^p(\boldsymbol{k}')|\overline{m',J^\pi M}\rangle\langle\overline{m',J^\pi M}|m,J^\pi M\rangle \times \langle m,J^\pi M|h^p(\boldsymbol{k})|0^+_{\rm i};\pi^+(\boldsymbol{k})\rangle \left[E_i+\omega-\frac{1}{2}(E^m_{J^\pi}+\overline{E}^{m'}_{J'^{\pi'}})\right]^{-1} + \{\text{crossed term}\}, \quad (6)$$

where $|0^+_{\rm i};\pi^+(\boldsymbol{k})\rangle$ and $|0^+_{\rm f};\pi^-(\boldsymbol{k}')\rangle$ are correspondingly the initial and final ground states of the target (A,Z) and daughter $(A,Z+2)$ nuclei together with incoming and outgoing pion states of the momenta $\boldsymbol{k}$ and $\boldsymbol{k}'$, $\omega = \sqrt{k^2+m^2_\pi}$ is the incident pion energy, $|m,J^\pi M\rangle$ and $|\overline{m',J^\pi M}\rangle$ are the intermediate nuclear states constructed as one-phonon QRPA excitations from the initial and final nuclei, $E^m_{J^\pi}$ and $\overline{E}^{m'}_{J'^{\pi'}}$ are their corresponding energies and E_i is the ground-state energy of the initial nucleus. The details of the QRPA formalism and final expressions that result by using the explicit form of the states and operators are quite lengthy and can be found in [5, 6] so we do not quote them here again.

3. Details of the calculations, results and discussion

We assumed the single-particle basis comprising $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $0d_{3/2}$, $1s_{3/2}$, $0d_{7/2}$, $1p_{1/2}$, $0d_{5/2}$, $1p_{1/2}$, $0f_{9/2}$, $1d_{5/2}$, $2s_{1/2}$ isotropic harmonic oscillator levels both for protons and neutrons (without a core). This basis is big enough to guarantee convergence of the results while maintaining reasonable computational time limits. The single-particle energies were obtained from a Coulomb-corrected Woods–Saxon potential[1] and the nuclear G-matrix elements were calculated by solving the Bethe–Goldstone equation with the realistic Bonn B potential with one-boson exchange [13]. We followed the standard procedure [5, 6] renormalizing the bare nuclear matter two-body G-matrix elements to fit the experimental pairing gaps ($g^{\rm pair}$ factors, for the pairing matrix elements). The results of the fitting are listed in Table I. Because of the lack of good experimental data for the energy of the Gamow–Teller resonance in the intermediate nucleus (^{56}Ni) we have not renormalized the particle–hole part of the interaction and set $g_{\rm ph}=1$. Anyhow, this factor influences the DCX amplitude only very slightly.

There is, however, no simple way to fit the particle-particle part of the interaction and, as previously, the corresponding renormalization factor $g_{\rm pp}$

[1] However, to simplify the calculation of radial integrals by an analytic formula, we use the corresponding wave functions of a harmonic oscillator — the resulting corrections to the cross-section are at the level of 1%.

TABLE I

The experimental pairing gaps calculated using the atomic mass evaluation of Audi and Wapstra [14] by the prescription of Moeller and Nix [15] and the corresponding renormalization constants of the pairing G-matrix elements.

	Nucleus	
	^{56}Fe	^{56}Ni
$\Delta_{\rm p}^{\rm exp}$ [keV]	1571.6	2078
$\Delta_{\rm n}^{\rm exp}$ [keV]	1425.1	2150
$g_{\rm p}^{\rm pair}$	1.015	1.038
$g_{\rm n}^{\rm pair}$	1.011	1.070

is treated as a parameter of the theory. Therefore we will discuss the reaction observables as a function of $g_{\rm pp}$. Its physical value should be, however, close to that of $g_{\rm ph}$ due to the correspondence between the particle–particle and particle–hole matrix elements (Pandya transformation). For the calculations

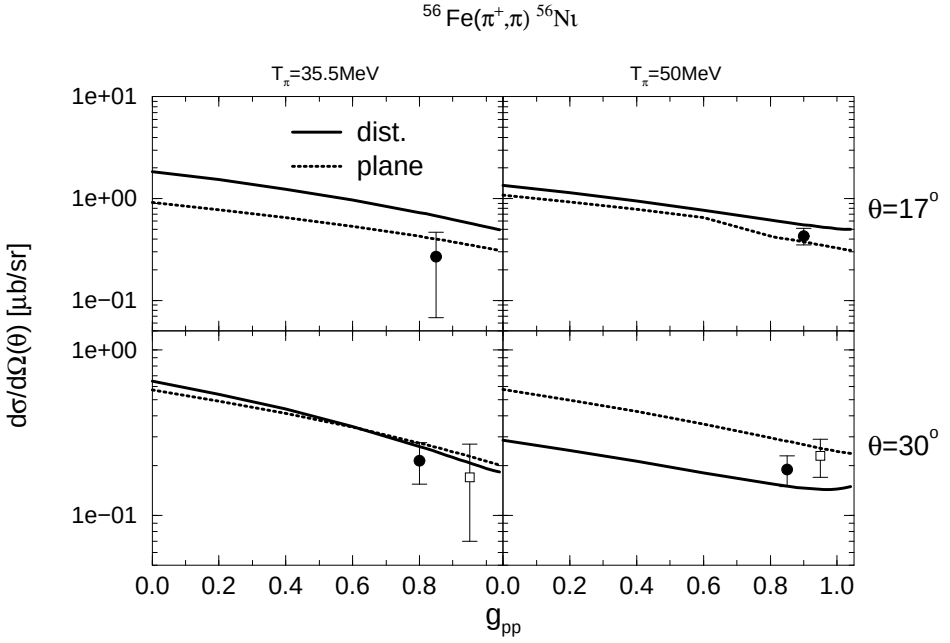

Fig. 2. Calculated differential cross-sections as a function of the particle–particle strength $g_{\rm pp}$ with comparison to experimental data (□) [1, 4], (•) [4]. Dashed curve is the result of the plane-wave approximation, the solid one with inclusion of distortion effects.

we use the parameters of the optical potential determined by a fit to data of pionic atoms, to π^+ elastic scattering and $\pi^\pm$ absorption on different targets over a broad energy range [7, 9]. The parameters are linearly interpolated where necessary. The data for the nuclear densities (two–parameter Fermi model) for ^{56}Fe are taken from [16].

Figure 2 shows the p-wave differential cross-section as a function of the particle–particle strength $g_{\rm pp}$ at $\vartheta = 17^\circ$ and $\vartheta = 30^\circ$ for the pion kinetic energy $T_\pi \cong 35.5\,\mathrm{MeV}$ and $T_\pi \cong 50\,\mathrm{MeV}$ and the corresponding data from measurements of the Tübingen experimental group at the Paul Scherrer Institute [1,4] (open squares □) and [4] (filled circles •). It seems at the first glance that plain p-wave calculations explain better the experimental data, but the Pandya relation between particle–article and particle–hole matrix elements suggests $g_{\rm pp} \approx g_{\rm ph} = 1$ and at this region the distorted p-wave calculation results are also within 1σ experimental error bars.

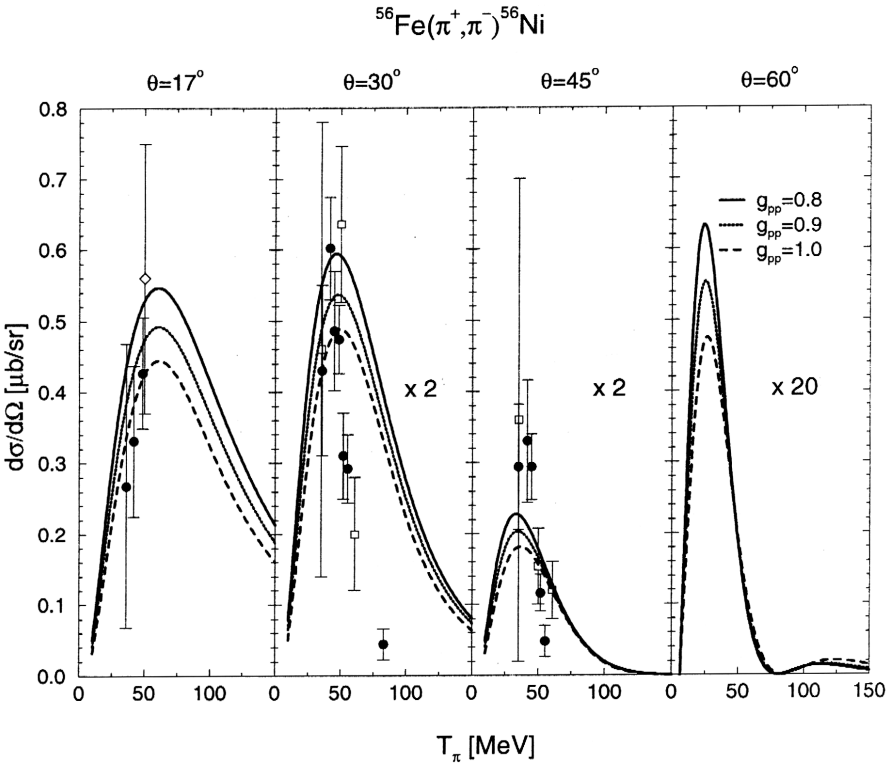

Fig. 3. Differential cross-sections for the plane-wave approximation as a function of the pion kinetic energy T_π calculated for the different values of the particle–particle strength $g_{\rm pp}$ and different scatering angles. Experimental data are taken from [1,4] (□), [4] (•) and [19] (◇).

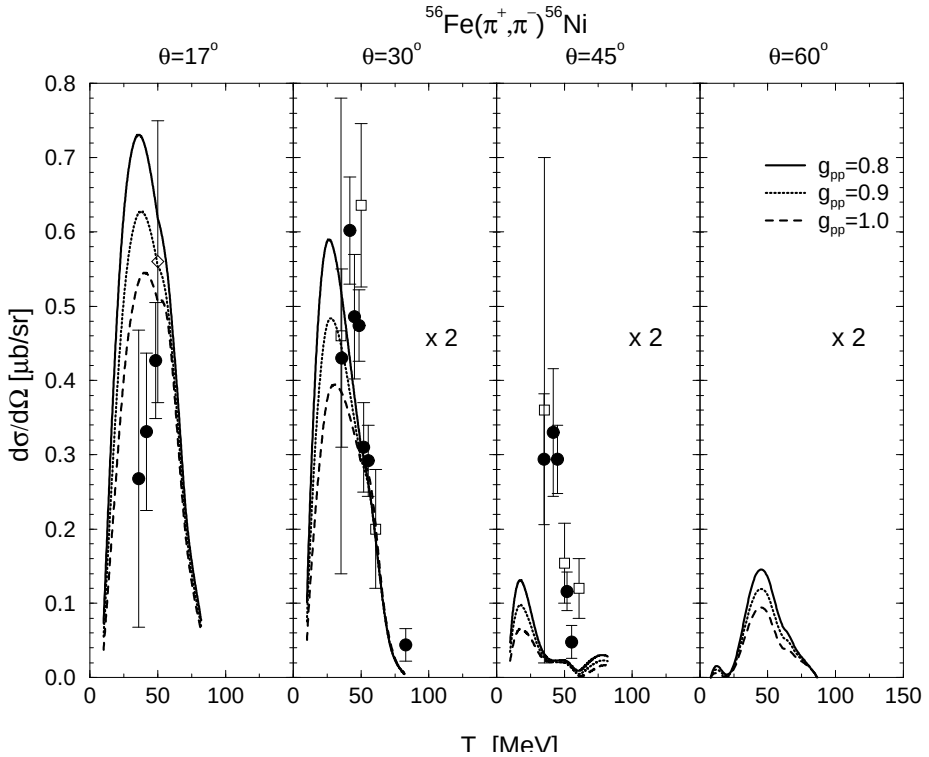

Fig. 4. Differential cross-sections for the distorted-wave approximation, calculated for the same parameters as in Fig. 3.

There is, however, much difference between these approaches what concerns the pion kinetic energy dependence of the differential cross-section. With plane waves (Fig. 3) the maximum of the energy distribution appears for $T_\pi \approx 50\,\mathrm{MeV}$ but it is too broad and does not account for the reduction of the cross-section in the high-energy region of 80–100 MeV, as suggested by the experimental data on ^{56}Fe and other nuclei (*cf. e.g.* Fig. 1 in [1]). With the distorted waves (Fig. 4) a more resonance-like shape is obtained and the maximum is shifted towards lower energies of about 35–40 MeV when calculated for smaller g_{pp} values. For $\vartheta = 30^\circ$ the distorted-wave calculations agree excellently with the experimental data, where the plane-wave calculations fail. Some discrepancy is seen for other angles, especially for $\vartheta = 45^\circ$, where the theoretical results underestimate the experiment. The source of this discrepancy can be seen while analyzing the angular dependence of the cross-section (Fig. 5). The lower the pion kinetic energy, the more forward-peaked the theoretical results (both plane-wave and distorted-wave) and the flatter the experimental data are. This calls cleanly for the necessity of inclusion of the s-wave sequential mechanism of the DCX reaction,

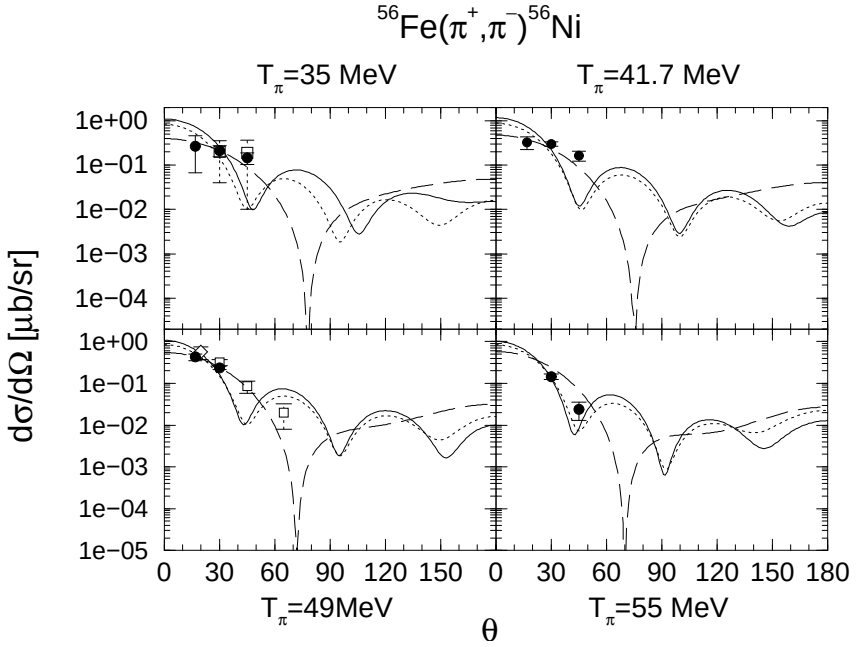

Fig. 5. Dependence of the differential cross-sections on the scattering angle ϑ calculated for different pion kinetic energy T_π within plane-wave approximation (dashed curve) and distorted-wave formalism (solid curve: $g_{pp} = 0.8$, dotted: $g_{pp} = 1.0$). Experimental data: (□) [1, 4], (•) [4] and (◇) [19].

that should gain strength for lower energies and interfere with the p-wave (absorption/emission) mechanism giving flatter angular distributions. For higher energies the latter mechanism prevails and already for $T_\pi = 55\,\mathrm{MeV}$ explains the experimental data rather well. An interesting difference between both approaches is seen between $\vartheta = 65°$ and 70°, where there is a local maximum for distorted-wave calculations and a minimum for the plane-wave ones. There exist one measurement due to Bilger [3] at $T_\pi = 50\,\mathrm{MeV}$, $\vartheta = 65.2°$ giving the value of the cross-section of 0.020 ± 0.012. This is much closer to the distorted-wave result than to the plane-wave calculations, that again shows that the latter do not account for all the physics governing the DCX process.

4. Conclusions, remarks and outlook

We have shown the importance of the distortion of the pion wave-functions for DCX reaction cross-section calculated in the QRPA framework. With our model we are able to describe semi-quantitatively recent experimental

data on ^{56}Fe. The gross features of the resonance-like shape of the cross-section as a function of pion energy can be reproduced at least qualitatively in terms of the conventional two-nucleon mechanism without invoking exotic mechanisms like dibaryons or multiple quark clusters. Nevertheless, as it can be infered from the angular distribution of the cross-section, the inclusion of the s-wave (sequential) mechanism is necessary.

For higher pion kinetic energies (above around 100 MeV) one should account for 2p–2h correlations that, most likely, constitute the most significant mechanism in building up collectivity in DCX processes. But in this region Δ-isobar degrees of freedom start playing a role in the reaction. Therefore, more elaborate theory that presented in this paper is necessary to push the calculations further up in energy. We did not attempt to do it here, anyway.

The question how far dibaryon mechanism competes with the conventional one is therefore still open. Further additions and improvements to the theory can be done, *e.g.* application of full QRPA with proton–neutron pairing [17] and/or proper treatment of Pauli principle within renormalized QRPA (RQRPA) formalism [18], that are expected to reduce the $g_{\rm pp}$ dependence of the results. The corresponding calculations are under way. We also plan to extend our study onto other cases where experimental data are available (*e.g.* Ca isotopes) or double-beta decay nuclei like ^{76}Ge.

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