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ACAUSAL PGT MODES AND THE NONLINEAR CONSTRAINT EFFECT*

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Dedicated to Andrzej Trautman in honour of his 64^{th} birthday

The effect of constraints on the initial value and acausal propagation problems in the Poincaré Gauge Theory is considered with the aid of the linearized theory and the Hamiltonian analysis. To linear order there are no difficulties, however non-linearities in any extra "if" constraints can cause serious problems involving a change in the number and type of constraints as well as acausal propagation modes. Specific examples are given. Only very special parameter choices are expected to avoid these problems. A similar story is predicted for most other gauge theories of gravity. This type of analysis holds promise as a strong test for alternate gravity theories.

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1. Introduction

With the exception of Einstein's geometric gravity theory, the fundamental interactions are described by gauge theories. Consequently "gauge theories of gravity" have been proposed. Here we focus on the Poincaré Gauge Theory (PGT) as developed by Hayashi and Shirafuji [1] and by Hehl [2, 3], but our considerations have a much wider application.

A viable theory should satisfy certain theoretical criteria such as having non-tachyonic propagating linearized modes carrying positive energy. Moreover theories should have an appropriate mathematical structure. This includes a well-posed initial value problem: the basic requirement is the Cauchy–Kovalevska theorem; beyond that, the propagation of dynamic modes should be described by hyperbolic evolution equations with well behaved characteristics.

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Kopczyński [4] provided an early hint of the difficulties. He noted that the one-parameter teleparallel theory (NGR [5]) had predictability problems. Further analysis [6] revealed that the problem was not generic: strangely, it occurred only for a special class of solutions. Cheng verified this via a Hamiltonian analysis using the Dirac constraint algorithm [7]. His investigation revealed a curious effect: the chain of constraints could "bifurcate" for certain field values, so that the number and/or type of constraints would depend on the phase space variables. We have long suspected that the PGT was vulnerable to this disease. Here we report on our discoveries.

The next section summarizes the PGT. In Section 3 the "problems" found by Dimakis [8] and Lemke [9,10] are described. These problems were reconsidered in [11], which is the basis for much of our presentation here. Section 4 summarizes the result of the linearized PGT analysis. The Dirac constraint algorithm is discussed in Section 5; a crucial feature of non-linear constraints is noted. Section 6 describes the PGT Hamiltonian analysis of Blagojević and Nicolić. An instructive example, the non-linear Proca field, is discussed in Section 7. Section 8 describes the new results of H. Chen, concerning a special PGT case with only one constrained mode. Section 9 reports on new computations of H.J. Yo regarding two specific dynamic PGT cases. The final section is our concluding discussion.

2. The Poincaré gauge theory

In the PGT ([1–3]) the gauge potentials for translations and Lorentz transformations are the orthonormal frame field (tetrads) $e_i{}^\alpha$ and the metric compatible connection $\Gamma_{i\alpha}{}^\beta$. The associated field strengths are the *torsion* $T_{ij}{}^\alpha = \partial_i e_j{}^\alpha + e_j{}^\beta \Gamma_{i\beta}{}^\alpha - (i \leftrightarrow j)$, and the *curvature* $R_{ij\alpha}{}^\beta = \partial_i \Gamma_{j\alpha}{}^\beta + \Gamma_{i\mu}{}^\beta \Gamma_{j\alpha}{}^\mu - (i \leftrightarrow j)$. Our metric signature is $(-+++)$, we use $i,j,k,\dots$ to denote coordinate indices, and $\alpha,\beta,\gamma,\dots$ for the frame indices. Indices denoted by $a,b,\dots$ refer to spatial coordinates.

The Lagrangian $\mathcal{V} = \mathcal{V}_{\mathrm{G}}(e,R,T)$ is assumed to be at most quadratic in the field strengths. Varying with respect to the potentials gives (vacuum) field equations of the form

$$\mathcal{F}_\alpha{}^i := D_j\,\mathcal{H}_\alpha{}^{ij} - \varepsilon_\alpha{}^i = 0, \qquad \mathcal{F}_{\alpha\beta}{}^i := D_j\,\mathcal{H}_{\alpha\beta}{}^{ij} - \varepsilon_{\alpha\beta}{}^i = 0, \tag{1}$$

where $\varepsilon_\alpha{}^i := e^i{}_\alpha \mathcal{V}_{\mathrm{G}} - T_{\alpha j}{}^\gamma\,\mathcal{H}_\gamma{}^{ji} - R_{\alpha j}{}^{\gamma\delta}\mathcal{H}_{\gamma\delta}{}^{ji}$, and $\varepsilon_{\alpha\beta}{}^i := \mathcal{H}_{[\beta\alpha]}{}^i$. The field momenta

$$\mathcal{H}_\alpha{}^{ij} := \frac{\partial \mathcal{V}_{\mathrm{G}}}{\partial T_{ji}{}^\alpha} = \frac{e}{l^2}\sum_{n=1}^{3} a_n \overset{(n)}{T}{}^{ji}{}_\alpha, \tag{2}$$

$$\mathcal{H}_{\alpha\beta}{}^{ij} := \frac{\partial \mathcal{V}_{\mathrm{G}}}{\partial R_{ji}{}^{\alpha\beta}} = -\frac{e a_0}{l^2} e^i{}_{[\alpha} e^j{}_{\beta]} + \frac{e}{\kappa}\sum_{n=1}^{6} b_n \overset{(n)}{R}{}^{ji}{}_{\alpha\beta}, \tag{3}$$

are linear in the field strengths. The three ${}^{(n)}T^{ji}{}_{\alpha}$ and the six ${}^{(n)}R^{ji}{}_{\alpha\beta}$ are the irreducible parts of the torsion and the curvature respectively. The a_i and b_i are free coupling constants.

3. The problems

For many ranges of its 10 parameters the PGT was regarded as experimentally and theoretically viable. Then Dimakis investigated the initial value problem [8]. He found parameter conditions necessary for the field equations to satisfy the Cauchy–Kovalevska theorem (and some additional *nonlinear* conditions necessary to achieve certain prescribed hyperbolic forms). These Cauchy–Kovalevska conditions assure the non-degeneracy of the kinetic Hessian matrix. One contradicts the "viable" condition, $2a_1 + a_2 = 0$, which was presumed to be necessary for a good theory. Soon thereafter Lemke [9] concluded that the same conditions were needed to prevent tachyonic shock waves in the theory. Because of the conflict with the "viable" condition, it was even proposed that the PGT should be discarded. Then Hecht, Lemke and Wallner [10] considered the question "Can the Poincaré Gauge Theory be Saved?" and arrived at an affirmative answer having found a "cure" for the PGT shock wave problem. Their idea was that the acausal modes can be gauged away because of a natural symmetry.

These issues have since been reconsidered (Hecht, Nester and Zhytnikov [11]). With the aid of the linearized theory and the Hamiltonian analysis, they concluded that all of the alleged problems stemmed from overlooking secondary constraints. The main points of their arguments are included in subsequent sections.

4. The linearized PGT

The linearized PGT analysis is revealing. In addition to the graviton there are possible propagating modes with $\text{spin}^{\text{parity}} = 2^+, 2^-, 1^+, 1^-, 0^+, 0^-$. All possible propagating "massive" modes, with $v < c$ (no tachyons) and $E \geq 0$ (no ghosts), were found in [1, 12]. The critical kinetic and mass parameter combinations appear in Table I.

The PGT has many distinct cases, with the torsion modes showing different behavior: some modes frozen and some dynamic. The requirements preclude the possibility of more than 3 good dynamic propagating "massive" torsion modes. A typical propagating mode has 3 vanishing parameter conditions, and the remaining parameters must satisfy several inequalities due to the "no ghost & no tachyon" conditions. The equations are then well posed and hyperbolic, having the Minkowski metric as their characteristic cones.

TABLE I

Critical parameter combinations and their associated constraints

J^p	Kinetic Parameter Combinations	Primary and Secondary Constraints	Mass Parameter Combinations
0^+	*(i)* a_2 *(ii)* b_4+b_6	$\phi^a{}_a,\ \chi^b{}_b$ $\phi^{a0}{}_a,\ \chi^{b0}{}_b$	$a_0,\ 2a_0+a_2$
1^+	*(i)* a_1+2a_3 *(ii)* b_2+b_5	$\phi_{[ab]},\ \chi_{[ab]}$ $\phi_{[a0b]},\ \chi_{[a0b]}$	$a_1-a_0,\ a_0/2+a_3$
2^+	*(i)* a_1 *(ii)* b_1+b_4	$\overline{\phi}_{(ab)},\ \overline{\chi}_{(ab)}$ $\overline{\phi}_{(a0b)},\ \overline{\chi}_{(a0b)}$	$a_0,\ a_1-a_0$
1^-	*(i)* $2a_1+a_2$ *(ii)* b_4+b_5	$\phi_{a0},\ \chi_{b0}$ $\phi_{ab}{}^b,\ \chi_{ab}{}^b$	$a_1-a_0,\ 2a_0+a_2$
0^-	b_2+b_3	$\phi^A,\ \chi^A$	$a_0/2+a_3$
2^-	b_1+b_2	$\phi^T{}_{abc},\ \chi^T{}_{abc}$	a_1-a_2

For each of these cases the kinetic Hessian matrix is *degenerate* but the propagating modes *always* satisfy a good flat space hyperbolic wave equation. Hence, there are *never* any acausal shocks. It appears that the PGT initial value problem *is well posed*—at least to the linear order.

5. Dirac constraint algorithm and constraint bifurcation

The Hamiltonian analysis is also a revealing approach. The Dirac constraint algorithm [13,14] applies to any theory. It *always* guarantees that the necessary time derivatives for the dynamic variables are all found, thereby satisfying the key requirement of the Cauchy–Kovalevska theorem. The starting point is a Lagrangian with a degenerate kinetic Hessian matrix. Consequently there are some primary constraints, $\phi_A \approx 0$ (i.e., they vanish "weakly"). The Hamiltonian, $H = H_0 + V^A\phi_A$, includes these with undetermined multipliers. The time derivative of any function has the form

$$\dot{f} \approx \{f, H_0\} + V^A\{f, \phi_A\}. \tag{4}$$

In particular, the primary constraints must be preserved in time:

$$\dot{\phi}_B \approx \{\phi_B, H_0\} + V^A\{\phi_B, \phi_A\} \approx 0. \tag{5}$$

This condition generally gives some information regarding the undetermined multipliers V^A and/or gives rise to some secondary constraints which then must also be preserved. After the constraints are all found they can be classified. The classification separates the dynamical degrees of freedom from those which are frozen and all of the gauge symmetries are identified. Gauge symmetries are associated with *first class* constraints which have vanishing ("on shell") Poisson brackets with all other constraints. The remaining (*second class*) constraints occur in pairs with non-vanishing Poisson brackets; each pair can be used to eliminate a nondynamical conjugate pair of variables.

The main feature for our work is that the "matrix" of the Poisson brackets of the constraints, $\{\phi_A, \phi_B\}$, may not have constant rank. This can happen only if the constraints are non-linear. This matrix is the key to whether one can determine the multipliers (which usually turn out to be the velocities missing in the Lagrangian). Strange behavior may occur as one approaches a point in the phase space where the rank changes (as we already noted above in connection with the NGR theory). At such points the number and type of constraints (and thus the number of gauges and physical degrees of freedom) are not constant. To appreciate this let us consider the PGT Hamiltonian analysis.

6. The PGT Hamiltonian analysis

For the PGT, we must certainly have the (first class) constraints associated with the local Poincaré gauge symmetry of space time. In addition, for each of the kinetic coefficient parameter combinations in Table I which vanishes, there is an *extra* degeneracy. Thus for the PGT there are numerous possible degenerate cases: any combination of the kinetic coefficient parameters could vanish. *All* of these possibilities have been systematically investigated by Blagojević and Nikolić [15] who developed a wonderful "if" constraint technique which enabled them to identify *all* possible constraints, at least in the "generic" cases. Associated with the torsion there are four primary "if" constraints:

$$\phi^a{}_a := \mathcal{H}_{a0}{}^a = -\frac{e}{l^2} a_2 T_{0a}{}^a, \tag{6a}$$

$$\phi_{[ab]} := \mathcal{H}_{[ab]0} + \frac{e}{3l^2}(a_1 - a_2)T_{ab0} = \frac{e}{3l^2}(a_1 + 2a_3)T_{0[ab]}, \tag{6b}$$

$$\overline{\phi}_{(ab)} := \overline{\mathcal{H}}_{(ba)0} = \frac{e}{l^2} a_1 \overline{T}_{0(ab)}, \tag{6c}$$

$$\phi_{a0} := \mathcal{H}_{0a0} + \frac{e}{3l^2}(a_1 - a_2)T_{ab}{}^b = \frac{e}{3l^2}(2a_1 + a_2)T_{0a0}. \tag{6d}$$

Associated with the curvature there are six primary "if" constraints:

$$\phi_{a0}{}^a := 2\mathcal{H}_{a0}{}^a{}_0 + \frac{e}{2\kappa}(b_4 - b_6)R_{ab}{}^{ab} - \frac{3e}{l^2}a_0 = \frac{e}{\kappa}(b_4 + b_6)R_{a0}{}^{a0}, \tag{7a}$$

$$\phi_{[a0b]} := 2\mathcal{H}_{[boa]0} - \frac{e}{\kappa}(b_2 - b_5)R_{[b}{}^c{}_{a]c} = \frac{e}{\kappa}(b_2 + b_5)R_{[b0a]0}, \tag{7b}$$

$$\overline{\phi}_{(a0b)} := 2\overline{\mathcal{H}}_{0(ab)0} - \frac{e}{\kappa}(b_1 - b_4)\overline{R}_{(bca)}{}^{c} = \frac{e}{\kappa}(b_1 + b_4)\overline{R}_{(b0a)0}, \tag{7c}$$

$$\phi_{ab}{}^{b} := 2\mathcal{H}_{ab}{}^{b}{}_{0} - \frac{e}{\kappa}(b_4 - b_5)R_{ab0}{}^{b} = \frac{e}{\kappa}(b_4 + b_5)R_{0ba}{}^{b}, \tag{7d}$$

$$\phi_{A} := 2\mathcal{H}_{A} - \frac{e}{\kappa}(b_2 - b_3)\epsilon^{abc}R_{abc0} = \frac{e}{\kappa}(b_2 + b_3)\epsilon^{abc}R_{0abc}, \tag{7e}$$

$$\phi^{T}{}_{abc} := 2\mathcal{H}^{T}{}_{abc0} + \frac{e}{\kappa}(b_1 - b_2)R^{T}{}_{abc0} = \frac{e}{\kappa}(b_1 + b_2)R^{T}{}_{0cab}. \tag{7f}$$

In each case, 'if' the critical kinetic parameter combination on the rhs vanishes the corresponding "if" constraint is included in the list of primary constraints.

For the generic PGT they found that there are only primary and secondary constraints (no tertiary constraints). The pattern is given in Table I. The Poisson bracket between either the paired primaries (if both exist), or a primary and the secondary it generates, was found to be generically nonvanishing and hence "second class". The value of these Poisson brackets are just the (generally nonvanishing) constant "mass" parameters of Table I *plus* some field dependent terms of non-linear origin. (The possibility of modifications due to these nonlinear terms was mentioned but they did not hint at their overwhelming importance.) Aside from such nonlinear effects, the Hamiltonian analysis is in complete accord with the linearized theory analysis of the propagating modes [1, 12].

7. An instructive example

Difficulties are to be expected in non-linear systems, based on past experience with coupling problems and acausal modes [16, 17]. Some time ago it was realized that there are generally problems with interacting higher spin fields. In addition to a change in the number and type of constraints and degrees of freedom (see, *e.g.* , [14, 17]), they are especially vulnerable to acausal propagation modes. An early example [16] is a good model for some effects we expect in the PGT.

Consider the non-linear Proca field:

$$\mathcal{L} = -\frac{1}{4}F^{ij}F_{ij} - \frac{1}{2}m^2A^iA_i - \frac{1}{4}\lambda(A^iA_i)^2, \tag{8}$$

where, $F_{ij} = \partial_i A_j - \partial_j A_i$ and m and λ are constants. We note the qualitative similarity to terms in the PGT Lagrangian $\sim (\partial\Gamma + \Gamma\Gamma) + (\partial e + \Gamma e)^2 + (\partial\Gamma + \Gamma\Gamma)^2$. The field equations are

$$\partial_i F^{ij} - m^2A^j - \lambda A^iA_iA^j = 0. \tag{9}$$

But this is not 4 independent dynamic field equations for $\ddot{A}_i$; in fact $\ddot{A}_0$ does not appear explicitly. Applying ∂_j to the field equation reveals that it implicitly imposes the condition

$$(m^2 + \lambda A^i A_i)\partial_j A^j + 2\lambda A^i A^j \partial_j A_i = 0. \tag{10}$$

One way to find the characteristics is to solve Eq. (10) for $\partial_j A^j$, substitute the result into the field equation (9) to get a second order equation for all components of A^i. The characteristics can then be found by standard procedures. The result is that, in addition to the usual null cone, there is a characteristic surface with normal n^i satisfying

$$(m^2 + \lambda A^j A_j) n^i n_i = -2\lambda (n_j A^j)^2. \tag{11}$$

This characteristic is spacelike if $\lambda > 0$, then the system has acausal propagation modes.

Instead the Hamiltonian formulation can be used. The canonical momenta $\pi^i := \partial\mathcal{L}/\partial\dot{A}_i$ are $\pi^b = F_0{}^b$, $\pi^0 \equiv 0$, and the Hamiltonian density is

$$\tfrac{1}{2}\pi^a\pi_a + \tfrac{1}{4}F^{ab}F_{ab} - A_0\partial_c\pi^c + \tfrac{1}{2}m^2(A^bA_b - A_0^2) + \tfrac{1}{4}\lambda(A^bA_b - A_0^2)^2. \tag{12}$$

The primary constraint $\pi^0 \approx 0$ gives rise to the secondary constraint

$$\chi := \partial_c\pi^c + m^2 A_0 + \lambda(A^bA_b - A_0A_0)A_0 \approx 0. \tag{13}$$

Their Poisson bracket has the value

$$\{\pi^0(x), \chi(y)\} = \delta^3(x-y)(m^2 + \lambda(A^bA_b - 3A_0A_0)). \tag{14}$$

Hence this constraint pair is second class — unless the rhs vanishes. The breakdown of the second class constraint condition (which is essentially the implicit function condition for the solvability of (13) for A_0) once again reveals the condition (in 3+1 form) for x^0=constant to be a characteristic surface.

Note that the $m = 0$ case of the system (8) has gauge freedom to linear order but not non-linearly. This example casts doubts upon conclusions based upon linearized theory especially for "massless" cases with gauge freedoms.

8. One mode suppressed

The special PGT case with $(2a_1 + a_2) = 0 = (b_4 + b_5)$ with all other parameters having generic values was considered by Chen [18]. Although this configuration suppresses only the 1^- mode, and thus is not physically acceptable due to "ghosts" and "tachyons", it nevertheless affords a simple illustration of the phenomena we wish to understand. There are only two relevant primary "if" constraints: ϕ_{a0} and $\phi_{ab}{}^{b}$ and they form a second class pair. For even greater simplicity he considered the specific parameter values $a_1 = a_3 = b_1 = b_2 = b_3 = b_6 = -1$, $a_2 = 2$ and $b_4 = b_5 = 0$. The significant quantity is the Poisson bracket of these two constraints:

$$\{\phi^{c}{}_{0}(x), \phi_{ab}{}^{b}(y)\} = (-\frac{2e}{\ell^2}\delta^{c}{}_{a} - \mathcal{H}_{0a}{}^{0c} + \mathcal{H}_{0b}{}^{0b}\delta^{c}{}_{a})\delta(x,y). \tag{15}$$

Generically, this combination is nonvanishing, the two constraints are second class and the two "velocity" multipliers are then determined. However the chain of constraints can 'bifurcate'. As one approaches those special field values where the bracket vanishes, the constraints change their type, and (unless certain additional quantities vanish) some of the "velocity" multipliers will become unbounded, which appears to be an acausal propagation mode. Although this seems to happen only for extreme field values, because the value of the bracket is not 4-covariant, it should occur when ordinary values are boosted to some suitable Lorentz transformed frame. In that frame the spacelike hypersurface would be tangent to the acausal characteristic.

9. Dynamic modes with spin 1^- or 2^-

Cases with only a 1^- or a 2^- propagating mode were also investigated. Only the results of the Poisson brackets (PB) of the primary second-class constraints are shown and compared with those of the linearized theory.

For the spin 1^- case the parameters were chosen to satisfy $a_1 + 2a_3 = 0$, $b_5 < 0$, $b_1 = b_2 = b_3 = b_4 = b_6 = 0$. Unspecified parameters have generic values. The corresponding if-constraints are $\phi_{[ab]}$, ϕ_A, $\phi^{T}{}_{abc}$, $\phi_{a0}{}^{a}$ and $\overline{\phi}_{(a0b)}$. The non-zero PB's of these constraints are of the form

$$\begin{aligned} \{\phi_A, \phi'_{[ab]}\} &= 2\delta_{xx'}\epsilon_{abc}\mathcal{H}^{cd}{}_{d0}, & \{\phi^{T}_{abc}, \phi'_{[de]}\} &= -\delta_{xx'}[\mathcal{H}_{cf}{}^{f}{}_{0}g_{a[d}g_{e]b}]^{T}, \\ \{\phi_{c0}{}^{c}, \phi'_{[ab]}\} &= 2\delta_{xx'}\mathcal{H}_{[b0a]0}, & \{\overline{\phi}_{(a0b)}, \phi'_{[cd]}\} &= \delta_{xx'}g_{\{a[d}\,\hat{\mathcal{H}}_{b\}0c]0}\,. \end{aligned} \tag{16}$$

Generically there is no first-class constraint in the spin 1^- cases. However, since the results of those PB's are beyond the 0^{th} order, they vanish in the linearized spin 1 theory. Therefore the linearized theory has extra first-class constraints and gauge freedoms (at least at the primary level).

For the spin 2^- case we considered the parameter choices

$$\begin{aligned} &a_1 + 2a_3 = 0, \quad 2a_1 + a_2 = 0, \quad a_0 - a_1 \leq 0, \\ &b_1 = b_4 = b_6 = 0, \quad b_3 = b_5 = -b_2 > 0. \end{aligned} \tag{17}$$

The corresponding primary if-constraints are ϕ_{a0}, $\phi_{[ab]}$, ϕ_A, $\phi_{a0}{}^a$, $\phi_{[a0b]}$ and $\overline{\phi}_{(a0b)}$. Non-linearly they are all 2nd-class. The only PB's with a 0^{th} order term are of the form

$$\begin{aligned}\{\phi_{[a0b]}, \phi'_{[cd]}\} &= \delta_{xx'} e[\frac{(a_0 - a_1)}{l^2} g_{d[a}g_{b]c} \\ &\qquad - \frac{2b_2}{\kappa}(R_{[c[ab]d]} + \tilde{R}_{[c[b}\, g_{a]d]} + \frac{1}{3} g_{d[a}g_{b]c} R_{ef}{}^{fe})]\,, \qquad (18)\end{aligned}$$

and they become the only non-vanishing PB's in the linearized theory:

$$\{\phi_{[a0b]}, \phi'_{[cd]}\} \xrightarrow{\text{Linearization}} \delta_{xx'} \frac{(a_0 - a_1)}{l^2} \eta_{d[a}\eta_{b]c}\,. \qquad (19)$$

Thus the number and type of constraints in the linear theory is different from that of the non-linear theory.

On the other hand it has recently been shown [19] that certain special PGT cases which have only spin 0 modes do have good propagating modes even in the fully nonlinear theory.

10. Discussion

Non-linearities in the constraints lead to a field dependence in the matrix of the PBs. This can cause a bifurcation in the constraint chain. The number and type of constraints can depend on the field values. This phenomenon has been linked to acausal propagation modes. If a theory is inherently non-linear, the non-linear theory is likely to be qualitatively different from the linear theory in the number and type of constraints.

In view of these difficulties we have noted with the PGT and anticipate with other gauge theories of gravity (*e.g.* , the MAG [20]), we have several alternatives:

1. We could abandon such theories (except for GR and the ECSK).
2. We could embrace the difficulties, using them to find strong conditions on the numerous parameters. Thereby we could end up with a more creditable fewer parameter PGT (it will include at least two spin 0 modes).
3. Perhaps this indicates the need for a more non-linear theory. After all the linearized spin 2 theory of gravity has problems which are only cured by the nonlinearities of GR.

In any case the type of analysis proposed here shows promise as a strong theoretical test for alternative theories of gravity: one can require that the matrix of PB's has constant rank or that under linearization there is no change in the number and type of constraints.

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