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NULL SURFACE CANONICAL FORMALISM*

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Dedicated to Andrzej Trautman in honour of his 64th birthday

The canonical formalism for general relativity on a null surface is compared with that on a space-like surface using Ashtekar variables, the self-dual connection and a densitized triad. The principal difference lies in the appearance of second class constraints. These arise in part because the metric on a null surface is singular, in part because on a null surface there is a preferred direction, and in part because a compact mapping will not map a null surface into a null surface. Second class constraints are eliminated by the use of Dirac brackets. It is shown that, in principle, this is particularly straightforward in this case.

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1. Introduction

There have been several attempts to construct a Hamiltonian for general relativity on a null surface [1-5]. Of these, only the work by D'Inverno and Vickers and by Goldberg, Robinson, and Soteriou (GRS) are successful in that all of the Einstein equations are obtained directly from the variation of the action from which the formalism begins. These succeed by introducing auxiliary variables whose values fix the nature of the surface t = constant: space-like, time-like, or null. The values of these auxiliary variables are determined by the variation of Lagrange multipliers in the action itself. In

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this talk, I focus on the formalism of GRS although the ideas are equally applicable to other formalisms.

The plan of this talk is first to describe briefly the Ashtekar formalism on a space-like surface. Then to discuss the differences which occur when the formalism is applied to a null surface. The Hamiltonian on the null surface will then be described together with the constraints. The constraints will be separated into first and second class and finally the treatment of the second class constraints will be described. The concluding remarks will discuss what needs to be accomplished if the quantization program is to move forward.

2. Space-like surface

In the Ashtekar formalism [6], the phase space variables are the pullback to a space-like surface of the self-dual connection and the dual of the pulled back self-dual two-forms constructed from an orthonormal frame. On the space-like surface, the latter form a densitized triad tangent to the surface. These variables arise naturally from the 3+1 decomposition of the action

$$S \;=\; \int d^4x\; R^A \wedge S^B g_{AB}\,. \tag{1}$$

$A, B = 1, 2, 3$ label the three independent self-dual components of the Riemann tensor and 2-forms and g_{AB} is an $O(3)$ metric which raises and lowers these indices. In a $3 + 1$ decomposition, this becomes

$$S \;=\; \int dtd^3x\{A^A{}_{i,0}E_A{}^i \;+\; B^A\mathcal{G}_A \;-\; \mathbf{N}\mathcal{H}_0 \;+\; N^i\mathcal{H}_i\}.$$

$B^A, N^i, \boldsymbol{N}$ behave like Lagrange multipliers which define the mappings generated by $\mathcal{G}_A, \mathcal{H}_i, \mathcal{H}_0$, the gauge, diffeomorphism, and scalar constraints:

$$\begin{aligned}
\mathcal{G}_A &= D_iE_A{}^i\,,\\
\mathcal{H}_i &= R^A{}_{ij}E_A{}^j\,,\\
\mathcal{H}_0 &= \eta_{ABC}R^A{}_{ij}E^{Bj}E^{Ck}\,.
\end{aligned}$$

The Hamiltonian is then just a linear combination of these constraints. The constraints are all first class — their Poisson brackets are equal to linear combinations of themselves and, therefore, vanish on the constraint surface of the phase space. Note that the action and Hamiltonian are complex. Therefore, to reconstruct the Einstein theory certain reality conditions are needed as well. However, these are beyond our present interests.

3. Null surface

The most striking feature of the canonical formalism on a null surface is the appearance of constraints in addition to those arising from gauge invariance or diffeomorphism invariance. In a broad sense, these null surface constraints arise from the fact that the metric on a null surface is singular. This has the effect that in a formalism making use of a system of non-singular triads, one of the triad vectors is not dynamical. Furthermore, on a null surface, there is a distiguished direction. This reduces the gauge group to the two complex parameter group of null rotations. Another feature of working on a null surface is that there are no small compact deformations which map a null surface to a neighboring null surface. The net effect of this is that together with the null surface constraints, one of the gauge constraints and the scalar constraint form a system of second class constraints.

The Poisson brackets between second class constraints do not vanish on the constraint surface. Furthermore, the matrix formed by the system of Poisson brackets is not singular. Therefore, second class constraints represent canonical pairs of phase space variables which are superfluous and which are not independent of the remaining variables. Following the Bergmann-Dirac quantization rules, these second class constraints should be eliminated from the theory before constructing the quantum algebra.

Another difference in the null surface formalism is that it is necessary to introduce an auxiliary variable in order to obtain all of the Einstein equations. This arises because if t = constant is a null surface, then $g^{00} = 0$. Variation of the action with respect to this variable is missing and, as a result, the corresponding field equation is missing. The auxiliary variable prevents that from occurring until after the variations have taken place.

The action we use is the same as in the previous section except that the 3+1 reduction is done by pullback to a null surface instead of a space-like surface. To construct the self-dual bivectors, we use a tetrad of null one-forms ($\nu^{\boldsymbol{i}}{}_i v_{\boldsymbol{j}}{}^i = \delta^{\boldsymbol{i}}{}_{\boldsymbol{j}}$, all indices have the range 1-3, repeated indices sum, and bold face indices refer to the one-forms and tetrads),

$$\begin{aligned}
\theta^{\mathbf{0}} &= (N + \alpha\nu^{\mathbf{1}}{}_i N^i)dt + \alpha\nu^{\mathbf{1}}{}_i dx^i\,, \\
\theta^{\boldsymbol{i}} &= \nu^{\boldsymbol{i}}{}_i(N^i dt + dx^i)\,, \\
\mathrm{e}_{\mathbf{0}} &= N^{-1}(\partial_t - N^i\partial_i)\,, \\
\mathrm{e}_{\boldsymbol{i}} &= (v_{\boldsymbol{i}}{}^i + \alpha\delta^1{}_{\boldsymbol{i}}N^{-1})\partial_i - \alpha\delta^1{}_{\boldsymbol{i}}N^{-1}\partial_t\,,
\end{aligned} \tag{2}$$

so that

$$ds^2 = \theta^{\mathbf{0}}\otimes\theta^{\mathbf{1}} + \theta^{\mathbf{1}}\otimes\theta^{\mathbf{0}} - \theta^{\mathbf{2}}\otimes\theta^{\mathbf{3}} - \theta^{\mathbf{3}}\otimes\theta^{\mathbf{2}}\,.$$

This definition of the metric identifies the tetrad as a null tetrad.

The self-dual bivectors on the null surface are then

$$\begin{aligned} S^1 &:= \tfrac{1}{2}[\theta^{\mathbf{1}} \wedge \theta^{\mathbf{0}} \ + \ \theta^{\mathbf{3}} \wedge \theta^{\mathbf{2}}] \,, \\ S^2 &:= \theta^1 \wedge \theta^{\mathbf{2}} \,, \\ S^3 &:= \theta^{\mathbf{3}} \wedge \theta^{\mathbf{0}} \,. \end{aligned} \tag{3}$$

Note the presence of the auxiliary variable α. It is needed to obtain all of the Einstein equations since

$$g^{00} \ = \ -\frac{2}{N}\alpha$$

so that $\alpha = 0 \Rightarrow g^{00} \ = \ 0$.

The self-dual Riemann tensor is defined from the self-dual connection

$$\begin{aligned} \Gamma^1 &:= \frac{1}{2}(\omega^{\mathbf{01}} \ + \ \omega^{\mathbf{23}}) \,, \\ \Gamma^2 &:= \omega^{\mathbf{21}} \,, \\ \Gamma^3 &:= \omega^{\mathbf{03}} \,. \end{aligned} \tag{4}$$

by

$$\tfrac{1}{2}R^A \ = \ d\Gamma^A \ + \ \eta^A{}_{BC}\Gamma^B \wedge \Gamma^C \,. \tag{5}$$

In terms of a 3+1 decomposition relative to the null surface

$$\Gamma^A \ = \ B^A dt \ + \ A^A{}_i dx^i \tag{6}$$

it follows that

$$\begin{aligned} R^A{}_{ij} &= 2A^A{}_{[i,j]} \ + \ 2\eta^A{}_{BC}A^B{}_j A^C{}_i \\ R^A{}_{0i} &= D_i B^A \ - \ A^A{}_{i,0} \,. \end{aligned} \tag{7}$$

Note, however, that when $\alpha = 0$,

$$S^3 \ = \ N\nu^{\mathbf{3}}{}_i dx^i \wedge dt$$

so that its pull back to the null surface vanishes. Since the triad densities defined by the dual of the pullback of the bivectors to the null surface are dynamical variables, this leads to three of the null surface constraints.

4. Hamiltonian

In terms of the variables defined in the 3+1 decomposition, the action, Eq. (1), takes the form

$$\begin{aligned} S &= \int d^4x\{\dot{A}^A{}_i\Sigma_A{}^i + B^A D_i\Sigma_A{}^i \ + \ N^i R^A{}_{ij}\Sigma_A{}^j \\ &\quad -\boldsymbol{N}v^i(R^1{}_{ij}\Sigma_{\mathbf{3}}{}^j + R^2{}_{ij}\Sigma_{\mathbf{1}}{}^j) + \ + \mu_i(\Sigma_{\mathbf{2}}{}^i \ - \ \alpha v^i) \ + \ \rho\alpha^2\}\,. \end{aligned} \quad (8)$$

In the above we have introduced the densitized triad variables $\Sigma_A{}^i$ to distinguish the null case from the space-like triad $E_A{}^i$:

$$\Sigma_A{}^i \ = \ \tfrac{1}{2}\eta^{ijk}S^B{}_{jk}g_{AB}$$

so that ($\nu = \det\nu^{\mathbf{i}}{}_i$)

$$\begin{aligned} \Sigma_{\mathbf{1}}{}^i &= -\nu v_{\mathbf{1}}{}^i\,, \\ \Sigma_{\mathbf{2}}{}^i &= \alpha v^i\,, \\ \Sigma_{\mathbf{3}}{}^i &= -\nu v_{\mathbf{3}}{}^i\,, \\ v^i &= \nu v_{\mathbf{2}}{}^i\,. \end{aligned} \quad (9)$$

Thus, $v^i, \Sigma_A{}^i$ are densities of weight one and $\boldsymbol{N} := N/\nu$ is a density of weight minus one. Note once again that $\alpha = 0$ implies that the dual of S^3, $\Sigma_{\mathbf{2}}{}^i \ = \ 0$. In this case, $\Sigma_1{}^i, v^i, \Sigma_3{}^i$ are a non-degenerate triad tangent to the null surface.

As in the space-like case, the action is already in canonical form, so we can identify the dynamical phase space variables as

$$(A^A{}_i, \Sigma_A{}^i)\,. \quad (10)$$

Here, unlike reference 3, the remaining variables - $B^A, v^i, \boldsymbol{N}, N^i, \mu_i, \alpha, \rho$ - are treated like Lagrange multipliers. Variation with respect to ρ leads to $\alpha = 0$ which forces t=constant surfaces to be null. Then variation with respect to α gives

$$\mu_i v^i \ = \ 0\,. \quad (11)$$

μ_i appears as one term in the propagation equation for $A^{\mathbf{2}}{}_i$. It is the condition on this Lagrange multiplier in Eq. (11) which leads to the Einstein equation which would otherwise be missing.

Varying the remaining multipliers leads to constraints on the phase space variables:

$$
\begin{aligned}
\mathcal{G}_A\ &:=\ \Sigma_A{}^i{}_{,i}\ +\ 2\eta_{ABC}A^B{}_i\Sigma_D{}^i g^{CD}\ =\ 0\,,\\
\mathcal{H}_i\ &:=\ -R^A{}_{ij}N^j\ =\ 0\,,\\
\mathcal{H}_0\ &:=\ v^i(R^1{}_{ij}\Sigma_3{}^j\ +\ R^2{}_{ij}\Sigma_1{}^j)\ =\ 0\,,\\
\phi_i\ &:=\ R^1{}_{ij}\Sigma_3{}^j\ +\ R^2{}_{ij}\Sigma_1{}^j\ =\ 0\,.\\
\mathcal{C}^i\ &:=\ \Sigma_2{}^i\ -\ \alpha v^i\ =\ 0\,.
\end{aligned}
\tag{12}
$$

It is easy to see that $\phi_i\Sigma_1{}^i = \mathcal{H}_i\Sigma_3{}^i\phi_i v^i = \mathcal{H}_0$ so that there are in total eleven constraints on the 18 dynamical variables: $\mathcal{G}_A, \mathcal{H}_i, \mathcal{H}_0$ are the usual gauge, diffeomorphism, and scalar constraints. $\mathcal{C}^i, \phi_i\Sigma_3{}^i$ result from use of the first order self-dual formalism on a null surface, hence are the null surface constraints.

In proceeding to the Hamiltonian, we may now forget about ρ and set $\alpha = 0$. (In references [3] and [4] this was done afterward.) However, in so doing we must keep not only the above constraints with $\alpha = 0$, but also the condition (11), $\mu_i v^i = 0$. As in the space-like case, the Hamiltonian is then a linear combination of the gauge, diffeomorphism, and scalar constraints, but with the addition of the null surface constraint $\Sigma_{\mathbf{2}}{}^i = 0$.

$$
H\ =\ \int d^3x\Big\{\mathbf{N}\mathcal{H}_0\ +\ N^i\mathcal{H}_i\ -\ B^A\mathcal{G}_A\ -\ \mu_i\Sigma_2{}^i\Big\}\,. \tag{13}
$$

Propagation of $\Sigma_2{}^i = 0\mathcal{G}_3 = 0$ lead to conditions on the Lagrange multipliers $v^i\mu_i$, repectively:

$$
\chi^i\ :=\ 2\delta^B{}_2D_j\Big(\mathbf{N}v^{[i}\Sigma_A{}^{j]}Q^A{}_B\Big) - 2A^3{}_jN^{[i}\Sigma_1{}^{j]} - B^3\Sigma_1{}^i\ =\ 0\,, \tag{14}
$$

$$
Q^A{}_B\ :=\ \delta_3^A\delta_B^1\ +\ \delta_1^A\delta_B^2,
$$

and

$$
\mu_i\Sigma_1{}^i\ =\ v^iR^2{}_{ij}\Sigma_3{}^j\,. \tag{15}
$$

Together with the propagation equations for $A^{\mathbf{1}}{}_iA^{\mathbf{2}}{}_i$ (15) leads to an identity when the symmetry properties of the Riemann tensor are taken into account. The remaining component, $\mu_i\Sigma_{\mathbf{3}}{}^i$, is just the null component of the conformal tensor [4]. Propagation of the remaining constraints does not lead to new constraints or additional conditions on multipliers.

Poisson brackets of $A^A{}_i\Sigma_A{}^i$ give us the propagation equations. However reality conditions are still needed to limit the solution of these equations to solutions of the Einstein equations [3,4].

5. Constraints

The constraints group themselves into first class constraints — those whose Poisson brackets vanish modulo the constraints — and second class — those whose Poisson brackets do not vanish on the constraint surface. There are five first class constraints:

$$\mathcal{G}_1 \;=\; 0, \qquad \mathcal{G}_2 \;=\; 0, \qquad \mathcal{H}'_i := \mathcal{H}_i \;+\; A^A{}_i \mathcal{G}_A \;=\; 0$$

and six second class constaints (recall that $\phi_i \Sigma_1{}^i = \mathcal{H}_i \Sigma_3{}^i \phi_i v^i = \mathcal{H}_0$):

$$\mathcal{G}_3 \;=\; 0, \qquad \mathcal{H}_0 \;=\; 0, \qquad \Sigma_3{}^i \phi_i \;=\; 0, \qquad \Sigma_2{}^i \;=\; 0\,.$$

The first class constraints generate the invariant mappings of the theory. In particular, $\mathcal{G}_1 \mathcal{G}_2$ generate the two complex parameter group of self-dual null rotations while $\mathcal{H}_i$ generate the three dimensional group of diffeomorphisms on the null surface.

Each first class constraint corresponds to two conditions on the phase space variables while each second class constraint corresponds to one condition on phase space variables. Therefore, the constraints correspond to 16 conditions on the 18 phase space variables.

Let us recall the origin of the second class constraints. First of all, $\alpha = 0$ forces t =constant to be a null surface and at the same time it requires $\Sigma_2{}^i = 0$. $\Sigma_3{}^i \phi_i = 0$ arises both from the use of a null surface and the complex structure which is introduced by the use of a self-dual connection. $v^i \phi_i = \mathcal{H}_0 = 0$ is second class because a null surface is special in that there are no compact infinitesimal mappings from a null surface to another null surface. Finally, $\mathcal{G}_3 = 0$ is second class because on a null surface there are the null directions which are left invariant by any gauge transformation. These last two conditions reduce the invariance of the formalism and therefore, reduce the number of first class constraints.

The Poisson bracket algebra of the first class constraints is not only closed, but is also a Lie algebra, the semi-direct product of the null rotations and the diffeomorphisms ($A' = 1, 2$) :

$$\begin{aligned}
\left\{ \int d^3x E^{A'} \mathcal{G}_{A'}, \int d^3x F^{B'} \mathcal{G}_{B'} \right\} \;&=\; 2 \int d^3x (E^2 F^1 \;-\; E^1 F^2) \mathcal{G}_2\,, \\
\left\{ \int d^3x E^{A'} \mathcal{G}_{A'}, \int d^3x X^i \mathcal{H}'_i \right\} \;&=\; - \int d^3x (\mathcal{L}_X E)^{A'} \mathcal{G}_{A'}\,, \\
\left\{ \int d^3x X^i \mathcal{H}'_i, \int d^3x Y^j \mathcal{H}'_j \right\} \;&=\; \int d^3x (\mathcal{L}_X Y)^i \mathcal{H}_i\,. \qquad (16)
\end{aligned}$$

As noted above, because the scalar constraint is among the second class constraints, the first class constraint algebra is a true Lie algebra. This may be helpful in constructing the quantum algebra.

The bracket algebra of the second class constraints is not as nice, but fortunately we do not need the exact expressions. However, if we define, $I = 1 \cdots 6$,

$$C_I \; := \; (\mathcal{G}_3, \mathcal{H}_0, \varSigma_3{}^j \phi_j, \varSigma_2{}^i)\,,$$

the Poisson bracket matrix has the structure

$$C \; := \; \begin{pmatrix} Q & R \\ -\tilde{R} & 0 \end{pmatrix} . \tag{17}$$

Each entry in the above is a 3×3 matrix. The inverse is easily found to be

$$C^{-1} \; := \; \begin{pmatrix} 0 & -\tilde{R}^{-1} \\ R^{-1} & R^{-1} Q \tilde{R}^{-1} \end{pmatrix} . \tag{18}$$

According to Dirac [7], the second class constraints are eliminated by defining a new bracket, now called the Dirac bracket, such that the bracket formed with the second class constraints and any other variable vanishes identically. Given two functions on the phase space, FG, the Dirac bracket is

$$\{F, G\}_D \; = \; \{F, G\} \; - \; \sum_{J,K} \{F, C_J\} C^{-1JK} \{C_K, G\}\,. \tag{19}$$

A similar result is obtained by Bergmann and Komar [8] who define new variables, starred variables, which have vanishing Poisson brackets with the second class constraints:

$$X^* \; = \; X \; - \; \sum_{IJ} \{X, C_I\} C^{-1IJ} C_J\,. \tag{20}$$

The Poisson brackets of the starred variables is equal to the Dirac brackets of the original variables. In either case, the second class constraints are then eliminated.

The structure of the Poisson bracket matrix shown above tells us that the Dirac brackets of $A^1{}_i, A^3{}_i, \varSigma_1{}^i, \varSigma_3{}^i$ are just the Poisson brackets. Therefore, in terms of these variables, one can solve the second class constraints for $A^2{}_i \varSigma_2{}^i$ which are then eliminated from further consideration.

$\varSigma_2{}^i = 0$ and $\mathcal{G}_3 = 0$ can be solved algebraically:

$$A^2{}_i \varSigma_1{}^i \; = \; \varSigma_3{}^i{}_{,i} - 2A^1{}_i \varSigma_3{}^i\,. \tag{21}$$

In principle, one can solve for the the remaining components of $A^2{}_i$ by integrating the second class part of the ϕ_i constraints. Our triad is defined so that ${\Sigma_{\mathbf{1}}}^i$ is tangent to the null generators of the null surface. Therefore, we can introduce a parameter along the null generators such that

$$ {\Sigma_{\mathbf{1}}}^i \;=\; \Sigma \frac{dx^i}{ds}\,, $$

then

$$ {\Sigma_1}^i \frac{\partial}{\partial x^i} \;=\; \Sigma \frac{d}{ds}\,. \tag{22} $$

Then one can formally integrate $v^i\phi_i = 0{\Sigma_3}^i = 0$ for the remaining components. However, to do so in general is exceedingly complicated. To give an indication of the nature of the solution, we can simplify the calculation. Asume that

$$ {\Sigma_1}^i \;=\; \Sigma {\delta^i}_1\,. \tag{23} $$

Then $(a = 2, 3)$

$$ \begin{aligned} {A^2}_a \;&=\; {Y_a}^b(s)\mathcal{A}_b(s)\,, \\ \mathcal{A}_b \;&=\; \mathcal{A}_b(\infty) \;+\; \int_\infty^s ds' Y_b^{-1c}(s')G_c(s')\,, \\ G_c \;&=\; {A^2}_{1,c} \;-\; 2{A^1}_c{A^2}_1 \;+\; \Sigma^{-1}[{A^2}_1{\Sigma_3}^1{A^3}_c \;+\; 2{A^1}_{[c,j]}{\Sigma_3}^j]\,, \\ {Y_a}^b(s) \;&=\; \exp\int_\infty^s ds'(2{A^1}_1 \;-\; \Sigma^{-1}{\Sigma_3}^j{A^3}_j)P\exp\int_\infty^s ds'\Sigma^{-1}{A^3}_a{\Sigma_3}^b(s')\,. \end{aligned} \tag{24} $$

The P in front of the exponential indicates path ordering of the product integrals. ${A^2}_i$ is no longer considered as an independent variable on the phase space. Wherever it appears in earlier expressions the above is to be substituted. On the other hand, if one is considering quantization, one need only determine the Poisson brackets of ${A^2}_i$ with the remaining variables using (21) and the above integral expressions.

6. Discussion

We have discussed the canonical formalism on a null surface using Ashtekar type variables — a self-dual connection and a densitized triad as phase space variables. The difference between the formulation on a space-like surface and a null surface has been stressed. Four extra constraints arise because one is working on a null surface. Together with these, the scalar constraint and one of the gauge constraints form a system of second class constraints. This occurs because a bubble deformation does not map a null surface into another null surface and because the null directions on a null surface are preserved under a gauge transformation. Finally, we showed how, in principle, the second class constraints can be removed while the remaining Poisson bracket algebra is unchanged. The remaining problem is to understand better the relationship of the reduced phase space variables with those which have been removed by solving the second class constraints. Then one can ask for the best way to treat the first class constraints which form a Lie algebra. The role of the first class constraints is to eliminate ${A^1}_i, {\Sigma_1}^i$, and half of ${A^3}_i, {\Sigma_3}^i$ as independent operators in the sought for quantum theory. This corresponds to the one degree of freedom one expects on a null surface.

There is another interesting direction. In the Ashtekar formalism on a space-like surface one is able to study the effect of degenerate triads. These of course lead to degenerate metrics [9]. The null surface formalism has a degenerate metric without having a degenerate triad. It would be interesting to understand the relationship of this degeneracy to that already studied.

It is my pleasure to have given this paper at a workshop to honor Andrzej Trautman. He was responsible for my first visit to Poland in 1961 and I have returned many times to discuss physics and to visit friends.

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