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WORMHOLES AND TIMETRAVEL*

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Dedicated to Andrzej Trautman in honour of his 64th birthday

We discuss a construction of a wormhole with the following properties: the wormhole connects to the same asymptotic region and is one-way traversable *i.e.* there exist timelike curves that start and end in the same asymptotic region and go through the wormhole. Moreover it is possible to satisfy the energy conditions. From any point in the asymptotic region there exist closed timelike curves.

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1. Introduction

In one of his books, the science fiction author Lem [1] describes an astronaut, traveling alone in his spaceship with a broken steering which he is unable to repair. The spacecraft is sucked into a strong gravitational field and suddenly there is a second person in the spaceship who turns out to be the same astronaut but from the next day. Then a discussion starts whether or not it makes sense to try to repair the spaceship together.

Already Einstein [2] worried that his theory of relativity might allow for spacetimes with closed timelike curves. Gödel [3] constructed a cosmological model where this phenomena can happen. More recently renewed interest focussed on the possibility of constructing such time-machines with the help of "wormholes". Wormholes are spacetimes with non-trivial topology in which a kind of tunnel exists connecting distant parts in the universe. These wormholes may not only serve as shortcuts in space but some also

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for timetravel. That the famous Schwarzschild black hole contains a wormhole was realized only one year after Einstein formulated his field equations: Flamm [4] recognized that the hypersurfaces of constant Killing time through Schwarzschild spacetime, when embedded in Euclidean space, may be illustrated by paraboloiods, revealing that these hypersurfaces consist of two asymptotic flat sheets. These sheets are connected by a narrow throat, described as a "bridge" by Einstein and Rosen [5] in 1935. The notion of "wormhole" was introduced in 1955 by Wheeler [6] in his famous book on Geometrodynamics.

The Schwarzschild wormhole however is non-traversable: the throat shrinks under its own gravitational attraction to a singularity thereby preventing the two asymptotic regions from being causally connected. Motivated by this deficiency Morris and Thorne [7] constructed traversable wormholes and pointed out that these may be used as time-machines. However, from general properties one can show that in these constructions anti-gravitating matter has to be present. In the last decade a large number of papers studied the properties of traversable wormholes. On the one hand the violations of the energy conditions for matter at the throat and on the other the occurrence of closed causal curves [8–18]. Most of the wormholes discussed in the literature are spherically symmetric and connect to distinct asymptotically flat regions. If however the wormhole is to connect to the same asymptotic of the universe, spacetime can be at best axially symmetric. Moreover, gravitational attraction will pull the two mouths of the hole together and spacetime will not be static.

In what follows we elaborate on a construction of a wormhole recently given by the authors [19] that overcomes the above mentioned difficulties: The wormhole is static, (one-way) traversable and connects to the same asymptotics. Moreover the energy conditions can be satisfied. On this spacetime closed timelike curves exist from any point in the asymptotic region.

There exist two important theorems about the existence of wormholes: Hawking [18] in his paper on chronology protection showed, loosely speaking, that for the construction of a time-machine one necessarily needs to violate the energy conditions. Friedman *et al.* [20] on the other hand proved a "topology protection theorem", by which it is impossible, under certain assumptions, to probe the non-trivial topology *i.e.* traveling or sending light rays through the wormhole from the asymptotic region. Both theorems do not apply to our construction: Hawking's theorem refers to spacetimes where closed causal curves exist from a certain time on (or up to), while our solution is an eternal time-machine. Friedman's conclusion require that spacetime is globally hyperbolic, a requirement which is obviously not met by our construction.

The exterior region of the wormhole is that of two charged shells held in static equilibrium by their electric repulsion. The interior is a Reissner–Nordström black hole. The transition between the two regions is achieved by two shells of charged matter. The matching can be made exact by applying the image method in analogy to the construction given by Lindquist [21], who considered the time-symmetric initial value problem for Einstein–Rosen manifolds.

2. The exterior solution

The exterior field of a system of charged bodies which are held in equilibrium by a balance between electrostatic repulsion and gravitational attraction is given by the Majumdar–Papapetrou solution of Einstein–Maxwell equations. In Cartesian coordinates this solution has the form

$$ds^2 = -V^{-2}dT_+^2 + V^2(dx^2 + dy^2 + dz^2)\,. \tag{1}$$

With an appropriate ansatz for the electromagnetic potential, $A = \pm\frac{1}{V}dT_+$, the Einstein–Maxwell field equations reduce to the three dimensional, flat space Laplace equation,

$$\triangle V(x,y,z) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) V(x,y,z) = 0\,. \tag{2}$$

This fact offers the possibility to overcome the problem of axisymmetry and hence to construct an exact wormhole solution: We cut out from the Majumdar–Papapetrou spacetime the interior of the histories of two (non-intersecting) spheres $S_i^+ (i = 1, 2)$ and require the potential function V to be constant on the surfaces. The problem is analogous to that of finding the electric potential outside two charged metal spheres. Such solutions can be found for any location and radii and arbitrary values of the potential on the spheres [22].

In what follows we give an explicit construction for the symmetric two-body problem. We choose the z-axis to point along the line of symmetry joining the two spheres S_i^+ with radii R and center them at $z = \pm d_1$. Moreover, we fix the value of the potential function on the spheres to

$$V|_{S_i^+} = V_0 = 1 + \frac{m_1}{R}. \tag{3}$$

In addition, we assume that $V(\vec{x})$ tends to one for $|\vec{x}| \to \infty$. This choice ensures that for large distances of the two spheres the field is that of two particles with $mass = charge = m_1$. The image masses m_n to make V constant on S_i^+ have to be located on the z-axis at $z = \pm d_n$, where

$$d_n = d_1 \; - \; \frac{R^2}{d_1 + d_{n-1}} \quad (n > 1)\,, \tag{4}$$

$$m_n = \; - \; \frac{m_{n-1} R}{d_1 + d_{n-1}} \quad (n > 1)\,. \tag{5}$$

The resulting expression for the metric potential $V(\vec{x})$, $\vec{x} = (x, y, z)$, is

$$V(\vec{x}) = 1 + \sum_{n=1}^{\infty} \left(\frac{m_n}{|\vec{x} + \vec{d}_n|} + \frac{m_n}{|\vec{x} - \vec{d}_n|} \right). \tag{6}$$

3. Wormhole geometry

Having found the exterior solution we match a Reissner–Nordström black hole to the interior of the spheres. This requires the introduction of two infinitely thin shells of charged matter at the transition surfaces S_i^+. The wormhole is obtained by gluing different asymptotic regions of one and the same extended Reissner–Nordström spacetime to the surfaces S_i^+. Hence, the metric interior to the shells has the form

$$ds_-^2 = -f(r_-) dT_-^2 + \frac{dr_-^2}{f(r_-)} + r_-^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \tag{7}$$

where

$$f(r_-) = 1 - \frac{2m}{r_-} + \frac{e^2}{r_-^2} \qquad (|e| \leq m)\,. \tag{8}$$

Let us take the timelike surface S_1^- defined by $r_- = RV_0$ lying outside the event horizon in one asymptotically flat region, say region I (see Fig. 1), of the given Reissner–Nordström spacetime and cut off the asymptotically flat part. In order to match the surface S_1^- to the exterior region at the surface S_1^+ we have to determine the identification of points on S_1^+ and S_1^-. Therefore we introduce a spherical polar coordinate system $(T_+, r_+, \vartheta, \varphi)$ centered at $z = -d_1$ such that sphere S_1^+ is given by $r_+ = R$. (Note that we have the choice of taking a right or left handed coordinate system pointing to the positive or negative direction of the z-axis.) The metric (1) of the exterior region takes the form

$$ds^2 = -V^{-2} dT_+^2 + V^2 (dr_+^2 + r_+^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2))\,. \tag{9}$$

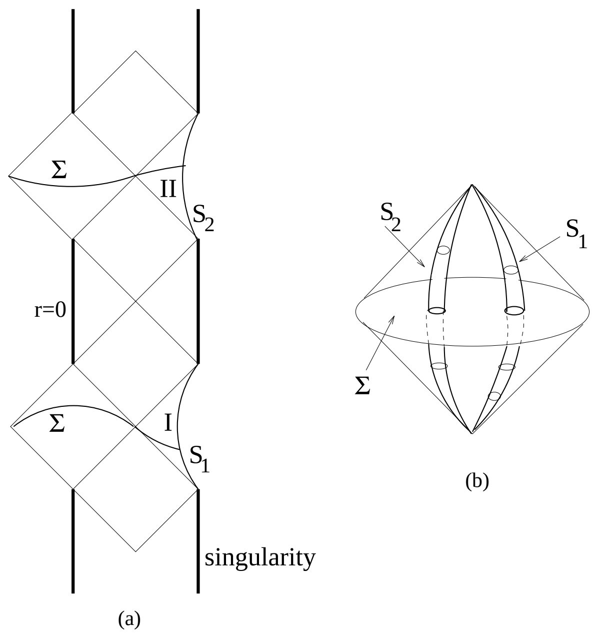

Fig. 1. Wormhole geometry: (a) interior Reissner–Nordström region, (b) schematic Penrose diagram of the exterior Majumdar–Papapetrou region.

We have not distinguished angular components of the coordinate patches (7) and (9) because now we identify points with equal values of ϑ, φ and equal proper time τ on the shells S_1^+ and S_1^-, $S_1^+ \equiv S_1^- \equiv S_1$. Hence, the induced metric on the shell S_1 is

$$ds^2|_{S_1} = -d\tau^2 + (RV_0)^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2)). \tag{10}$$

To construct a traversable wormhole we repeat this procedure for a surface S_2^- in the asymptotic region II of the given extended Reissner–Nordström spacetime lying in the causal future of S_1^-. This leads to a second shell S_2. Although the coordinate system (7) does not cover regions I and II, it is not necessary to explicitly write down different coordinates which cover the whole spacetime. By symmetry all results such as energy density and pressures of the shells are valid for both.

4. Energy density and pressures of the shells

Consider shell S_1. Denoting by n the unit normal to S_1 (directed towards the Majumdar–Papapetrou region), and by $u = d/d\tau$ the shell's velocity, the components of these vectors with respect to the different coordinate systems

(7) and (9) are given by

$$u^\alpha_+ = V_0 \quad (1,0,0,0)\,, \qquad n^\alpha_+ = \frac{1}{V_0} \quad (0,1,0,0)\,, \tag{11}$$

$$u^\alpha_- = \frac{1}{\sqrt{f(RV_0)}} \quad (1,0,0,0)\,, \qquad n^\alpha_- = \sqrt{f(RV_0)} \quad (0,1,0,0)\,. \tag{12}$$

Applying the usual formalism of thin shells the stress-energy tensor S_{ab} of the shell can be expressed by the jump in the extrinsic curvature $[K_{ab}]$ on this surface [23]. In terms of the intrinsic coordinates $(\tau, \vartheta, \varphi)$ S_{ab} is diagonal, the energy density σ depends on the angle ϑ and the tangential pressures are numerically equal and constant on the surface:

$$\sigma(\theta) = -\frac{1}{4\pi}[K^\vartheta_\vartheta]_{S_1} \tag{13}$$

$$= -\frac{1}{4\pi V_0^2}\frac{\partial V}{\partial r_+}\bigg|_{S_1} - \frac{1}{4\pi R V_0}\left(1 - \sqrt{1 - \frac{2m}{RV_0} + \frac{e^2}{(RV_0)^2}}\right)\,, \tag{14}$$

$$p = \frac{1}{8\pi}\Big([K^\tau_\tau] + [K^\vartheta_\vartheta]\Big)_{S_1} \tag{15}$$

$$= -\frac{1}{8\pi R V_0}\left(\frac{1 - \frac{m}{RV_0}}{\sqrt{1 - \frac{2m}{RV_0} + \frac{e^2}{(RV_0)^2}}} - 1\right)\,. \tag{16}$$

The properties of the energy density and pressure can be inferred by decreasing the mass and charge parameters m and $|e|$ (note that $|e| \leq m$) of the inner Reissner–Nordström region,

$$\lim_{m,e\to 0} \sigma(\theta) = -\frac{1}{4\pi V_0^2}\frac{\partial V}{\partial r_+}\bigg|_{r_+=R}\,, \tag{17}$$

$$\lim_{m,e\to 0} p = 0\,. \tag{18}$$

We can see that the sign of the surface energy density crucially depends on the sign of the normal derivative of $V(\vec{x})$ on the surfaces S_i.

In [19] we have shown explicitly that for $\frac{R}{d_1} \leq \frac{1}{3}$ this quantity is strictly positive on the surfaces S_i for arbitrary values of θ and positive mass parameter m_1. We also argued that numerical analysis indicate a critical value for $\frac{R}{d_1}$ beyond which the energy density changes sign at the inner pole. Now we are able to prove that this is not so. This stronger result can be obtained by applying the classical maximum principles developed for elliptic partial differential operators [24]. From the boundary point lemma together with the weak and strong maximum principles it follows that the outward normal derivative of the function $V(\vec{x})$ (pointing to larger radial coordinate values) is strictly negative. As a consequence, for $m, e = 0$ the energy density σ is strictly positive, *i.e.* $\sigma(\theta) \geq \sigma_0 > 0$. This proves that for all $R < d_1$ and for sufficient small values of the parameter m and $|e|$ not only is the energy density σ positive but all energy conditions are satisfied.

5. Causal structure

From the Penrose diagram of the extended Reissner–Nordström spacetime and the schematic drawing of the exterior Majumdar–Papapetrou region (Fig.1) one sees that the wormhole differs from previous models [7]: one wormhole mouth lies in the future of the other, or to put it differently, the "throat" is not a timelike hypersurface but (according to the symmetry) a spacelike slice half-way through the Reissner–Nordström wormhole. In addition, any spacelike slice which avoids the singularities (*e.g.* hypersurface Σ in Fig. 1) cuts S_1 and S_2 and connects two separated asymptotic regions.

Because any point in region II can be connected by causal curves through the wormhole from any point in region I (Fig. 1) an observer starting from the outside and entering the wormhole through S_1 is able to re-emerge at S_2 arbitrary far in the past. If the time gap resulting from the wormhole traversal is large enough he is able to travel back to his starting point in the exterior region and meet his "former self". This makes clear that any event in the Majumdar–Papapetrou region lies on a closed causal curve and therefore the wormhole is an "eternal" time machine.

Notice that the condition of continuity of the induced metric on the surfaces S_1 and S_2 does not fix the identification uniquely. There remains the possibility to introduce a constant but arbitrary shift in time. Hence, one is able to arrange the wormhole construction in a way that for example observers freely falling through the wormhole along the z-axis (starting with a given initial velocity at $z = 0$) come back to their starting point in space and time.

Multiple traversable and non-traversable wormhole geometries may be obtained by introducing additional shells in the Majumdar–Papapetrou region and connecting them to the inner Reissner–Nordström solution.

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REFERENCES

[1] S. Lem, *Dzienniki Gwiazdowe*, Czytelnik, Warszawa 1971.

[2] A. Einstein, *Berliner Berichte* **29**, 1030 (1914).

[3] K. Gödel, *Rev. Mod. Phys.* **21**, 447 (1949).

[4] L. Flamm, Z. Phys. **XVII**, 448 (1916).

[5] A. Einstein, N. Rosen, *Phys. Rev.* **48**, 73 (1935).

[6] J.A. Wheeler, *Geometrodynamics*, Academic Press Inc., London 1962.

[7] M.S. Morris, K.S. Thorne, *Am. J. Phys.* **56**, 395 (1988).

[8] M.S. Morris, K.S. Thorne, U. Yurtsever, *Phys. Rev. Lett.* **61**, 1446 (1988).

[9] V.P. Frolov, I.D. Novikov, *Phys. Rev.* **D42**, 1057 (1990).

[10] V.P. Frolov, *Phys. Rev.* **D43**, 3878 (1991).

[11] J.L. Friedman, M. Morris, *Phys. Rev. Lett.* **66**, 401 (1991).

[12] M. Visser, *Phys. Rev.* **D39**, 10, 3184 (1989); *Nucl. Phys.* **B328**, 203 (1989); *Phys. Rev.* **D47**, 554 (1993); *Lorentzian Wormholes: From Einstein to Hawking*, AIP Press (1995).

[13] T.A. Roman, *Phys. Rev.* **D47**, 1370 (1993).

[14] S. Kar, *Phys. Rev.* **D49**, 862 (1994).

[15] R. Balbinot, C. Barrabès, A. Fabbri, *Phys. Rev.* **D49**, 2801, (1994).

[16] F. Schein, P.C. Aichelburg, W. Israel, *Phys. Rev.* **D54**, 3800 (1996).

[17] D. Hochberg, M. Visser, *Phys. Rev.* **D56**, 4745 (1997).

[18] S.W. Hawking, *Phys. Rev.* **D46**, 603 (1992).

[19] F. Schein, P.C. Aichelburg, *Phys. Rev. Lett.* **77**, 20, 4130 (1996).

[20] J.L. Friedman, K. Schleich, D.M. Witt, *Phys. Rev. Lett.* **71**, 1486 (1993).

[21] R.W. Lindquist, *J. Math. Phys.* **4**, 938 (1963).

[22] E. Durand, Electrostatique II: Problemes generaux conducteurs (Masson et C^{ie} 1966, p.207).

[23] C. Barrabès, W. Israel, *Phys. Rev.* **D43**, 4, 1129 (1991).

[24] Gilbarg, N.N. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer Verlag 1977.