# NEW HIGGS FIELD ANSATZ <br> FOR EFFECTIVE GRAVITY IN FLAT SPACE TIME* 

A. Geitner

Uni Konstanz Fakultät für Physik, L.S. Dehnen Fach M678, D-78457 Konstanz, Germany
e-mail: geitner@spock.physik.uni-konstanz.de
M. Hanauske

Institut für Physik, Johann Wolfgang Goethe-Universität
Rober-Mayer Str.10, Postfach 111932
D-60054 Frankfutr am Main, Germany
e-mail: 06994415680-0001@t-online.de
and E. Hitzer
c/o RIMS, Kyoto University
Kitashirakawa, Sakyo-ku, 606 Kyoto, Japan
e-mail: hitzer@kurims.kyoto-u.ac.jp
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Dedicated to Andrzej Trautman in honour of his $64^{\text {th }}$ birthday
Regarding Pauli's matrices as proper Higgs fields one can deduce an effective approximation for gravity in flat space. In this work we extend this approximation up to the second order, reaching complete agreement in the special case of gravitational waves. Unification in view, we introduce isospinorial degrees of freedom. In this way the mass spectrum and chiral asymmetry can be generated with the help of an additional scalar Higgs field. The Higgs modes corresponding to gravity are discussed.

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[^0]
## 1. Introduction

In a sequence of previous works [1] a semiclassical theory for effective gravity in flat space time was proposed. Key ingredients were: (a) a new $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge of the handed chirally represented particle/antiparticle doublet, (b) replacing the usual scalar Higgs field of the standard model by assigning the Pauli matrix fields ${ }^{1} \tilde{\sigma}(x)$ the role of a nonscalar Higgs field. By analyzing the resulting new set of field equations one finds that (c) the excitations of this new Higgs field effectively describe Einstein's gravity in lowest order, (d) passive gravitational and inertial mass of particles and antiparticles alike are generically identical, and (e) the gravitational constant $G$ is defined through choosing the ground state.

These results need to be extended in primarily two directions: (1) going from the lowest linear order to the by now experimentally verified higher (2-nd) order and (2) by taking the other fundamental interactions into account. New works in these two areas will be presented in the subsequent two sections. ${ }^{2}$

## 2. The second order ${ }^{3}$

The expression for the kinetic energy of the new nonscalar Higgs field $\tilde{\sigma}(x)$

$$
\begin{equation*}
\operatorname{tr}\left[\left(\eta^{\rho \beta} \eta^{\mu \nu} D_{\rho} \tilde{\sigma}_{\mu \mathrm{R}}\right)\left(D_{\beta} \tilde{\sigma}_{\nu \mathrm{L}}\right)\right]+\text { (index permutations) } \tag{2.1}
\end{equation*}
$$

in [1] is quadratic in $\tilde{\sigma}(x)\left(D_{\rho}\right.$ is the new $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge covariant derivative and $\stackrel{(0)}{\tilde{\sigma}}^{\mu} \mathrm{R} / \mathrm{L}=\frac{v}{4}\left(1, \pm \sigma_{i}\right), v=$ const. defines the groundstate $)$. The resulting field equations are therefore only linear in $\tilde{\sigma}$, respectively its excitations $\varepsilon^{\mu}{ }_{\nu} \cdot\left(\tilde{\sigma}_{\mu}=\left(\delta^{\mu}{ }_{\nu}+\varepsilon^{\mu}{ }_{\nu}\right) \stackrel{(0)}{\tilde{\sigma}^{\nu}}\right)$. Nonlinear effects like a gravitational self interaction, peri helion shift, gravitational waves, etc. require to generalize the Lagrange density so as to include 2-nd order effects.

### 2.1. Generalization

The principle applied here is to replace in (2.1):

$$
\begin{equation*}
\eta^{\rho \beta} \rightarrow \operatorname{atr}\left(\tilde{\sigma}_{\mathrm{R}}^{\rho} \tilde{\sigma}_{\mathrm{L}}^{\beta}\right), \ldots \tag{2.2}
\end{equation*}
$$

where $a$ is at this stage just a constant to be determined suitably. In this way the number of terms in (2.1) increases greatly and it proved suitable

[^1]to utilize the package Mathtensor of Mathematica in order to deduce and expand the field equations. As the comparison with gravity in higher orders is the main concern of this section, everything is simplified by neglecting matter- and gauge fields. Similarly no attention was paid to the possible handedness of the excitations $\varepsilon^{\mu}{ }_{\nu}$.

The expansion of the Higgs field equations

$$
\begin{equation*}
\mathcal{H}_{\mu \nu}=\stackrel{(1)}{\mathcal{H}}_{\mu \nu}+\stackrel{(2)}{\mathcal{H}}_{\mu \nu}+\ldots=0 \tag{2.3}
\end{equation*}
$$

in terms of $\varepsilon^{\mu}{ }_{\nu}$ :
yields order by order:

$$
\begin{equation*}
\stackrel{(1)}{\mathcal{H}}_{\mu \nu}=0 \quad \stackrel{\text { see }[1]}{\longrightarrow} \quad \square \stackrel{(1)}{\varepsilon}_{\mu \nu}={\stackrel{(1)}{\varepsilon_{\mu}}{ }_{|\alpha| \nu}^{\alpha}+{\stackrel{(1)}{\varepsilon_{\nu}}}^{\alpha}{ }_{|\alpha| \mu}, ~}_{\text {a }} \tag{2.5}
\end{equation*}
$$

$$
\begin{equation*}
\stackrel{(2)}{\mathcal{H}}_{\mu \nu}=0 \tag{2.6}
\end{equation*}
$$

As previously shown $[1]{ }^{(1)} \varepsilon^{\alpha}{ }_{\alpha} \stackrel{(1)}{\varepsilon}=0$ corresponds to 1 st order energy-momentum conservation. Expecting the same for the 2nd order should yield $\stackrel{(2)}{\varepsilon}=0$. Further analysis proved that ${ }^{(1)}{ }_{\alpha}{ }_{\alpha \beta}{ }^{(1)}{ }_{\varepsilon}^{\alpha \beta}$ 位 $=0$ should hold in addition, in order to achieve some reasonable comparison with gravity. Whether both of these 2 nd order trace conditions are to be explained by ways of some conservation law remains to be investigated.

$$
\text { 2.2. Comparing } \stackrel{(2)}{\mathcal{H}}_{\mu \nu}=0 \text { with } G R
$$

The strategy adopted is to extract an effective metric from (2.6)

$$
\begin{equation*}
g_{\mathrm{eff}}^{\mu \nu}=\underbrace{\eta^{\mu \nu}+A \stackrel{(1)}{\varepsilon}_{\mu \nu}^{\prime}}_{\text {lin.case }}+C \stackrel{(2)}{\varepsilon} \mu \nu^{(2)} D{\stackrel{(1)}{\varepsilon} \mu_{\alpha}}^{(1)} \alpha \nu \tag{2.7}
\end{equation*}
$$

and to express $\stackrel{(2)}{\mathcal{H}}_{\mu \nu}=0$ explicitly in terms of it. This then allows to formally compare (2.6) with Einstein's equations in vacuum

$$
\begin{equation*}
\stackrel{(1)}{R}_{\mu \nu}=0 \quad \text { (lin. case [1]) } \tag{2.8}
\end{equation*}
$$

and

$$
\begin{equation*}
\stackrel{(2)}{R}_{\mu \nu}=0, \tag{2.9}
\end{equation*}
$$

where $R_{\mu \nu}$ is calculated in terms of the Christoffel symbols formally associated with $g_{\text {eff }}^{\mu \nu}$.

The result is an agreement of (2.6) with (2.9) up to an antisymmetric divergence term

$$
\begin{align*}
B_{\mu \nu} & =X_{\mu \nu \sigma}{ }^{\mid \sigma}, \quad X_{\mu \nu \sigma}=-X_{\mu \sigma \nu}  \tag{2.10}\\
X_{\mu \nu \sigma} & =-v \varepsilon_{[\nu \rho} \varepsilon_{\mu}{ }^{\rho}{ }_{\mid \sigma]}-v \varepsilon_{\mu \rho} \varepsilon_{[\nu}{ }^{\rho}{ }_{\mid \sigma]}-v \varepsilon_{\rho}{ }^{\gamma} \eta_{\mu[\nu} \varepsilon_{\sigma]}{ }^{\rho}{ }_{\mid \gamma}-v \varepsilon_{[\sigma \rho} \eta_{\mu \nu]} \varepsilon^{\rho \gamma}{ }_{\mid \gamma} . \tag{2.11}
\end{align*}
$$

The necessary restrictions on $g_{\text {eff. }}^{\mu \nu}$ are simply $A \neq 0$ and $D=\frac{1}{2} A^{2}-1$. $C$ remains free. The antisymmetry of $X$ means that it plays no role in the energy-momentum conservation. By further restricting $A=C=2$ it becomes possible to write

$$
\begin{equation*}
g_{\mathrm{eff}}^{\mu \nu} \mathbf{1}=\left(\frac{4}{v}\right)^{2} \tilde{\sigma}_{\mathrm{R}}^{(\mu} \tilde{\sigma}_{\mathrm{L}}^{\nu)} \tag{2.12}
\end{equation*}
$$

Furthermore (2.6) may be rewritten as

$$
\begin{aligned}
\stackrel{(2)}{S}_{\mu \nu}{ }^{\alpha}{ }_{\mid \alpha} & =\text { const. }\left({ }^{(2)}{ }_{(\mu \nu)}(\tilde{\sigma})-\frac{1}{2} \stackrel{(2)}{t}(\tilde{\sigma}) \eta_{\mu \nu}\right) \\
t_{\mu}{ }^{\nu} & =\sum_{R, L} \frac{\partial \mathcal{L}_{\mathrm{Higgs}}}{\partial \tilde{\sigma}^{\alpha}{ }_{\mid \nu}} \tilde{\sigma}^{\alpha}{ }_{\mid \mu}-\delta_{\mu}{ }^{\nu} \mathcal{L}_{\mathrm{Higgs}}
\end{aligned}
$$

if $A^{2}=2 C$. This strongly resembles analogous expressions in GR [4] and shows the analogy with respect to self interaction in both theories. In the case of gravitational waves, it finally turns out that by applying the so-called short wave formalism [5] the $B_{\mu \nu}$-discrepancy averages out to zero.

## 3. Isospinorially extended theory ${ }^{4}$

### 3.1. The Lagrangian

We start with a chirally symmetric representation in which we put leptons and quarks in a 4 -iso spinor $\psi_{\mathrm{L} / \mathrm{R}}=\left(\begin{array}{l}\nu \\ e \\ u \\ d\end{array}\right)_{\mathrm{L} / \mathrm{R}}$, where each entry itself is a 2 -spinor $\nu_{\mathrm{L} / \mathrm{R}}=\binom{\nu}{\bar{\nu}}$. Most parts of the Lagrangian have now the

[^2]same form as in the isoscalar version:
The fermionic term: $\quad \mathcal{L}_{M}=\tilde{\psi}_{\mathrm{L}}^{\dagger} \tilde{\sigma}_{\mathrm{L}}^{\mu} D_{\mu} \tilde{\psi}_{\mathrm{L}}+$ h.c. $+(\mathrm{L} \leftrightarrow \mathrm{R})$.
The Higgs field term:
\[

$$
\begin{aligned}
\mathcal{L}_{H 1}= & \operatorname{tr}\left(D_{\alpha} \tilde{\sigma}_{\mathrm{L}}^{\mu} D^{\alpha} \tilde{\sigma}_{R \mu}-D_{\alpha} \tilde{\sigma}_{\mathrm{L}}^{\alpha} D_{\mu} \tilde{\sigma}_{R}^{\mu}-D_{\alpha} \tilde{\sigma}_{\mathrm{L}}^{\alpha} D_{\mu} \tilde{\sigma}_{R}^{\mu}\right) \\
& -\mu^{2} \operatorname{tr}\left(\tilde{\sigma}_{\mathrm{L}} \tilde{\sigma}_{\mu \mathrm{R}}\right)-\frac{\lambda}{12} \operatorname{tr}\left(\tilde{\sigma}_{\mathrm{L}}^{\mu} \tilde{\sigma}_{\mu \mathrm{R}}\right)^{2}
\end{aligned}
$$
\]

The Yukawa coupling term: $\quad \mathcal{L}_{I}=-k\left(\tilde{\psi}_{\mathrm{L}}^{\dagger} \tilde{\sigma}_{\mathrm{L} \mu} \tilde{\sigma}_{\mathrm{R}}^{\mu} \tilde{\psi}_{\mathrm{R}}+\right.$ h.c. $)$
To be able to generate all fermionic and bosonic masses we need an additional scalar Higgs field, whose Lagrange density is

$$
\mathcal{L}_{H 2}=\left(D_{\alpha} \phi\right)^{\dagger}\left(D^{\alpha} \phi\right)-\frac{\bar{\mu}}{2} \phi^{\dagger} \phi-\frac{\bar{\lambda}}{4}\left(\phi^{\dagger} \phi\right)^{2} .
$$

And of course we need the kinetic term for the gauge bosons, which has the usual form since we do not yet couple them to gravity:

$$
\mathcal{L}_{F}=-\frac{1}{16 \pi} F_{\mu \nu a} F^{\mu \nu a}
$$

This Lagrangian is invariant under $\mathrm{U}(1) \times \mathrm{SU}(2)_{\text {spin }} \times \mathrm{SU}(2)_{\text {isospin }}$ transformations with the generators

$$
\tau_{i}=\frac{1}{2} \rrbracket_{\text {spin }}\left(\begin{array}{cc}
\sigma_{i} & 0 \\
0 & \sigma_{i}
\end{array}\right), i=0 \ldots 3, \quad \tau_{i}=\frac{1}{2} \rrbracket_{\text {isospin }} \sigma_{i-3}, i=4 \ldots 6,
$$

where $\sigma_{i}$ are the usual Pauli-matrices ( $\sigma_{0} \equiv \mathbf{1}$ ).

### 3.2. Spontaneous symmetry breaking

First we choose a Basis:
Spin space as usual $\sigma_{\mathrm{R} / \mathrm{L}}^{\mu}=\left(\sigma^{0}, \pm \sigma^{i}\right)$,
Isospin space $N^{a}=\left(\begin{array}{cc}\sigma^{a} & 0 \\ 0 & 0\end{array}\right), a=0 \ldots 3, N^{a}=\left(\begin{array}{cc}0 & 0 \\ 0 & \sigma^{a}\end{array}\right)$,
$a=4 \ldots 7$,
where possible quark-lepton mixing is neglected for simplicity.
We now write the tensor-field as ground- and exited state:

$$
\tilde{\sigma}_{\mathrm{L} / \mathrm{R}}^{\mu}=\stackrel{(0)}{\tilde{\sigma}_{\mathrm{L} / \mathrm{R}}^{\mu}}+\varepsilon_{\mathrm{L} / \mathrm{R} \nu \mathrm{a}}^{\mu} \sigma_{\mathrm{L} / \mathrm{R}}^{\nu} N^{a} .
$$

The Dirac equation is best reproduced if we choose following ground-state:

$$
\stackrel{(0)}{\tilde{\sigma}}_{\mathrm{L} / \mathrm{R}}^{\mu}=\sigma_{\mathrm{L} / \mathrm{R}}^{\mu} \stackrel{(0)}{N}_{\mathrm{L} / \mathrm{R}} \text { with } \stackrel{(0)}{N}_{\mathrm{L} / \mathrm{R}}=\operatorname{diag}\left(n_{1}, n_{2}, n_{3}, n_{4}\right)_{\mathrm{L} / \mathrm{R}},
$$

where $\stackrel{(0)}{N}_{\mathrm{L} / \mathrm{R}}$ cannot depend on $\mu$ because of the isotropy of space. Now we can generate chiral asymmetry by choosing $n_{R 1}=-n_{R 2}$. As result we have a right-handed neutrino, that does not couple to the $W^{ \pm}$bosons, but to the $Z$-Boson, which is not in contradiction to experiments, since these give only evidence to the fact, that right-handed neutrinos do not participate in week decay (see Wu-experiment [6]).

For simplicity we now choose the following ground-state, which sets both quark masses to be equal:

$$
N_{\mathrm{L}}=\left(\begin{array}{ll}
l \mathbf{1} & 0 \\
0 & q \mathbf{1}
\end{array}\right), \quad N_{\mathrm{R}}=\left(\begin{array}{ll}
-l \sigma^{3} & 0 \\
0 & q \mathbf{1}
\end{array}\right) .
$$

A direct consequence of the parity violation is the fact, that the neutrino receives a negative mass from the tensorial Higgs field. This can be compensated by the scalar Higgs field, whose ground-state we choose as

$$
\stackrel{0}{\phi=v}\left(\begin{array}{l}
1 \\
0 \\
0 \\
0
\end{array}\right)
$$

### 3.3. Fermionic field equations

The field equations for the fermions are

$$
i \tilde{\sigma}_{\mathrm{R} / \mathrm{L}}^{\mu} D_{\mu} \psi_{\mathrm{R} / \mathrm{L}}+\frac{i}{2}\left(D_{\mu} \tilde{\sigma}_{\mathrm{R} / \mathrm{L}}^{\mu}\right) \psi_{\mathrm{R} / \mathrm{L}}-k \tilde{\sigma}_{\mathrm{R} / \mathrm{L}}^{\mu} \tilde{\sigma}_{\mu \mathrm{L} / \mathrm{R}} \psi_{\mathrm{L} / \mathrm{R}}-\tilde{k} \phi\left(\phi^{\dagger} \psi_{\mathrm{L} / \mathrm{R}}\right)=0 .
$$

To be able to compare this to the Dirac equation of the standard theory we need to renormalize the fermionic spinor components:

$$
\tilde{\nu}_{L, R}=\sqrt{l} \nu_{L, R}, \tilde{e}_{L, R}=\sqrt{l} e_{\mathrm{L}, \mathrm{R}}, \tilde{u}_{L, R}=\sqrt{q} u_{L, R}, \tilde{d}_{L, R}=\sqrt{q} d_{L, R}
$$

For the ground state this gives

$$
\begin{aligned}
& 0=i \sigma_{\mathrm{R}}^{\mu} \partial_{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{R}}-k\left(\begin{array}{c}
l \tilde{\tilde{\nu}} \\
l \tilde{e} \\
q \tilde{u} \\
q \tilde{d}
\end{array}\right)+\bar{k} \frac{v^{2}}{l}\left(\begin{array}{c}
\tilde{\nu} \\
0 \\
0 \\
0
\end{array}\right)_{\mathrm{L}}-\frac{1}{2}\left(g_{1} \omega_{\mu 0} \sigma_{\mathrm{R}}^{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{R}}\right. \\
& \left.+g_{2} \omega_{\mu 3} \sigma_{\mathrm{R}}^{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
-\tilde{\tilde{u}} \\
\tilde{u} \\
-\tilde{d}
\end{array}\right)_{\mathrm{R}}+g_{2} \omega_{\mu 1} \sigma_{\mathrm{R}}^{\mu}\left(\begin{array}{c}
0 \\
0 \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{R}}+i g_{2} \omega_{\mu 2} \sigma_{\mathrm{R}}^{\mu}\left(\begin{array}{c}
0 \\
0 \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{R}}\right)+\frac{1}{4} \Omega_{i}{ }^{i}\left(\begin{array}{c}
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{R}}
\end{aligned}
$$

and

$$
\begin{aligned}
& 0=i \sigma_{\mathrm{L}}^{\mu} \partial_{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{L}}-k\left(\begin{array}{c}
-l \tilde{\nu} \\
l \tilde{e} \\
q \tilde{u} \\
q \tilde{d}
\end{array}\right)_{R}-\bar{k} \frac{v^{2}}{l}\left(\begin{array}{c}
\tilde{\nu} \\
0 \\
0 \\
0
\end{array}\right)_{R}-\frac{1}{2}\left(g_{1} \omega_{\mu 0} \sigma_{\mathrm{L}}^{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{L}}\right. \\
& \left.+g_{2} \omega_{\mu 3} \sigma_{\mathrm{L}}^{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
-\tilde{e} \\
\tilde{u} \\
-\tilde{d}
\end{array}\right)_{\mathrm{L}}+g_{2} \omega_{\mu 1} \sigma_{\mathrm{L}}^{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{L}}+i g_{2} \omega_{\mu 2} \sigma_{\mathrm{L}}^{\mu}\left(\begin{array}{c}
\tilde{\nu} \\
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{L}}\right)-\frac{1}{4} \Omega_{i}{ }^{i}\left(\begin{array}{c}
\tilde{e} \\
\tilde{u} \\
\tilde{d}
\end{array}\right)_{\mathrm{L}}
\end{aligned}
$$

where $\Omega_{i}^{i}$ is the trace of the spin-gauge bosons $\Omega_{\mu a}=\omega_{\mu 3+a}$ with $i=1 \ldots 3$.
Obviously for the masses of the fermions follows

$$
m_{\nu}=\frac{\tilde{k} v^{2}}{l}-4 k l \stackrel{!}{=} 0, m_{e}=4 k l, \quad m_{u}=m_{d}=4 k q
$$

After implementing the Weinberg mixture, which is the same as in the standard model, one gets the Dirac equations which differ from the standard model in following points:

- There exists a right-handed neutrino, but it couples to the $Z$-boson only (which may have measurable consequences for the boson's lifetime).
- All fermions couple to the spin-gauge bosons. Since these are Planck massive (see below) this plays no role in the low energy limit.
- The coupling constants of the right- and left-handed quarks can be influenced separately by the choice of ground-state, but a "complete" asymmetry as for the neutrino would be linked to zero or negative mass.


## Boson masses

The mass-square matrix $M^{2 \mu}{ }_{\nu}{ }^{i j}$ for the gauge bosons is

$$
\begin{aligned}
& 2 \pi \hat{g}_{(i)(j)}\left(4 \operatorname{tr}\left\{\left[\tau^{(i},{\stackrel{\circ}{\tilde{\sigma}_{\mathrm{L}}}}_{\mathrm{L}}\right]\left[\tau^{j)}, \stackrel{\circ}{\sigma}_{\lambda \mathrm{R}}\right]+(\mathrm{L} \leftrightarrow \mathrm{R})\right\}\left(\delta_{\rho}^{\mu} \delta_{\nu}^{\lambda}-\frac{1}{2} \delta_{\nu}^{\mu} \delta_{\rho}^{\lambda}\right)\right. \\
& \left.+\delta_{\nu}^{\mu} \stackrel{\circ}{\phi}\left\{\tau^{i}, \tau^{j}\right\} \stackrel{\circ}{\phi}\right)
\end{aligned}
$$

with $\hat{g}_{(i)(j)}=2 \pi g_{(i)} g_{(j)}$. This leads to Planck massive spin-gauge bosons [1]. Unfortunately it seems impossible to generate the $Z$-boson mass with the tensorial Higgs fields, so that all masses of the electroweak gauge bosons have to be generated with help of the scalar Higgs field.

### 3.4. Tensor-field excitations

To investigate the structure of field equations of the tensor-field we neglect the gauge-bosons. The first order field equations for the excitations $\varepsilon_{\mu}^{\nu a}$ of the tensor field are:

$$
\begin{aligned}
& \partial_{\alpha} \partial^{\alpha} \varepsilon_{\mathrm{R}}^{\mu \nu a}-2 \partial_{\alpha} \partial^{\mu} \varepsilon_{\mathrm{R}}^{\alpha \nu a}-\frac{\mu^{2}}{4 q^{2}} \eta^{\mu \nu}\left(q \delta^{a 4}-l \delta^{a 3}\right)\left(\varepsilon_{\mathrm{L} \alpha}^{\alpha(-l 3+q 4)}+\varepsilon_{\mathrm{R} \alpha}^{\alpha(l 0+q 4)}\right) \\
& =\frac{i}{8}\left(\psi_{\mathrm{L}}^{\dagger} N^{a} \sigma_{\mathrm{L}}^{\nu} \partial^{\mu} \psi_{\mathrm{L}}-\text { h.c. }\right)-\frac{k}{4}\left(\psi_{\mathrm{L}}^{\dagger} N^{a} \sigma_{\mathrm{L}}^{\nu} \sigma_{\mathrm{R}}^{\mu} \stackrel{(0)}{N_{\mathrm{R}}} \psi_{\mathrm{R}}+\text { h.c. }\right)
\end{aligned}
$$

with the left-handed equation respectively. Here $\varepsilon_{\mu}{ }^{\nu(a x+b y)}$ means $a \varepsilon_{\mu}{ }^{\nu x}+$ $b \varepsilon_{\mu}{ }^{\nu y}$ and the source is developed to 0th order only. These equations can be divided in two classes.

The first class consists of excitations with isospin-index $a \varepsilon\{0,3,4,7\}$ and is a gravitation like interaction (see also [1]), where each kind of fermions generates its own gravitation. With the redefined isospinors (3.3) and the 0 -th order energy-momentum tensor the equation for the "quark-gravity" is

$$
\begin{aligned}
& \partial_{\alpha} \partial^{\alpha} \varepsilon_{\mathrm{L}}^{\mu(4+7)}-2 \partial_{\alpha} \partial^{\mu} \varepsilon_{\mathrm{L}}^{\alpha \nu(4+7)}-\frac{\mu^{2}}{4 q} \eta^{\mu \nu}\left(\varepsilon_{\mathrm{L} \alpha}^{\alpha(-l 3+q 4)}+\varepsilon_{\mathrm{R} \alpha}^{\alpha(l 0+q 4)}\right) \\
& \quad=\frac{1}{2 q}\left(T^{\mu \nu}\left(\tilde{u}_{\mathrm{R}}\right)-\frac{\eta^{\mu \nu}}{2} T_{\alpha}^{\alpha}\left(\tilde{u}_{\mathrm{R}}\right)+\frac{m_{e}}{4}\left(\tilde{u}_{\mathrm{L}}^{\dagger} \sigma_{\mathrm{L}}^{[\nu} \sigma_{\mathrm{R}}^{\mu]} \tilde{u}_{\mathrm{R}}+\text { h.c. }\right)\right)
\end{aligned}
$$

We get similar equations for other combinations of the excitations $\varepsilon_{\mu}{ }^{\nu a}$ with $a \varepsilon\{0,3,4,7\}$ with the other fermions as source. Herein the neutrino is making an exception since its (zero) mass is partially generated by the scalar Higgs field, i.e. it provides in the $(0+3)$ equation an additional source term for the gravitation-like interaction

$$
\frac{\eta^{\mu \nu}}{2} m_{e}\left(\tilde{\nu}_{\mathrm{L}}^{\dagger} \tilde{\nu}_{\mathrm{R}}+\tilde{\nu}_{\mathrm{R}}^{\dagger} \tilde{\nu}_{\mathrm{L}}\right)
$$

thus violating the equivalence-principle. However, this term cancels out in the classical limit.

The second class consists of excitations with isospin-index $a \varepsilon\{1,2,5,6\}$. These fields carry electrical charge. The source terms of these equations have the form of energy-momentum tensors but they contain the fermions in a mixed form:

$$
\begin{aligned}
\partial_{\alpha} \partial^{\alpha} \varepsilon_{\mathrm{L} \mu}^{\nu(1+i 2)} & -2 \partial_{\alpha} \partial_{\mu} \varepsilon_{\mathrm{L}}^{\alpha \nu(1+i 2)}=\frac{-i}{4 l}\left(\left(\tilde{\nu}_{\mathrm{R}}^{\dagger} \sigma_{\mathrm{R}}^{\nu} \partial_{\mu} \tilde{e}_{\mathrm{R}}-\text { h.c. }\right)\right. \\
& \left.-2 k l\left(\tilde{\nu}_{\mathrm{L}}^{\dagger} \sigma_{\mathrm{L} \mu} \sigma_{\mathrm{R}}^{\nu} \tilde{e}_{\mathrm{R}}-\tilde{\nu}_{\mathrm{R}}^{\dagger} \sigma_{\mathrm{R}}^{\nu} \sigma_{\mathrm{L} \mu} \tilde{e}_{\mathrm{L}}\right)\right)
\end{aligned}
$$

In the equations of the first class we see that quarks and leptons couple with different coupling-constants to gravity. Moreover, the fact that each kind of fermions produces a different gravity and couples to its own gravity only (as can be seen by investigating the Dirac equation for the excited Higgs field) is a contradiction to experiment. Due to the different distribution of u- and d-quarks in the Earth, this would cause a measurable violation of the equivalence principle of the order of $\kappa \simeq 10^{-6}$ in the Eötvös-type experiments (for the same materials even the "famous" result of Fischbach [7] is 2 orders of magnitude lower).

### 3.5. Transition to a uniform gravitational field

By putting certain constraints on the Higgs field it is possible to construct one uniform gravity for all fermions. We want, that in the Dirac equation all fermions couple to the same Higgs-field exited by the energy-momentum tensors of all fermions and that all other excitations can be neglected. This can be done by constraining the excitations to be multiples of the groundstate.

We then get a Dirac-equation and one Higgs field equation that have the same form as in the iso-scalar case. It is not clear if the classical limit applies to neutrinos also, since they are chirally asymmetric. If we neglect the effects of this asymmetry in the fermionic energy momentum the classical limit is exactly analogous to the iso-scalar case [1].

Unfortunately we have not yet been able to find an appropriate Lagrange density to realize these constraints.

## 4. Conclusions

This work has shown how a nonscalar Higgs field can be interpreted as a gravity transmitting field on a flat background spacetime. In the 2-nd order appears a discrepancy to Riemannian gravity (but not in the case of gravitational waves) which needs to be interpreted. One way may be to include torsion [3].

The isospinorial extension allowed to generate the chiral asymmetry as well. But an additional scalar Higgs field had to be introduced, and new predictions like a possible coupling of a right handed neutrino to $Z$ bosons need to be investigated. Beyond this only the excitations proportional to the groundstate seem relevant for gravity.

Whether the additional scalar Higgs field is really essential and how the excitations might, if at all, be properly constrained is left to future research, as well as attempts of quantization of this theory.

## REFERENCES

[1] H. Dehnen, E. Hitzer, Int. J. Theor. Phys. 33, 575 (1994); 34, 1981 (1995); E. Hitzer, The Higgs-Field Theoretic Extension of the Spin-Gauge Theory of Gravity (Thesis), Hartung-Gorre, Konstanz 1996, ISBN 3-89649-012-5.
[2] D. Ketterer, Diplomarbeit, Univ. Konstanz (1995); J. Fibich, Diplomarbeit, University of Konstanz (1996).
[3] M. Hanauske, Diplomarbeit, University of Konstanz (1997); http://kaluza.physik.uni-konstanz.de/DE/MH/.
[4] F. DeFelice, C.J.S. Clarke, Rel. on Curved Manifolds, CUP, 1990.
[5] C. Misner, K. Thorne, J. Wheeler, Gravitation, Freeman and Co. 1973.
[6] C.S. Wu, et al., Phys. Rev. 105, 1413 (1957).
[7] E. Fischbach, Phys. Rev. Lett. 56, 3 (1986).


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[^1]:    ${ }^{1}$ Already spacetime dependent because of (a).
    ${ }^{2}$ See [2] for previous works in direction (2).
    ${ }^{3}$ This section is based on [3].

[^2]:    ${ }^{4}$ Compare gr-qc/9712074 for further details.

