Vol. 29 (1998) *ACTA PHYSICA POLONICA B* No 4

MINIMAL COUPLING AND THE EQUIVALENCE PRINCIPLE IN QUANTUM MECHANICS*

C. Lämmerzahl

Laboratoire de Physique des Lasers, Institut Galilée, Université Paris 13
93430 Villetaneuse, France
and Fakultät für Physik, Universität Konstanz
Fach M 674, 78457 Konstanz, Germany
e–mail:claus@spock.physik.uni-konstanz.de

(Received January 28, 1998)

Dedicated to Andrzej Trautman in honour of his 64th birthday

The role of the Equivalence Principle (EP) in classical and quantum mechanics is reviewed. It is shown that the weak EP has a counterpart in quantum theory, a Quantum Equivalence Principle (QEP). This implies that also in the quantum domain the geometrization of the gravitational interaction is an operational procedure similar to the procedure in classical physics. This QEP can be used for showing that it is only the usual Schrödinger equation coupled to gravito–inertial fields which obeys our equivalence principle. In addition, the QEP applied to a generalized Pauli equation including spin results in a characterization of the gravitational fields which can be identified with the Newtonian potential and with torsion. Also, in the classical limit it is possible to state beside the usual EP for the path an EP for the spin which again may be used for introducing torsion as a gravitational field.

PACS numbers: 04.20. Cv

1. Introduction

The Equivalence Principle (EP) is a statement of universality in the sense that a certain physical effect occurs for all members of a class of physical objects. Equivalently, the EP is a statement of independence in the sense that this physical effect occurs independent of the chosen member of the class of physical objects. The importance of an EP lies in the fact that it

* Presented at the Workshop on Gauge Theories of Gravitation, Jadwisin, Poland, September 4–10, 1997.

serves as a tool to characterize or to distinguish the gravitational interaction from other interactions. Indeed, by means of the EP we can *define* what we mean by a gravitational field.

The validity of the EP is an important matter of discussion in connection with the attempt of unifying gravity with the other three interactions or with the program of quantising gravity. Any violation of this basic principle of General Relativity has of course dramatic implications for our present understanding of the physical nature of gravitation and of the cosmic evolution of our universe.

According to the physical domain under consideration there may be many versions of EPs. Each EP depends on the class of chosen physical objects and for each class EPs of various strength may be formulated.

1.1. The EP in classical physics

The *weak EP* (or the universality of free fall) requires that in a gravitational field (*i*) *all* structureless point particles follow the same path and (*ii*) this path is uniquely characterized by the initial position $x(t_0)$ and initial velocity $\dot{x}(t_0)$. This leads to an equation of motion for the path of the form $\ddot{x}^\mu = f^\mu(x, \dot{x})$ where the dot means the derivative with respect to some parameter. Since in this equation there appears no parameter characterizing properties of the particle, gravity acts universal and can be geometrised. Here gravity is connected with the function $f^\mu(x, \dot{x})$. Gravity may still be a Finslerian geometry, for example.

In order that gravity is an affine geometry one has to apply the *strong EP* or Einstein's elevator which means that gravity can be transformed away by choosing an appropriate frame of reference (this also means that gravity can be simulated by choosing some frame of reference). The strong EP requires that for each point x_0 there is a frame so that for all particles $\ddot{x} = 0$ in x_0. This specifies the functions $f^\mu(x, \dot{x})$ and leads to the path equation $\ddot{x}^\mu + \Gamma^\mu_{\nu\rho}(x)\dot{x}^\nu\dot{x}^\rho \sim \dot{x}^\mu$ thus introducing the notion of a (projective) connection. Here gravity is described by the functions $\Gamma^\mu_{\nu\rho}(x)$. (Another version of the strong EP requires that for all x_0 there is a frame so that locally all physical phenomena are as in gravity free–space. However, this version is not operational since one has to know in advance non–gravitational physics which is not possible because gravity cannot be shielded.)

If gravity should be describable within the framework of a Riemannian geometry, that is, by a space–time metric, then *Einstein's EP* has to be applied, [1, 2]. Einstein's EP requires that the weak EP, local Lorentz invariance and local position invariance holds. Local Lorentz invariance means that gravity is described by a metric and possibly additional scalar fields. However, these scalar gravitational fields are ruled out by local position invariance.

Einstein's EP implies the strong EP, and the strong EP implies the weak EP. Therefore we have a hierarchy of EPs, each leading to a more specific geometrical frame for the description of gravity.

The main point of the above discussion is that the EP is a means (*i*) to *define* operationally the gravitational interaction, (*ii*) to *geometrise* the gravitational interaction, and (*iii*) to fix the *equation of motion of matter*.

1.2. An EP in the quantum domain?

Due to the very different nature of physical phenomena in the quantum regime one may ask whether there is any hope at all to find an EP in the quantum domain. In classical physics the equivalence principle is a strictly local notion since it involves the notions of a point or a path of a point particle. In the quantum domain physics is described by fields which are nonlocal objects since they are spread out over all space. Consequently, one has to doubt whether local notions in a formulation of an EP will make sense in the quantum domain.

The EP in QM can be discussed with respect to (*i*) the minimal coupling procedure [3], (*ii*) the path (WKB–path, Heisenberg equations of motion [4], path integral [5, 6]), (*iii*) the phase shift in neutron interferometry [7, 8], (*iv*) the question whether gravitationally induced effects can be transformed away [3], and (*v*) with respect to the structure of the solution and of the Green function. Therefore there are many attempts to discuss this notion or to transform this notion into the quantum domain. However, these approaches are (*i*) either valid for a restricted domain only (classical approximation), or (*ii*) disprove that this notion can be carried over to the quantum domain, or (*iii*) are valid only for homogeneous gravitational fields, or (*iv*) are rather formal statements which have no clear operational realization. For example, the solution of the Schrödinger equation in a homogeneous gravitational field depends on mass contrary to the analogous situation for point particles. Also, the phase shift in neutron interferometry in a homogeneous gravitational field depends on the mass, too, again violating the weak EP [7, 8]. Also the strong EP is not valid for neutron interferometry since curvature effects which are present due to the extension of the interferometer, cannot be transformed away by choosing a distinguished reference frame.

To be more explicit, the phase shift in neutron interferometry (see Fig. 1) is described using the Schrödinger equation coupled to the Newtonian potential and performing a WKB approximation [7, 10]. The measured quantity is the intensity of the neutron beam leaving the interferometer at one port, $I = \frac{1}{2} I_0 (1 - \cos \phi_{\rm grav}^{\rm neutron})$, with the phase

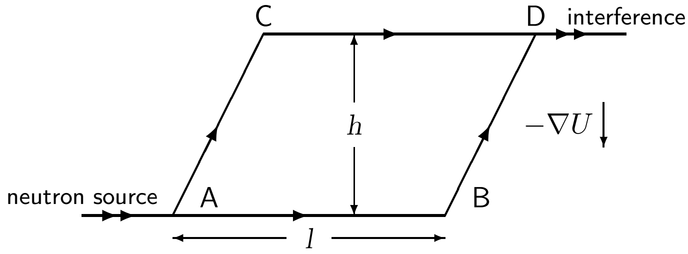

Fig. 1. Neutron interference used in [7] to demonstrate the gravitational influence on the neutron's wave function. A and D are beam splitter and recombiner, respectively, and B and C are mirrors.

$$\phi_{\rm grav}^{\rm neutron} = \frac{1}{\hbar} \oint \boldsymbol{p}(\boldsymbol{x}) \cdot d\boldsymbol{x} = \frac{m\boldsymbol{g} \cdot \boldsymbol{h} l}{\hbar v} + U_{ab} h^a h^b \frac{ml}{\hbar v} + \mathcal{O}(g^2, gU_{ab}) \qquad (1)$$

($\boldsymbol{g} := \nabla U$). The phase shift depends on the mass m of the neutron so that there is clearly no weak EP, and it depends on the curvature $U_{ab} := \partial_a \partial_b U$ so that also the strong EP is not valid. We want to stress the most characteristic feature of this derivation of the phase shift: it treats the beam splitting as a splitting in *configuration space*.

1.3. General remarks

In order to geometrize gravity it is enough to have *one effect* for which an EP is valid. This is important because there are of course effects in classical and quantum mechanics which depend on the mass of the considered object. For example, the elastic scattering of point particles or the spacing between the interference fringes in a double slit interference experiment. Therefore all EPs are valid for distinguished situations only which may be very difficult to specify experimentally.

The importance of the problem whether there is still some EP in the quantum domain concerns the notion of the gravitational field and the space–time geometry in the quantum domain: if and only if there is an EP then also in the quantum domain one can define uniquely and without any approximation what we mean by the gravitational field in this domain. Only in this case we are allowed to geometrise gravity in the domain of quantum physics. We will show in the following that there is an EP in quantum theory which is slightly modified compared to the weak EP in classical mechanics. However, it has the same logical structure than all other EPs. Therefore, *also in the quantum domain gravitation is operationally definable and geometrizable.* Here we will also include spin in our considerations.

There are two points concerning the search for an EP within a certain theory: On the one hand, if a certain theory is given then we may ask which EP is valid for this theory. On the other hand we may also ask which theory is singled out of a wide range of theories by the required validity of an EP. That is what we are doing in Sects. 3.1 and 3.2.

Another point is the following: In the quantum domain one usually employes a minimal coupling procedure in order couple gravity to the Klein-Gordon or the Dirac equation. However, in doing so one has to know in advance the physics in the gravity–free world. Since there is always a gravitational field all over the universe, this is neither an operational nor a self–contained approach. However, in the following we are able to show that at least in the non–relativitistic domain for first quantised matter *the minimal coupling procedure is equivalent to the QEP* which we are going to formulate.

2. A Quantum Equivalence Principle

In order to give motivation for our formulation of a Quantum Equivalence Principle (QEP) we first discuss the phase shift for atom beam interferometry in gravitational fields. This structure of the corresponding result allows us to state a QEP. In addition, we are able to show that this QEP can also be applied to neutron interferometry. For more discussions, see [10].

2.1. Atom beam interferometry in gravitational fields

The important point is that in atom beam interferometry the interference is described by a closed loop in *momentum space* and not in configuration space: it is the momentum space where the splitting occurs (see Fig. 2). The observed quantity is the number of atoms in the excited state leaving the apparatus at port I, for example:

$$I_2 = \int |a_{e,\boldsymbol{p}+\hbar\boldsymbol{k}}(t_2)| d^3p = \frac{1}{2}\,(1-\cos\phi)\, I_1\,. \tag{2}$$

From the dynamics of the quantum field we can calculate the phase shift due to the gravitational interaction [10]:

$$\phi^{\rm atom}_{\rm grav} = -k^a T^2 \left(g_a - U_{ab} T \left(\frac{\hbar k^b}{2m} + \langle \widehat{v}^b \rangle_0 - g^b \frac{37}{12} T \right) \right)\,. \tag{3}$$

Here $\langle \widehat{v}^b \rangle_0$ is the mean value of the velocity operator at the moment of the first laser pulse.

Eq. (3) is an exact quantum result, we made no approximation in $\hbar$ as, for example, in the description of the neutron interferometer. (3) is also

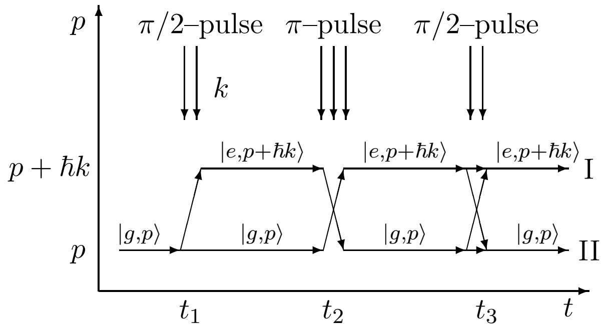

Fig. 2. Atom beam interferometry as closed loop in momentum space. $|g,p\rangle$ is an atom in the ground state with momentum p, $|e,p\rangle$ is an atom in the excited state. The laser pulses change the internal state as well as the momentum of the atoms thus acting as "mirrors" in momentum space. For the Ramsey–Bordé interferometer we just have to replace t by x.

exact in $\boldsymbol{g}$ and of first order in U_{ab}. The first term $\phi_{\rm grav} = -\boldsymbol{k}\cdot\boldsymbol{g}\; T^2$ describes the phase shift due to the acceleration. An astonishing feature is, that although there appears no $\hbar$, this term is an exact quantum result. The reason for that is that we use only the quantities T and $\boldsymbol{k}$ which are given by the experimental setup. Only if we *formally* introduce classical notions then $\hbar$ as well as the mass m comes in: defining formally a "length" by means of $\boldsymbol{l} = \langle\widehat{\boldsymbol{v}}\rangle_0 T$ and a "height" by $\boldsymbol{h} = (\hbar\boldsymbol{k}/m)T$ then the above acceleration induced phase shift can be rewritten as $\phi_{\rm grav} = -m\boldsymbol{h}\cdot\boldsymbol{g}l/(\hbar|\langle\widehat{\boldsymbol{v}}\rangle_0|)$ which has exactly the form of the first term of the phase shift (1) [7]. However, these notions of a "length" and of a "height" have no operational meaning in atom interferometry since there is no beam splitting in configuration space (for an atomic beam with a width of ca. 1 cm the "splitting" may be of about 10 μm). For more discussions of (3) see [10].

The *weak EP is exactly fulfilled for this acceleration induced phase shift* since there appears no mass. This also means that a mass dependence of the equations of motion or of the solutions is no indication for a break-down of the EP on the level of observations.

The part $\phi_{\rm grav} = U_{ab}k^a\langle\widehat{v}^b\rangle_0 T^3$ describes the phase shift due to the Newtonian part of the Riemannian curvature. It is the quantum version of the geodesic deviation equation and gives a genuine quantum measurement of the space–time curvature.

Since the total phase shift (3) depends on m the weak EP is not valid. However, if we consider a phase shift for an infinitesimal loop in momentum space, that is for $\boldsymbol{k}\to 0$, and normalize it by dividing through its modulus

k, then the weak EP turns out to be valid:

$$\lim_{k\to 0}\frac{1}{k}\phi_{\rm grav} = -\bar{k}^a T^2 \left(g_a + U_{ab} T \left(\langle \widehat{v}^b \rangle_0 - g^b \frac{37}{12} T \right)\right) \tag{4}$$

($\bar{\boldsymbol{k}} = \boldsymbol{k}/k$). This expression is valid in an arbitrarily curved space-time.

Therefore also in interference experiments the influence of the Newtonian potential on the interference fringes does not depend on the used quantum matter. This means that with the help of the above procedure we have an operational means to distinguish the gravitational interaction in the quantum domain in the same way as the usual EP does in the classical domain, and thus to assign the gravitational interaction a geometrical nature.

We use this result to proceed in formulating an EP for the quantum domain.

2.2. A Quantum Equivalence Principle

Quantum Equivalence Principle (QEP): *For all given initial states the input independent result of a physical experiment is independent of the characteristic parameters (like mass, charge) of the quantum system.*

Here we mean by "input" the characterization of the experimental preparation or of the influence on physical system. In our case it is the wave vector $\boldsymbol{k}$ which influences the motion of the atoms. "Input independent" means that the influence given by k approaches zero, but the resulting expression is to be normalized by k thus giving a finite expression.

Note that for this EP we do not use any local notion. We are using only the notions "quantum systems" and "influence". As the EP in classical physics, this EP is a statement of universality for observations thus leading to an operational definition of the gravitational field and its geometrization on the quantum level. Our QEP is a generalization of the weak EP; the strong EP is not applicable to quantum mechanics. It is clear that for charged particles in an electromagnetic field the QEP is not valid, see [10].

2.3. The Quantum EP in neutron interferometry

After having shown that the weak EP is valid for atom beam interferometry, we have to reconsider the phase shift in neutron interferometry. It has been shown [10] that the corresponding phase shift (1) can be reformulated by

$$\phi_{\rm grav}^{\rm neutron} = \boldsymbol{g}\cdot\boldsymbol{G}\, TT' + \mathcal{O}(g^2, U_{ab})\,, \tag{5}$$

where $\boldsymbol{G}$ is the reciprocal lattice vector of the mono-crystal on the Ewald sphere near the momentum of the incoming neutron.

Since the calculation for deriving this formula is similar to that for the derivation of the phase shift in atom interferometry, the above result is the exact phase shift in the same way as (3) is. Indeed, it has been remarked by Bordé [14] that neutron scattering at crystals has a mathematical structure which is similar to that of the scattering of atoms by laser beams. The difference to the usual WKB treatment is, that here we do not consider neutron interferometry as a loop in configuration space, but instead as loop in momentum space.

Therefore, for the phase shift in neutron interferometry the same remarks concerning the validity of the EP hold: The QEP is valid also in neutron interferometry, and especially for homogeneous gravitational fields, the usual weak EP is valid. This is in contrast to [8, 9].

3. Equivalence principle and minimal coupling

After having shown that the usual Schrödinger equation coupled in the usual way to the Newtonian potential obeys our QEP, we now want to proceed the other way around: We ask for that quantum equation of motion which results from the requirement of validity of our QEP. In order to answer that question we first start from a very general structure of a Schrödinger equation for a scalar field. In this case the minimal coupling procedure is equivalent to the validity of our QEP. Next we use a very general approach starting from basic principles leading to a generalized Dirac and Pauli equation. In this case we have to discuss the role of the spin in the formulation of our QEP. As a main result it follows that there is a QEP so that beside the Newtonian potential torsion naturally comes out as a geometrical field in an operational way.

3.1. A general Schrödinger equation

We start from an ansatz describing a general Schrödinger equation

$$H = \frac{1}{2m}\widehat{\boldsymbol{p}}^2 + V(\widehat{\boldsymbol{x}}) + \frac{1}{2}(\boldsymbol{V}(\widehat{\boldsymbol{x}}) \cdot \widehat{\boldsymbol{p}} + \widehat{\boldsymbol{p}} \cdot \boldsymbol{V}(\boldsymbol{x})) + \frac{1}{2}W^{ab}\widehat{p}_a\widehat{p}_b \,. \tag{6}$$

For the sake of simplicity we restrict our equation to be of second order only. In this ansatz the fields $V(\boldsymbol{x})$, $\boldsymbol{V}(\boldsymbol{x})$, and W^{ab} are to be determined with the help of our QEP. We also assume that W^{ab} does not depend on the position and that $\boldsymbol{V}(\boldsymbol{x})$ is linear in the position. Neither these assumptions nor the restriction to a second order equation influence our way of reasoning. Our scheme can be carried through in full generality.

From this general Schrödinger equation we can calculate in the same way as above the input independent phase shift. Then we *require that the QEP holds.* This implies [10] that V/m, $\boldsymbol{V}$, and mW^{ab} must be independent of the mass m appearing in the kinetic term of the general Schrödinger equation (6). It is convenient to introduce new functions $U := V/m$, $U^{ab} := mW^{ab}$ which, due to our QEP, are independent of m and thus independent of the chosen quantum system. Insertion of these new functions into (6) gives

$$H = \frac{1}{2m}(\delta^{ab} + U^{ab})\widehat{p}_a\widehat{p}_b + mU(\widehat{\boldsymbol{x}}) + \frac{1}{2}(\boldsymbol{V}(\widehat{\boldsymbol{x}}) \cdot \widehat{\boldsymbol{p}} + \widehat{\boldsymbol{p}} \cdot \boldsymbol{V}(\widehat{\boldsymbol{x}}))\,. \tag{7}$$

The expression $\delta^{ab} + U^{ab}$ can be transformed to δ^{ab} by means of a coordinate transformation. Clearly, U has to identified with the Newtonian potential. The QEP forces the m in front of the Newtonian potential to be the same as in the kinetic term. The term $\frac{1}{2}(\boldsymbol{V}(\widehat{\boldsymbol{x}})\cdot\widehat{\boldsymbol{p}}+\widehat{\boldsymbol{p}}\cdot\boldsymbol{V}(\widehat{\boldsymbol{x}})) = \Lambda^a_b\frac{1}{2}\{\widehat{p}_a, \widehat{x}^b\}$ describes an expanding, rotating or shearing frame. For an antisymmetric Λ^a_b, for example, we get the Sagnac–effect $\phi_{\text{Sagnac}} = 2T^2k^a\Lambda^b_a\langle\widehat{v}_b\rangle_0 = 2T^2\boldsymbol{k}\cdot(\boldsymbol{\omega}\times\boldsymbol{v})$. A formal use of a “length” and “height” as above, again leads to the usual formula for the Sagnac effect $2\frac{m}{\hbar}\boldsymbol{\omega}\cdot\boldsymbol{A}$ which contains $\hbar$ and the mass m.

Therefore we have the result that for a general Schrödinger equation the requirement, that the QEP should hold, implies the usual structure of the gravito–inertial interaction in the quantum domain. The QEP amounts to an operational justification of the minimal coupling procedure at least in the non-relativistic domain. A corresponding treatment of the relativistic case should also be done.

3.2. A general approach with spin

In this approach we take the quantum field as fundamental physical object and require dynamical principles for this quantum field which is described by means of a multicomponent complex valued field. We require the following dynamical properties: (*i*) there should be a well posed Cauchy–problem, (*ii*) the superposition principle should hold, (*iii*) it should propagate with a finite speed, and (*iv*) should obey a conservation law. The mathematical consequence of these requirements is a generalized Dirac equation (for a review see [12])

$$0 = i\widetilde{\gamma}^\mu(x)\partial_\mu\varphi(x) - M(x)\varphi(x) \tag{8}$$

which is a first order hyperbolic system of partial differential equations with still unspecified matrices $\widetilde{\gamma}^\mu$ and M. In general, the $\widetilde{\gamma}^\mu$ do not fulfill a Clifford algebra. One can introduce the deviation $X^{\mu\nu}$ from the usual Clifford algebra $\widetilde{\gamma}^\mu\widetilde{\gamma}^\nu + \widetilde{\gamma}^\nu\widetilde{\gamma}^\mu = g^{\mu\nu}\mathbf{1} + X^{\mu\nu}$ where $g^{\mu\nu} := \frac{1}{4}\text{tr}(\widetilde{\gamma}^\mu\widetilde{\gamma}^\nu)$. However, it can be shown that the notion of a generalized Clifford algebra always exists [15]. This deviation from the usual Clifford algebra can be geometrically interpreted: The null cones of the generalized Dirac equation which are derived from the characteristic surfaces, split in more than one components thus leading to a birefringence behaviour of null propagation. Similarly, there is also no longer a single mass shell [11, 15].

In the non–relativistic limit we get a *generalized Pauli–equation* in a non–rotating frame (see [11] for the details)

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\varphi &= -\frac{\hbar^2}{2m}\left(\delta^{ij} - \frac{\delta m_{\rm i}^{ij}}{m} - \frac{\delta \bar{m}_{\rm i k}^{ij}\sigma^k}{m}\right)\nabla_i\nabla_j\varphi + \left(cA^i_j + \frac{1}{m}a^i_j\right)\sigma^j i\hbar\nabla_i\varphi \\ &\quad + \Big[e\phi(x) + \frac{e}{2m}\boldsymbol{H}\cdot\boldsymbol{\sigma} + (1+\boldsymbol{C}\cdot\boldsymbol{\sigma})mU(x) + \delta m_{{\rm g}ij}U^{ij}(x) \\ &\quad + \hbar c\boldsymbol{T}\cdot\boldsymbol{\sigma} + mc^2\boldsymbol{B}\cdot\boldsymbol{\sigma}\Big]\varphi\,. \end{aligned} \tag{9}$$

In this equation there appear several anomalous terms which are not present in the usual Pauli equation coupled to the Newtonian potential: the terms $\delta m_{\rm i}^{ij}$ and $\delta\bar{m}_{\rm ik}^{ij}$ describe an anomalous inertial mass which may depend on the spin of the particle, A^i_j and a^i_j give rise to a spin–momentum coupling, $\boldsymbol{C}$ characterizes an anomalous spin coupling to the Newtonian potential, $\delta m_{{\rm g}ij}$ is the anomalous gravitational mass tensor, $\boldsymbol{T}$ represents an extra spin-coupling term, and $\boldsymbol{B}$ may be called a spin–dependent "rest mass". These anomalous terms violate Einstein's EP [11]. ϕ and $\boldsymbol{H}$ are the scalar electrostatic potential and the magnetic field, respectively.

Interference experiments

We describe two types of interference experiments which are useful for our attempt to apply the QEP. For a spin–flip interference experiment we use an atomic beam in a defined spin state, say $|\frac{1}{2},\frac{1}{2}\rangle$. A beam splitter splits via a spin–flip this state into a superposition of the states $|\frac{1}{2},\frac{1}{2}\rangle$ and $|\frac{1}{2},-\frac{1}{2}\rangle$. After a time Δt again a spin–flip is applied recombining these two states. Measuring again the spin along the given axis, gives an interference pattern depending on the difference of the energies accumulated by these two states during the time Δt. The corresponding phase is given by

$$\phi^{\rm spin} = \frac{2}{\hbar}\left(\frac{\delta\bar{m}_{\rm ik}^{ij}}{2m^2}p_ip_j - \frac{\hbar}{m}a^i_kp_i - cA^i_kp_i + mc^2B_k + C_kmU + cT_k\right)S^k\Delta t\,. \tag{10}$$

A second type of experiment measures the acceleration, compare [19]. The corresponding phase shift for a spherically symmetric gravitational field, disregarding curvature, is

$$\phi^{\rm accel} = T^2 \frac{GM}{r_0^3} \left(k_i r_0^i + \frac{\delta m_{{\rm g}ij}}{m} \frac{r_0^i r_0^j}{r_0^2} k_l r_0^l - \frac{6}{5} \frac{\delta m_{{\rm g}ij}}{m} r_0^i k^j \right.$$
$$\left. + \frac{2}{5} \frac{\delta m_{{\rm g}ii}}{m} k_l r_0^l - \frac{\delta m_{{\rm i}ij} + \delta \bar{m}_{{\rm i}ijk} S^k}{m} r_0^i k^j + C_j S^j k_i r_0^i \right) , \quad (11)$$

where S is the spin of the quantum particle.

Before applying our QEP we present the equations of motion of the classical limit of the generalized Pauli equation.

The classical limit

From Eq. (9) we get as classical acceleration

$$a^i = - \left(\delta^{ij} + \frac{\delta m_{\rm i}^{ij}}{m} + \left(\frac{\delta \bar{m}_{\rm ik}^{ij}}{m} + \delta^{ij} C_k \right) S^k \right) \partial_j U - \delta^{ij} \frac{\delta m_{{\rm g}kl}}{m} \partial_j U^{kl} . \quad (12)$$

We recognize that on the quantum level (10,11) there are more possibilities to violate the weak EP than on the classical level.

We also can calculate the dynamical behavior of the spin expectation value in the classical approximation:

$$\frac{d}{dt} \boldsymbol{S} = \boldsymbol{\Omega} \times \boldsymbol{S} \quad (13)$$

with

$$\Omega_i := \frac{1}{2m} \frac{\delta \bar{m}_{\rm ii}^{kl}}{m} p_k p_l + \left(\frac{1}{m} a_i^j + c A_i^j \right) p_i + mc^2 B_i + c T_i + C_i m U(x) . \quad (14)$$

In order to test the complete set of anomalous parameters we need the equation of motion for the path *and* of the spin. Both quantities are needed in order to determine the structure of space–time. This point of view is the basis of Riemann–Cartan theories as has been stressed *e.g.* by Trautman [20] and Hehl [17].

3.3. Equivalence principles

Based on our QEP we can now state two different EPs. This is possible because our quantum system now has two properties: mass and spin. One QEP requires that the measured effects are independent of the mass, and the second requires the independence from mass *and* spin.

QEP$_m$: The input independent phase shift should be independent of the mass of the particle.

If we apply QEP_m to the acceleration induced phase shift ϕ^{accel} then we get as necessary conditions δm_{i}^{kl}, $\delta\bar{m}_{\text{ii}}^{kl}$, C_k, $\delta m_{\text{g}ij} = 0$. QEP_m applied to the spin–flip induced phase shift ϕ^{spin} gives B_i, A_j^i, $\delta\bar{m}_{\text{ii}}^{kl} = 0$. Taking both results together we have that all anomalous terms but $\boldsymbol{T}$ and a_j^i have to vanish. This is also clear from the fact that the coupling to $\boldsymbol{T}$ and a_j^i in (9) are the only one which do not involve the mass of the quantum object and thus can be regarded as coupling to a geometrical field. Indeed, it is possible to absorb the a_j^i-term in [9] into the kinetic term by replacing $\nabla_i \to \nabla_i - \frac{i}{\hbar}\delta_{ij}\, a_l^j\, \sigma^l$ and neglecting terms of second order in the anomalous terms. According to the non–relativistic limit of the Dirac equation in a Riemann–Cartan space–time [13], these terms can be interpreted as time– and space–components of the axial torsion. Since the Dirac equation in Riemann–Cartan space–time was obtained by minimally coupling of the corresponding Lagrangian (see *e.g.* [17]), our QEP again singles out this procedure, at least in the non–relativistic domain considered here.

As a second possibility we can regard spin to be on the same level as mass being a property of quantum objects. Then we can take as our equivalence principle the requirement that the input independent phase shift should be independent of the mass and the spin of the particle. This we call $\text{QEP}_{m,S}$.

QEP$_{m,S}$: The input independent phase shift should be independent of the mass *and* the spin of the particle.

Then we get the same results as above but in addition that $\boldsymbol{T}$ and a_j^i have to vanish, too.

Therefore, if we regard spin on the same level as mass, then the requirement of a QEP rules out torsion. However, if we regard mass to be more fundamental in this connection, then torsion is still allowed as a geometrical field interacting with quantum objects. Therefore, there is a QEP, namely QEP_m, so that the Newtonian potential (or the space–time metric) as well as torsion can be introduced operationally as geometric fields. The requirement that the QEP should hold for the generalized Pauli equation implies that all couplings except the Newtonian potential and the axial torsion should vanish. Consequently, *at least in the non–relativistic regime, Riemann–Cartan geometry is a consequence of a QEP.*

There is another possibility to formulate an EP, namely on the level of the equations of motion for the path and for the spin in the quasiclassical approximation. The structure of the equation of motion for the spin (13) suggests an EP for the spin which also naturally leads to the introduction of torsion as the only geometrically interpretable interaction with the spin.

EP for the spin: For any initial state of the quantum system, the spin motion does not depend on the characteristic parameters of the quantum system (like mass, charge, *etc.*).

A similar EP for the spin has been first suggested by Adamowicz and Trautman [16]. The only terms in (13) which do not depend on the mass or on the charge of the particle are the terms a^i_j and $\boldsymbol{T}$. Since these terms can be interpreted as parts of axial torsion, the requirement of the EP for the spin implies that all interactions except the Newtonian potential and the axial torsion should vanish leading again to a Riemann–Cartan geometry.

Also within a constructive axiomatic approach it was possible to introduce torsion from the equation of motion for the spin [18]. Of course, torsion violates local Lorentz invariance so that Einstein's EP is not valid.

4. Conclusion

We have shown that also in the quantum domain it is possible to state an EP which distinguishes the gravitational fields from other fields. Therefore, also in the quantum domain gravity is geometrisable and the notion of a space–time structure makes operational sense in the quantum domain. We were able to include spin into our scheme: With the help of a QEP requiring that also spin effects should be independent of the mass, we operationally defined a Riemann–Cartan geometry in the quantum domain. At least in the non–relativistic domain considered here, the QEP is equivalent to the minimal coupling procedure.

I thank very much Ch. Bordé and F.W. Hehl for discussions and the organizers of the workshop for the invitation. Financial support of the CNRS (France) and the WE–Heraeus–Stiftung is acknowledged.

REFERENCES

[1] K.S. Thorne, D.L. Lee, A.P. Lightman, *Phys. Rev.* **D7**, 3563 (1973).

[2] C.M. Will: *Theory and Experiment in Gravitational Physics*, Revised Edition, Cambridge University Press, Cambridge 1993.

[3] J. Audretsch, F.W. Hehl, C. Lämmerzahl, in J. Ehlers, G. Schäfer, *Relativistic Gravity Research with Emphasis on Experiments and Observations*, *Springer Lecture Notes in Physics*, **410**, Springer–Verlag, Berlin 1992, p. 368–407.

[4] H. Rumpf, in *International School of Cosmology and Gravitation on Spin, Torsion, Rotation and Supergravity*, deSabbata (ed.), Plenum, 1979.

[5] H. Kleinert, *Acta Phys. Pol.* **B29**, 1033 (1998).

[6] H. Kleinert, A. Pelster, *Acta Phys. Pol.* **B29**, 1015 (1998).

[7] R. Colella, A.W. Overhauser, S.A. Werner, *Phys. Rev. Lett.* **34**, 1472 (1975).

[8] D. Greenberger, *Ann. Phys.* **47**, 116 (1968); D. Greenberger, A.W. Overhauser: *Rev. Mod. Phys.* **51**, 43 (1979).

[9] J.L. Staudenmann, S.A. Werner, R. Colella, A.W. Overhauser, *Phys. Rev.* **A21**, 1419 (1980).

[10] C. Lämmerzahl, *Gen. Relativ. Gravitation* **28**, 1043 (1996).

[11] C. Lämmerzahl, *Classical Quantum Gravity* **17**, 13 (1998).

[12] J. Audretsch, C. Lämmerzahl, in *Semantical Aspects of Space–Time Geometry*, U. Majer, H.-J. Schmidt (eds.) BI Verlag, Mannheim 1993.

[13] C. Lämmerzahl, *Phys. Lett.* **A228**, 223 (1997).

[14] Ch.J. Bordé, in *Atom Interferometry*, P.R. Berman (ed.), Academic Press, Boston 1996 p.257.

[15] C. Lämmerzahl, in *Recent Developments in Gravitation and Mathematical Physics*, A. Garcia *et al.* (eds.) Science Network Publishing, Konstanz 1998 [online: http://kaluza.physik.uni-konstanz.de/2MS].

[16] W. Adamowicz, A. Trautmann, *Bull. Acad. Pol. Sci. Ser. Sci. Math. Astron. & Phys.* **23**, 339 (1975).

[17] F.W. Hehl, P. von der Heyde, G.D. Kerlick, *Rev. Mod. Phys.* **48**, 393 (1976); F.W. Hehl, J. Lemke, E.W. Mielke, in *Geometry and Theoretical Phyiscs*, J. Debrus , A.C. Hirshfeld (eds.), Springer-Verlag, Berlin 1991, p.221.

[18] J. Audretsch, C. Lämmerzahl, *Classical Quantum Gravity* **5**, 1285 (1988).

[19] M. Kasevich, S. Chu, *Phys. Rev. Lett.* **67** 181 (1991); *Appl. Phys.* **B54**, 321 (1992).

[20] A. Trautman, *Sov. Phys. Usp.* **89**, 319 (1966).