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PROBLEM OF COSMOLOGICAL SINGULARITY AND GAUGE THEORIES OF GRAVITATION*

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Dedicated to Andrzej Trautman in honour of his 64th birthday

The problem of cosmological singularity of the general relativity theory is discussed in the frame of gauge approach to gravitation. Equations of isotropic cosmology obtained in the frame of gauge theories of gravitation are given. Some regular cosmological solutions of these equations are discussed.

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1. Problem of cosmological singularity and attempts of its resolution

The problem of cosmological singularity (PCS) is one of the most important problems of relativistic cosmology and classical general relativity theory (GR). This problem was extensively discussed in literature (see for example [1–4]). Many physicists and cosmologists are inclined to believe that classical GR must be revised in the case of extremely high energy densities ρ, pressures p, temperatures T. The singularity "must mean for cosmology that the classical Einsteinian theory is inapplicable in the beginning of cosmological expansion of the Universe … . The singularity does not lead to absurdity of the whole theory, but it is a boundary pillar limiting its applicability" (Zel'dovich [5]). There were many attempts to resolve PCS. Corresponding works can be divided into two groups.

The first group includes papers remaining in the frame of GR (metric theories of gravity) but taking into account some effects for gravitating matter which violate the energy dominance conditions for energy-momentum

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tensor (see for example [6–14]). Some regular cosmological solutions were found in this way. So by taking into account quantum effects for matter fields (by using classical description of gravitational field), namely by considering the vacuum polarization by gravitational field and conformal anomalies of massless scalar fields, regular cosmological solutions with the Friedmann asymptotics in the past and in the future were obtained [9]. However, as the effective gravitational Lagrangian contains terms quadratic in the curvature tensor and corresponding cosmological equations contain higher derivatives, there are other solutions without Friedmann asymptotics, for which the correspondence principle with Newton's theory of gravity in case of gravitating systems with rather small energy densities is not satisfied. Supposing that at extreme conditions matter is in the particular state with equation of state $p = -\rho$ of gravitating vacuum some regular solutions were obtained. For the first time one of such solutions for closed model was found in Ref. [12] by using certain pfenomenological equation of state for matter, this model preceded inflationary cosmological models. Note that the presence of usual matter besides gravitating vacuum, generally speaking, leads to singularity of inflationary models [15]. However, if the energy density of gravitating vacuum is sufficiently large, the vacuum gravitational repulsion effect prevents singularity in the case of closed models that allows to build regular inflationary closed models in the frame of GR [14]. In case of regular cosmological solution with the vacuum initial stage [8] there is the problem of the origin of this stage. In order to resolve the problem of the origin of the Universe the radical idea of the quantum birth of the Universe was developed (see, for example [16]). The corresponding scenario includes the birth of the closed Universe from quantum fluctuations described in the initial stage by the de Sitter solution. Unlike the noted cosmological models with the beginning in time some models of eternally oscilating closed models are obtained by means of certain modification of Einstein gravitational equations [13]. The evolution of these models during the transition from compression to expansion has the de Sitter character.

The second group of papers is connected with theories generalizing Einsteinian theory of gravity. As it is known the gauge approach to gravity leads to generalization of GR [30]. At present there are different gauge theories of gravity depending on the choice of gauge group and gravitational Lagrangian (Poincaré gauge theory (PGT), metric-affine gauge theory (MAGT), its simplest variant — gauge theory of gravity in the Weyl–Cartan space-time (WCGT) *etc.*). As the gauge invariance principle is the basis of modern theories of fundamental physical interactions, it seems important to investigate PCS in the frame of gauge theories of gravity. One of the first attempts in this direction was undertaken by W. Kopczyński and A. Trautman in the frame of the simplest PGT — the Einstein–Cartan theory. Some nonsingular models with spinning matter were built [17]. Note that obtaited results depend essentially on classical model of spinning matter, and in the case of usual spinless matter the Einstein–Cartan theory is identical to GR and hence singular Friedmann models of GR are its exact solutions. The next step to investigate the PCS in the frame of PGT was made in Ref. [18] by using sufficiently general gravitational Lagrangian including both a scalar curvature and terms quadratic in the curvature and torsion tensors. The conclusion about possible existence of limiting energy density for usual spinless gravitating matter was obtained, the regularity in the metrics of corresponding cosmological solutions was ensured by gravitational repulsion effect nearby limiting energy density. Investigations of other authors confirm this conclusion [24]. Further study of homogeneous isotropic models in PGT has showed that this theory possesses important regularizing properties and leads to gravitational repulsion effect wich can be caused by different physical factors [20–23]. Analogous investigations in MAGT (WCGT) show that these gauge theories of gravity lead to the same consequences [25–26].

In this paper investigations of PCS in the frame of gauge theories of gravity are discussed. Equations of isotropic cosmology in PGT, MAGT (WCGT) are given in Section 2. Some regular cosmological solutions are discussed in Section 3.

2. Equations of isotropic cosmology in PGT and MAGT

Homogeneous isotropic cosmological models will be considered in the frame of gauge theories of gravity (PGT, MAGT, WCGT) based on sufficiently general gravitational Lagrangian including both a scalar curvature and terms quadratic in the gravitational field strengths. The gravitational Lagrangians of PGT and MAGT can be written in the following symbolic form

$$L_G^{(\mathrm{PGT})} = f_0 F + \sum_{(i)} f_i F^2 + \sum_{(k)} a_k S^2\,, \tag{1}$$

$$L_G^{(\mathrm{MAGT})} = f_0 F + \sum_{(i)} f_i F^2 + \sum_{(k)} a_k S^2 + \sum_{(p)} k_p Q^2 + \sum_{(s)} m_s SQ\,, \tag{2}$$

where $f_0 = (16\pi G)^{-1}$ (G is Newton's gravitational constant), f_i, a_k, k_p, m_s are indefinite coefficients. All invariants quadratic in the curvature F, torsion S, nonmetricity Q tensors built without using of the Levi–Civita symbol are taken into account in (1) and (2). The number of parameters f_i in (1) and (2) is different because of various symmetry properties of the curvature tensor in PGT and MAGT [30]. The number of gravitational equations, their structure are different in PGT and MAGT. However, physical consequences of PGT and MAGT practically coincide in the case of homogeneous isotropic models.

Let us consider the geometric structure of homogeneous isotropic space in MAGT defined by three tensors: metric $g_{\mu\nu}$, torsion $S^{\lambda}{}_{\mu\nu}$ and nonmetricity $Q_{\mu\nu\lambda}$[1]. In spherical coordinates $\left(x^1=r,\ x^2=\vartheta,\ x^3=\varphi\right)$ the metrics is given by

$$g_{\mu\nu}=\mathrm{diag}\left(1,\ -\frac{R^2\left(t\right)}{1-kr^2},\ -R^2\left(t\right)r^2,\ -R^2\left(t\right)r^2\sin^2\vartheta\right),\qquad\left(k=0,\pm1\right). \tag{3}$$

By using the diagonal orthonormalized tetrad $h^i{}_\mu$ corresponding to the metric (3)

$$\begin{gathered}h^i{}_\mu=\mathrm{diag}\left(1,\ \tfrac{R(t)}{\sqrt{1-kr^2}},\ R\left(t\right)r,\ R\left(t\right)r\sin\vartheta\right),\\ \left(g_{\mu\nu}=\eta_{ik}h^i{}_\mu h^k{}_\nu,\ \eta_{ik}=\mathrm{diag}\left(1,\ -1,\ -1,\ -1\right)\right),\end{gathered}$$

and supposing that the theory is invariant under transformations of spacial inversions we have the following nonvanishing components of torsion [18] $S^{\lambda}{}_{\mu\nu}=-S^{\lambda}_{\nu\mu}$: $S^1{}_{10}=S^2{}_{20}=S^3{}_{30}=S(t)$. The structure of nonmetricity tensor $Q_{\mu\nu\lambda}=Q_{\nu\mu\lambda}$ in the considered case is defined by three functions $Q_i(t)$ $(i=1,2,3)$; we have the following nonvanishing tetrad components of $Q_{\mu\nu\lambda}$ [31]:

$$\begin{aligned}Q_{\hat1\hat1\hat0}&=Q_{\hat2\hat2\hat0}=Q_{\hat3\hat3\hat0}=Q_1\left(t\right),\\ Q_{\hat0\hat0\hat0}&=Q_2\left(t\right),\\ Q_{\hat0\hat1\hat1}&=Q_{\hat0\hat2\hat2}=Q_{\hat0\hat3\hat3}=Q_3\left(t\right).\end{aligned} \tag{4}$$

Then nonvanishing components of the curvature tensor $F^{\mu\nu}_{\rho\sigma}=-F^{\mu\nu}_{\sigma\rho}$ are determined by three functions A, B, and C [26]:

$$\begin{aligned}F^{01}_{01}&=F^{02}_{02}=F^{03}_{03}=A-C,\\ F^{10}_{10}&=F^{20}_{20}=F^{30}_{30}=A+C,\\ F^{12}_{12}&=F^{13}_{13}=F^{23}_{23}=B,\end{aligned}$$

[1] μ, ν, …are holonomic (world) indices; i, k …are anholonomic (tetrad) indices. Numerical tetrad indices are denoted by means of a sign "ˆ" over them.

where

$$
\begin{aligned}
A &= \frac{\left[\dot{R} - 2RS_q\right]^{\cdot}}{R} + \frac{1}{4}Q_3\left(Q_1 + Q_2\right), \\
B &= \frac{k + \left[\dot{R} - 2RS_q\right]^2}{R^2} - \frac{1}{4}Q_3^2, \\
C &= \frac{1}{2}\frac{\dot{R} - 2RS_q}{R}\left(Q_1 + Q_2\right) + \frac{1}{2}\frac{\left(RQ_3\right)^{\cdot}}{R}, S_q = S - \frac{1}{4}Q_1 + \frac{1}{4}Q_3, \quad (5)
\end{aligned}
$$

and a dot denotes differentiation with respect to time and also ${F^{[01]}}_{01} = A$, ${F^{(01)}}_{01} = -C$ and ${F^{(12)}}_{12} = 0$. The Bianchi identity is reduced to the following relation

$$
\dot{B} + 2H\left(B - A\right) + 4AS_q + CQ_3 = 0, \qquad \left(H = \frac{\dot{R}}{R}\right). \qquad (6)
$$

In the frame of PGT we have to put in (5)–(6) $Q_i = 0$ and hence $C = 0$. Supposing that gauge theory of gravity leads to cosmological equations for the metric in the form of differential equations of the same order as in GR, the following solution of gravitational equations of MAGT for homogeneous isotropic models in the Weyl–Cartan space time was obtained [26]:

$$
\begin{aligned}
Q_1 \ + \ Q_2 = 0, \qquad Q_3 &= 0, \qquad C = 0\,, \\
A &= -\frac{1}{12f_0}\frac{\rho + 3p + \frac{\beta}{2}\left(\rho - 3p\right)^2}{1 - \beta\left(\rho - 3p\right)}\,, \\
B &= \frac{1}{6f_0}\frac{\rho - \frac{\beta}{4}\left(\rho - 3p\right)^2}{1 - \beta\left(\rho - 3p\right)}\,, \\
S - \frac{1}{4}Q_1 &= -\frac{1}{4}\frac{d}{dt}\ln\left|1 - \beta\left(\rho - 3p\right)\right|, \qquad (7)
\end{aligned}
$$

where $\beta = -\dfrac{f}{3f_0^2}$, f is a certain linear combination of coefficients f_i. The solution (7) is valid if certain restrictions for indefinite parameters of gravitational Lagrangian take place. Besides relations (7) gravitational equations lead to additional relation which allows to determine the torsion S and nonmetricity Q_1 functions. In dependence on restrictions for coefficients a_k, k_p, m_s there are three following possibilities:

1. $Q_1 = 0$, $S \neq 0$: solution (7) describes in this case isotropic models in the Riemann–Cartan space-time and it was obtained at the first time in the frame of PGT [18];

2. $S = 0$, $Q_1 \neq 0$: the solution (7) corresponds to isotropic models in the Weyl space-time and it was obtained by using gravitational Lagrangian of MAGT of type (1) [25];

3. $S = \sigma Q_1$, $(\sigma \neq 0)$. The cofficient σ is certain function of a_k, k_p, m_s. According to (7) we have

$$\begin{aligned} Q_1 &= \frac{1}{1-4\sigma}\frac{d}{dt}\ln|1-\beta\,(\rho-3p)|, \qquad \left(\sigma\neq\tfrac{1}{4}\right) \\ S &= \frac{\sigma}{1-4\sigma}\frac{d}{dt}\ln|1-\beta\,(\rho-3p)|\,. \end{aligned} \tag{8}$$

Solutions (7)–(8) describe homogeneous isotropic models in the Weyl–Cartan space-time. Note that the same solutions take place in the frame of WCGT.

Cosmological equations for the metrics can be obtained by using the solution (7) and equating expressions for curvature functions A and B (5) and (7):

$$\frac{\left[\dot{R}+R\left(\ln\sqrt{|1-\beta\,(\rho-3p)|}\,\right)^{\cdot}\right]^{\cdot}}{R} = -\frac{1}{12f_0}\frac{\rho+3p+\frac{\beta}{2}\,(\rho-3p)^2}{1-\beta\,(\rho-3p)}\,, \tag{9}$$

$$\frac{k}{R^2}+\left\{\frac{d}{dt}\ln\left[R\sqrt{|1-\beta(\rho-3p)|}\,\right]\right\}^2 = \frac{1}{6f_0}\frac{\rho-\frac{\beta}{4}\,(\rho-3p)^2}{1-\beta\,(\rho-3p)}. \tag{10}$$

Note that unlike GR terms of L_G quadratic in the curvature tensor *do not lead to high derivatives in cosmological equations* (9)–(10). The conservation law in usual form follows from (9)–(10)

$$\dot{\rho}+3H\,(\rho+p)=0. \tag{11}$$

The generalised cosmological Friedmann equation (GCFE) (10) deduced at first in PGT [18] and conservation law (11) determine the evolution of functions $R(t)$ and $\rho(t)$ if equation of state of matter $p=p(\rho)$ is known. We have the following integrals of Eqs (10)–(11)

$$R=\exp\left(-\frac{1}{3}\int\frac{d\rho}{\rho+p\,(\rho)}\right)\,, \tag{12}$$

$$\begin{aligned} t_2-t_1 \;=\; &\int\limits_{R_1}^{R_2}\left[1+\frac{\beta}{2}(\rho+9p)-\frac{9}{2}\beta(\rho+p)\frac{d\,p}{d\,\rho}\right][1-\beta(\rho-3p)]^{-1/2} \\ &\times\left\{-k\,[1-\beta(\rho-3p)]+\frac{1}{6f_0}\left[\rho-\frac{\beta}{4}(\rho-3p)^2\right]\,R^2\right\}^{-1/2}d\,R\,. \end{aligned} \tag{13}$$

The integrals (12)–(13) together with formulas for noneinsteinian characteristics (torsion, nonmetricity) give cosmological solutions for homogeneous isotropic models in the frame of gauge theories of gravity.

3. Regular cosmological solutions in gauge theories of gravity

According to GCFE (10) and integral (13) the behaviour of metric coincides practically with that of GR in the case of sufficiently small energy densities $\rho \ll |\beta|^{-1}$. This means that cosmological solutions in gauge theories of gravity possess the Friedmann asymptotics. However, in the case of extremely high energy densities $\rho \sim |\beta|^{-1}$ predictions of GR and gauge theories of gravity differ essentially. In particular, gauge theories of gravity lead to gravitational repulsion effect at extreme conditions and to possible existence of limiting energy density for usual gravitating systems. As the behaviour of cosmological solutions (13) depends essentially on an equation of state of matter, we need to know this equation and its evolution at extreme conditions to build more realistic cosmological models. Since we do not know this equation, various models with phenomenologically given equations of state were investigated.

The simplest cosmological models of the hot Universe including two noninteracting components — radiation ($p_r = \frac{\rho_r}{3}$) and dust ($p_d = 0$) were considered in Ref. [18]. In the beginning of cosmological expansion the contribution of dust to energy density is negligible, but dust energy density gives the main contribution during later stages of cosmological expansion. The regularity in the metric ($R > 0$) is ensured by the existence of finite limiting energy density at the bounce. From mathematical point of view this means that corresponding solutions (13) of GCFE (10) possess extremal points for $R(t)$ and $\rho(t)$.

Putting $t = 0$ at an extremum we will analyse near this point the behaviour of functions $R(t)$ and $\rho(t)$ written in the form

$$R(t) = R_e(1 + r_1 t^2 + \ldots), \qquad \rho(t) = \rho_e + \rho_1 t^2 + \ldots\,, \tag{14}$$

where $\rho_1 = -3(\rho_e + p_e)r_1$, $p_e = p(\rho_e)$, $R_e > 0$. According to (10) the extremal parameters R_e, ρ_e, p_e satisfy the conditions

$$\rho_e - 3p_e = \beta^{-1}, \qquad r_1 = -\frac{f_0}{8f}\left[1 - 3\frac{dp}{d\rho}\bigg|_{\rho=\rho_e}\right]^{-1} \quad \left(p \neq \frac{\rho}{3}\right) \tag{15}$$

or

$$\rho_e-\frac{\beta}{4}\left(\rho_e-3p_e\right)^2=\frac{6kf_0}{R_e^2}\,Z \qquad \left(Z\equiv 1-\beta(\rho_e-3p_e)\neq 0\right)\,,$$
$$\left[1+\frac{3\beta}{2Z}(\rho_e+p_e)\left(1-3\frac{d\,p}{d\,\rho}\bigg|_{\rho=\rho_e}\right)\right]^2 r_1=\frac{k}{2R_e^2}\left[1-\frac{3\beta}{2Z}(\rho_e+p_e)\right.$$
$$\left.\times\left(1-3\frac{d\,p}{d\,\rho}\bigg|_{\rho=\rho_e}\right)\right]-\frac{1}{8f_0Z}\left(\rho_e{+}p_e\right)\left[1-\frac{\beta}{2}(\rho_e-3p_e)\left(1-3\frac{d\,p}{d\,\rho}\bigg|_{\rho=\rho_e}\right)\right]. \tag{16}$$

The formulas (15)–(16) can be generalized easily for multicomponent models [14]. The character of gravitational interaction near the point $R_e(\rho_e)$ depends on the sign of r_1. If $r_1>0$ the gravitational interaction has the character of repulsion and the solution (14) describes the regular transition from compression to expansion. Similar situation takes plase in the case of discussed above two-component models if $\beta>0$, limiting energy density is defined by means of the first relation (15). Note that for $t\to 0$ the curvature, torsion and nonmetricity diverge: $A,\,B\;\sim t^{-2},\quad S,\,Q_1\sim t^{-1}$. Analogous situation takes plase in the case of other equations of state of matter $p=p(\rho)$ for which $p<\dfrac{\rho}{3}$. In order to avoid singularity of curvature and torsion (nonmetricity) in cosmological solutions of gauge theories of gravity, the regularizing role of different physical factors was studied. It was shown that spontaneous symmetry breaking by gravitational field [22], viscosity [23], scalar fields [28], gravitating vacuum [20, 27, 28] etc. can play the role of regularizing factors, which change effective equation of state and in consequence obtained cosmological solutions are regular in the metric, curvature, torsion (nonmetricity). Note that torsion and nonmetricity near extremal points in these cosmological solutions are small although the evolution of the scale factor differs significantly from that of GR. The behaviour of the metric in this case depends essentially on terms of L_G quadratic in the curvature tensor; however, this dependence differs significantly from that of a metric theory in a Riemannian space-time (*cf.* [9]).

From the physical point of view regular solutions for inflationary cosmological models are most interesting. We shall consider corresponding results obtained in the frame of gauge theories of gravity [19, 20, 27, 28]. Taking into consideration the gravitating vacuum of the Higgs fields which can be manifested in the beginning of cosmological expansion, we write the energy density and pressure in the form: $\rho=\rho_m+\rho_v$, $p=p_m+p_v$, where the values of matter and vacuum are denoted by indices m and v respectively. We shall put that for limiting time intervals the vacuum energy density is constant, $\rho_v=\text{const}>0$ and $p_v=-\rho_v$. In the frame of PGT and

MAGT (WCGT) the gravitating vacuum is described by the de Sitter metric satisfying the GCFE and vanishing torsion and nonmetricity [19, 20]. By using various equations of state of matter different inflationary cosmological solutions were considered. Two-component isotropic models including radiation ($p_m = \dfrac{\rho_m}{3}$) and gravitating vacuum were studied [20]. In this case the GCFE has the form of the cosmological Friedmann equation of GR with some effective gravitational constant

$$k + \dot{R}^2 = \frac{\rho_m + \tilde{\rho}_v}{6\tilde{f}_0} \qquad \Big(\tilde{f}_0 = f_0(1 - 4\beta\rho_v), \quad \tilde{\rho}_v = \rho_v(1 - 4\beta\rho_v)\Big) \,. \tag{17}$$

If vacuum energy density is sufficiently large, $\rho_v > \frac{1}{4\beta}$, effective gravitational constant $\widetilde{G} = \dfrac{1}{16\pi\tilde{f}_0}$ in (17) is negative which leads to vacuum gravitational repulsion effect, which prevents cosmological singularity. In this case limiting energy density for matter $\rho_{me} = |\tilde{\rho}_v| + \frac{6k\tilde{f}_0}{R_e^2} > 0$ and acceleration coefficient $r_1 = \frac{\rho_v}{6f_0} - \frac{k}{2R_e^2} > 0$ are determined from (16)[2].

By using qualitative theory of dynamic systems of the second order, solutions for inflationary isotropic models (13) were analysed in the case of equations of state for matter in the form of linear dependence between pressure and energy density with $p_m < \frac{\rho_m}{3}$ [27] and $p_m > \frac{\rho_m}{3}$ [28]. Multicomponent inflationary models including radiation, dust, massless scalar fields and gravitating vacuum, for which effective equation of state changes with the time, were analysed in Ref. [29]. Inflationary cosmological solutions with regular metric, curvature, torsion, nonmetricity were found in the case $\rho_v > \frac{1}{4\beta}$. All solutions noted above for inflationary models have the de Sitter asymptotics. The transition to the Friedmann asymptotics for these models includes the first order phase transition in the form of a jump, $\rho_v \to \rho_v' < \frac{1}{4\beta}$ followed by continuous transition to the Friedmann asymptotics.

Note that values of limiting energy density and limiting temperature in considered inflationary models depend on equation of state of matter and on value of the parameter β and can be less than Planckian values. Physical processes, in particular phase transitions in the beginning of cosmological expansion, depend essentially on the values of limiting energy density and limiting temperature. It is interesting to investigate indicated processes in dependence on these values.

[2] The derivative $\frac{dp}{d\rho}$ in Eqs (13), (15), (16) must be replaced by $\frac{dp_m}{d\rho_m}$.

It is interesting also to note that Eqs (16) have the solution for closed model ($k = +1$) in the limit to GR ($\beta \to 0$):

$$\rho_{me} + \rho_v = \frac{6f_0}{R_e^2}, \quad r_1 = -\frac{1}{24f_0}\left(\rho_{me} + 3p_{me} - 2\rho_v\right). \tag{18}$$

If the vacuum energy density is sufficiently large

$$\rho_v > \tfrac{1}{2}\left(\rho_{me} + 3p_{me}\right), \tag{19}$$

we have $r_1 > 0$ which corresponds to prevention of singularity by virtue of gravitational repulsion effect provoked by the gravitating vacuum. After the phase transition "vacuum $\to$ matter" the evolution has the Friedmann character and the expansion must be changed by compression (r_1 in (18) is negative if $\rho_v \to 0$). If the phase transition "matter $\to$ vacuum" takes place during the compression stage with ρ_v satisfied to (19) we shall have regular transition from compression to expansion. This means that regular closed inflationary model can be built in the frame of GR [14].

In conclusion note that besides cosmological models the integral (13) contains solutions corresponding to some hypothetical superdense gravitating systems [21] having a lower limit for admissible values of energy density $\rho \sim |\beta|^{-1}$. Some regular solutions for these systems were obtained [28, 26], the dynamics of superdense gravitating systems is essentially noneinsteinian and in considered isotropic approximaton has the character of pulsations between minimum and maximum values of energy density.

4. Conclusion

Investigations of homogeneous isotropic models in the frame of gauge theories of gravitation show that the behaviour of gravitating systems at extremal conditions in these theories differs essentially from that of GR. Gauge theories of gravitation (PGT, MAGT, WCGT) possess important regularizing properties and allow to build regular cosmological models. Analogous conclusion was obtained also in the case of homogeneous anisotropic models [32] and probably is valid in the case of arbitrary gravitating systems.

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