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COSMIC CENSORSHIP IN A KERR-LIKE COLLAPSE SCENARIO* **

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Dedicated to Andrzej Trautman in honour of his 64th birthday

In this contribution we discuss a recent result which shows that a gravitational collapse cannot in generic situations lead to the formation of a final state resembling the Kerr solution with a naked singularity. This result supports the validity of the cosmic censorship hypothesis.

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1. Introduction

The cosmic censorship hypothesis put forward by Penrose [1] says that a physically realistic gravitational collapse, which develops from a regular initial state, can never result in a naked singularity, *i.e.* all singularities formed in such a collapse should always be enclosed within an event horizon and hence invisible to outside observers. This hypothesis plays a fundamental role in the theory of black holes and has been recognized as one of the most important open problems in classical general relativity. There exist various examples of exact solutions of Einstein's equations that represent the collapse of a regular initial configuration to a *naked* singularity (see *e.g.* Ref. [2] and references cited therein). All these examples, however, represent special situations that need not arise in a realistic collapse, and so they cannot, without proofs of stability, have any bearing on the hypothesis of Penrose. For instance, most of them (with the exception of the case explored numerically by Shapiro and Teukolsky [3]) deal only with the collapse of *nonrotating*

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matter. But as realistic collapsing objects — such as, for example, massive stars at the final stage of evolution — are expected to be generally rotating, one should attempt to test the cosmic censorship hypothesis in more general collapse scenarios, where angular momentum is present as well. One of the known exact solutions of Einstein's equations which might be relevant to such a situation is that discovered by Kerr [4]. This solution depends on two parameters m and a, and represents the exterior gravitational field of a rotating body with mass m and angular momentum $L = ma$, as measured from infinity in units chosen so that $c = G = 1$.

In the context of cosmic censorship, the most interesting feature of the Kerr solution is the ringlike curvature singularity appearing in the central part of the solution whenever $m \neq 0$ and $a \neq 0$ — this singularity can be interpreted as the final product of collapse of a rotating object. There is, however, an essential difference between the Kerr singularity existing in the case $a \leq m$ and that occurring as the parameter a exceeds the mass m. Namely, if $a \leq m$, then there exists an event horizon and the whole ring singularity is always hidden behind this horizon, whereas if $a > m$, there is *no* event horizon and the singularity is visible for all observers (see Ref. [5], pp.161–168). Accordingly, since some collapsing stars may have an angular momentum greater than the square of their mass (for the Sun, $L \sim m^2$), it is of importance in view of cosmic censorship to know whether a physically realistic collapse of a rotating object, which starts off from a regular initial state, can ever lead to the formation of a final state resembling the Kerr solution with a naked singularity. Just this question has been raised by Penrose in his original article on cosmic censorship [1] as the basic open problem in the context of his hypothesis. In this contribution we shall present a certain recent result [6] concerning this problem.

2. Kerr-like collapse scenario

Let (M, g) be a space-time which admits a weakly asymptotically simple and empty conformal completion $(\tilde{M}, \tilde{g})$ [5]. These space-times are ideally suited to model collapse scenarios of isolated objects. Suppose that the space-time (M, g) contains a rotating object which undergoes complete gravitational collapse. Assume also that the collapse starts off from a *regular* initial state. To make this notion precise, we shall assume that the collapse develops from initial data given on some partial Cauchy surface S of (M, g) for which the following two conditions (a) and (b) are satisfied:

(a) $I^-(S) = D^-(S)$;

(b) all null geodesics generating the future null infinity $\mathcal{J}^+$ of the completion $(\tilde{M}, \tilde{g})$ intersect the closure of the future Cauchy development $D^+(S, \tilde{M})$.

Condition (a) is an obvious requirement ensuring that all the possible pathologies of (M, g) produced by the collapse — such as naked singularities or, say, causality violations — can only occur to the future of the surface S. Condition (b) means that (M, g) is to be partially future asymptotically predictable from S as defined by Tipler [7]. This requirement is needed in order to exclude the possibility of the occurrence of *trivial* future Cauchy horizons of the surface S. Such Cauchy horizons do not signal the possible occurrence of naked singularities or causality violations and their presence can be due to a poor choice of a partial Cauchy surface. One example of such a poor choice of a partial Cauchy surface may be the surface $t = -(1 + x^2 + y^2 + z^2)^{1/2}$ in Minkowski space.

In order to consider the question of whether or not the collapse can lead to any naked singularity, one should first impose on the space-time (M, g) a condition ruling out artificial naked singularities which could easily be created just by removing some regular points from the region $J^+(S) \cap J^-(\mathcal{J}^+, \tilde{M})$. A very useful condition of this type has been proposed in Ref. [8]. This condition is based on the idea that physically essential singularities should always be associated with large curvature strengths, which should in turn lead to the focusing of Jacobi fields along null geodesics approaching the singularities. Let $\eta : [0, s) \to M$ be an affinely parametrized, achronal, incomplete null geodesic; and let Z_1 and Z_2 be two linearly independent spacelike vorticity-free Jacobi fields along η. The exterior product of these Jacobi fields defines a spacelike area element, whose magnitude at a value t of the affine parameter on η we denote by $A(t)$. If we now introduce the function $z(t)$ defined by $A(t) \equiv z^2(t)$, then one can show [9] that $z(t)$ satisfies

$$\frac{d^2 z}{dt^2} + \frac{1}{2}(R_{ab} K^a K^b + 2\sigma^2) z = 0, \tag{1}$$

where K^a is the tangent vector to η and σ is the shear scalar. The geodesic η is said to satisfy the *inextendibility condition* [8] if there exist an affine parameter value $t_1 \in (0, s)$ and a solution $z(t)$ of Eq. (1) along $\eta(t)$ with initial conditions: $z(t_1) = 0$ and $\dot{z}(t_1) = 1$, such that $\lim_{t \to s} z(t) = 0$. One can show [8] that if η does satisfy this condition, then there is *no* reasonable (continuous) extension of the space-time M in which η could be continued beyond the value s of its affine parameter t, and this means that η should then terminate in a genuine singularity at $t = s$. Thus we can avoid the occurrence of artificial naked singularities in the space-time (M, g) by assuming that the following condition (c) holds:

(c) every past (future) incomplete, past (future) endless null geodesic generating an achronal subset of (M, g) satisfies the above inextendibility condition.

Naturally, one cannot *a priori* exclude the existence of some "true" singularities in (M, g), *i.e.* those not caused only by the removal of regular points, such that null geodesics approaching them fail to satisfy the inextendibility condition. It seems, however, likely that such singularities would not be accompanied by large curvature strengths, and so one can hope that they might be ignored on physical grounds. Indeed, it is worth noting here that null geodesics terminating at the so-called *strong curvature singularities* [9], which are often regarded to be the *only* physically reasonable singularities, will always satisfy, just by definition, the above inextendibility condition. It should be, however, stressed that the inextendibility condition will hold for a considerably larger class of singularities than only those of the strong curvature type. This is because the inextendibility condition can hold even if the curvature near singularities would remain bounded [10], whereas strong curvature singularities can exist only if the curvature in their neighbourhood diverges strong enough [11]. One can therefore expect that the condition (c) assumed above should involve no essential loss of generality of our considerations. (The curvature strength of singularities satisfying the inextendibility condition is examined in detail in Ref. [10].)

Let us now return to the collapse scenario. Suppose that the collapsing object was not able to dissipate enough the angular momentum to form an event horizon and the final state of collapse is a region $K \subset J^+(S) \cap J^-(\mathcal{J}^+, \tilde{M})$ resembling just the Kerr solution with a naked singularity. Clearly, the formation of the nakedly singular region K must lead to the occurrence of future Cauchy horizon, $H^+(S)$, of the initial surface S and one can certainly assume that $K \subset I^+[H^+(S)]$. To simplify our considerations, we shall also assume that the formation of the region K is the *only* reason for occurring of the horizon $H^+(S)$. That is, we shall assume that the following condition (d) holds:

(d) every null geodesic generating the Cauchy horizon $H^+(S)$ intersects the boundary $\dot{K}$ of the region K.

Of course, the above assumptions do not guarantee that the region K will be similar in any nontrivial sense just to the Kerr solution with a naked singularity. But, as is well known [5], the most striking feature of the Kerr solution with a naked singularity is the presence of *closed timelike curves* passing through *every* point of the space-time. One can therefore ensure the existence of a clear similarity between the region K and this Kerr solution by imposing the requirement that the chronology condition of (M, g) fails to be satisfied everywhere in K. To make things precise, we shall thus define the naked Kerr-like region K as follows:

$$K = \{x \in J^+(S) \cap J^-(\mathcal{J}^+, \tilde{M}) |\, x \in I^+(x)\}\,.$$

3. Cosmic censorship theorem

The following theorem shows that the collapse scenario considered above cannot be realized in generic space-times.

Theorem. Under the conditions (a)–(d) stated above, the naked Kerr-like region K can never form in the space-time (M, g) if the following two additional conditions (e) and (f) are satisfied:

(e) the null convergence condition holds on (M, g), *i.e.* $R_{ab}V^aV^b \geq 0$ for every null vector V^a of (M, g);

(f) the generic condition holds for (M, g), *i.e.* every null geodesic λ of (M, g) admits a point at which $K_{[a}R_{b]cd[e}K_{f]}K^cK^d \neq 0$, where K^a is the tangent vector to λ.

Conditions (e) and (f) are reasonable requirements for *any* physically realistic model of a classical space-time [5]. Note that condition (e) may be obtained, using Einstein's equations, from the weak energy condition: $T_{ab}K^aK^b \geq 0$ for any timelike vector K^a, which is known to be fulfilled by all the observed classical matter fields. Condition (f) essentially requires that every null geodesic should encounter some matter or randomly oriented radiation, and so one can expect that this condition should always hold in physically realistic (generic) space-times. It should, however, be stressed here that most of the known exact solutions of Einstein's equations, due to their special symmetries, *do* violate condition (f); for example, it fails to be satisfied for null geodesics generating the Cauchy horizons in the Kerr solution. Thus the above theorem does not exclude the possibility that the naked Kerr-like region K could form in some highly symmetric models of collapse of rotating matter. This theorem shows, however, that the existence of the region K could not certainly be a *stable* property of such models if they would be slightly perturbed just enough to satisfy the generic condition (f). One can thus expect that, according to the cosmic censorship hypothesis, the naked Kerr-like region K should never form in realistic collapse situations. Since we have defined the Kerr-like region K to be the set of all points at which the chronology condition fails to be satisfied, the above result gives also some support to the validity of the chronology protection conjecture of Hawking [12] and suggests that there may exist some deeper connection between this conjecture and the cosmic censorship hypothesis.

Here we shall only give the main ideas of the proof of the above theorem; for the detailed proof we refer the reader to Ref. [6].

In very brief outline, the proof runs as follows. First, one establishes that if the assertion of the theorem were false, then there would have to exist some past endless, past incomplete generator η of the Cauchy horizon $H^+(S)$ with future endpoint on $\mathcal{J}^+$. By condition (d) the generator η must intersect the boundary $\dot{K}$ of the Kerr-like region K. This enables one to show that the causal simplicity condition of (M,g) must break down, due to the chronology violation inside K, at some point $p \in \eta \cap \dot{K}$, such that $E^{-}(p) \neq \dot{J}^{-}(p)$. Using this, one then constructs a certain sequence $\{\eta_i\}$ of achronal null geodesic segments converging to the generator η. As all the geodesic segments η_i are achronal, by the well-known argument none of them can have a pair of conjugate points. This implies, in turn, that any Jacobi field along any η_i cannot be refocused. As the sequence $\{\eta_i\}$ converges to the generator η, this must then imply, by continuity, that any Jacobi field along the generator η cannot be refocused as well. But the past incomplete generator η must satisfy, by condition (c), the inextendibility condition, which requires that at least one Jacobi field along η should be refocused. In this way one obtains a contradiction, which completes the proof.

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