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SECOND ORDER QCD CONTRIBUTIONS TO POLARIZED SPACELIKE AND TIMELIKE PROCESSES *

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We will give an outline of the computation of the QCD corrections to the spin structure function $g_1(x,Q^2)$ and the spin fragmentation function $g_1^H(x,Q^2)$ which are measured in deep inelastic electron-proton scattering and in electron-positron annihilation respectively. In particular we show how to deal with the γ_5-matrix and the Levi–Civita tensor, appearing in the amplitudes of the parton subprocesses, when the method of N-dimensional regularization is used.

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1. Deep inelastic electron–proton scattering

Deep inelastic electron-proton scattering proceeds via the following reaction (see Fig. 1)

$$e^-(l_1,\sigma_1) + P(p,s) \to e^-(l_2,\sigma_2) + \text{`X'}\,. \tag{1.1}$$

Here 'X' denotes any inclusive hadronic final state and V in Fig. 1 stands for the neutral intermediate vector bosons given by γ, Z. For simplicity we will assume that the momentum transfer is very small with respect to the mass of the Z-boson so that the process in Fig. 1 is dominated by the one photon exchange mechanism only. In the case the proton is polarized parallel ($\rightarrow$) or anti-parallel ($\leftarrow$) with respect to the spin of the incoming electron we obtain the cross section

$$\frac{d^2\sigma(\rightarrow)}{dx\,dy} - \frac{d^2\sigma(\leftarrow)}{dx\,dy} = \frac{4\pi\alpha^2}{Q^2}\Big[\{2-y\}g_1(x,Q^2)\Big]\,, \tag{1.2}$$

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where $g_1(x,Q^2)$ denotes the longitudinal spin structure function. Further we have defined the scaling variables

$$x = \frac{Q^2}{2p\,q}\,, \qquad y = \frac{p\,q}{p\,l_1}\,, \qquad q^2 = -Q^2 < 0\,. \tag{1.3}$$

The spin structure functions show up in the antisymmetric part of the

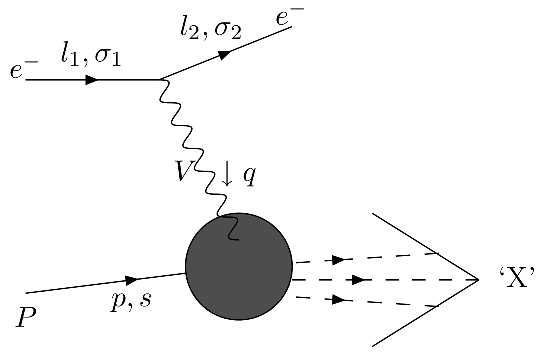

Fig. 1. Kinematics of polarized deep inelastic electron–proton scattering.

hadronic tensor

$$W_{\mu\nu}(p,q,s) = \frac{1}{4\pi}\int d^4z\, \mathrm{e}^{iq\cdot z}\langle p,s \mid J_\mu(z)J_\nu(0) \mid p,s\rangle\,, \tag{1.4}$$

which is given by

$$W^A_{\mu\nu}(p,q,s) = \frac{m}{2p\,q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\Big[s^\beta g_1(x,Q^2) + (s^\beta - \frac{s\,q}{p\,q}p^\beta)g_2(x,Q^2)\Big]\,. \tag{1.5}$$

Here $g_2(x,Q^2)$ denotes the transverse spin structure function which is kinematically suppressed in cross section (1.2). Since the leading power corrections are of twist two, one can give a parton model description of the longitudinal structure function which can be written as

$$g_1(x,Q^2) = \frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2 \int_x^1 \frac{dz}{z}\Big[\Delta f_q^{\rm S}\left(\frac{x}{z},\mu^2\right)\Delta\mathcal{C}^{\rm S}_{1,q}(z,\frac{Q^2}{\mu^2})$$
$$+\Delta f_g^{\rm S}\left(\frac{x}{z},\mu^2\right)\Delta\mathcal{C}^{\rm S}_{1,g}(z,\frac{Q^2}{\mu^2}) + n_f\Delta f_{q,k}^{\rm NS}\left(\frac{x}{z},\mu^2\right)\Delta\mathcal{C}^{\rm NS}_{1,q}(z,\frac{Q^2}{\mu^2})\Big]\,. \tag{1.6}$$

Unfortunately there does not exist such a simple formula for $g_2(x,Q^2)$ because of twist three contributions which are not power suppressed with respect to the twist two parts. Hence one cannot give a simple parton model

interpretation for the transverse spin structure function and we will therefore not discuss it in the subsequent part of this paper.
In Eq. (1.6) we have used the following notation. The charge of the light quarks is denoted by e_k and n_f stands for the number of light flavours. The spin parton densities $\Delta f_i(z,\mu^2)$ $(i=q,g)$ depend on factorization scale μ which is put to be equal to the renormalization scale. The spin parton coefficient functions $\Delta\mathcal{C}_{1,i}$ depend on the same scale μ. The quark parton densities and the quark coefficient functions can be split in non-singlet (NS) and singlet (S) parts with respect to the flavour group. The singlet and non-singlet combinations of parton densities are given by

$$\Delta f_q^{\rm S}(z,\mu^2) \;=\; \sum_{k=1}^{n_f}[\Delta f_k(z,\mu^2)+\Delta f_{\bar k}(z,\mu^2)]\,, \tag{1.7}$$

and

$$\Delta f_{q,k}^{\rm NS}(z,\mu^2) \;=\; \Delta f_k(z,\mu^2)+\Delta f_{\bar k}(z,\mu^2)-\frac{1}{n_f}\Delta f_q^{\rm S}(z,\mu^2)\,, \tag{1.8}$$

respectively. Since it turns out that the equations are easier to study when one performs a Mellin transform defined by

$$F^{(n)}=\int_0^1 dz\, z^{n-1}F(z)\,, \tag{1.9}$$

we will present all the following formulae in this representation.
The parton densities and the coefficient functions above satisfy the renormalization group equations. Let us first define the differential operator

$$D \underset{D}{=} \mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\,, \qquad \beta(g)=-\beta_0\frac{g^3}{16\pi^2}+\cdots\,. \tag{1.10}$$

Using this notation the renormalization group equations for the parton densities read

$$\begin{aligned} D\,\Delta f_{q,k}^{\rm NS,(n)} &= -\Delta\gamma_{qq}^{\rm NS,(n)}\Delta f_{q,k}^{\rm NS,(n)}\,, \qquad k=u,d\cdots\,,\\ D\,\Delta f_i^{\rm S,(n)} &= -\Delta\gamma_{ij}^{\rm S,(n)}\Delta f_j^{\rm S,(n)}\,, \qquad i,j=q,g\,, \end{aligned} \tag{1.11}$$

and for the coefficient functions

$$\begin{aligned} D\,\Delta\mathcal{C}_{1,q}^{\rm NS,(n)} &= \Delta\gamma_{qq}^{\rm NS,(n)}\Delta\mathcal{C}_{1,q}^{\rm NS,(n)}\,,\\ D\,\Delta\mathcal{C}_{1,i}^{\rm S,(n)} &= \Delta\gamma_{ji}^{\rm S,(n)}\Delta\mathcal{C}_{1,j}^{\rm S,(n)}\,, \qquad i,j=q,g\,. \end{aligned} \tag{1.12}$$

From the equations above it follows that the structure function is a renormalization group invariant *i.e.*

$$D\, g_1^{(n)}(Q^2) = 0\,, \tag{1.13}$$

which implies that it is a physical quantity independent of the scale μ.

The anomalous dimensions and the coefficient functions are calculable order by order in perturbation theory. Let us first sketch the derivation of the anomalous dimensions before we pay attention to the coefficient functions. In [1] the anomalous dimensions appearing in the spin dependent quantities have been derived from the calculation of the operator matrix elements (OME's). For an alternative derivation see [2]. These OME's are obtained by sandwiching local operators between quark and gluon states. These operators appear in the lightcone expansion of the product of the electromagnetic currents in Eq. (1.4). Suppressing some irrelevant Lorentz indices the expansion reads as follows

$$J(x)J(0) \underset{x^2\to 0}{=} \sum_{n=0}^{\infty}\sum_i c_{1,i}^{(n)}(x^2) O_i^{(n)}(0)\,, \qquad i=q,g\,. \tag{1.14}$$

Here n denotes the spins of the local operators $O_i^{(n)}$ and $c_{1,i}^{(n)}(x^2)$ are the Fourier transforms of the coefficient functions in position space. The operators of twist two, which can also be split into singlet and non-singlet parts, are given in the literature (see *e.g.* Eqs (2.5)–(2.7) in [1]). Since, in the Bjørken limit, the integrand in Eq. (1.4) is dominated by the light cone we can replace the current–current product by the above expansion so that one has to compute the renormalized OME's

$$A_{ij}^{r,(n)}\left(\frac{-p^2}{\mu^2}\right) = \langle j(p) \mid O_i^{(n)} \mid p(j)\rangle\,, \tag{1.15}$$

where $i,j = q,g$ and p denotes the external momentum of the quarks and gluons and $r = \mathrm{NS},\mathrm{S}$. In [1] and recently also in [3] the above operator matrix element (OME) has been computed up to second order in the strong coupling constant α_s. The calculation proceeds as follows. After having derived the operator vertices (see Appendix A in [1]) one has to compute the Feynman graphs (see Figs 1–6 in [1]) which correspond to the unrenormalized (bare) OME's. The latter reveal ultraviolet singularities which are regularized by N-dimensional regularization. The unrenormalized OME's indicated by a hat can be written in the form

$$\hat{A}_{ij}^{r,(n)}\left(\frac{-p^2}{\mu^2},\frac{1}{\varepsilon}\right) = \delta_{ij} + \hat{a}_s S_\varepsilon\left(\frac{-p^2}{\mu^2}\right)^{\varepsilon/2}\Big[\frac{1}{\varepsilon}\Delta\gamma_{ij}^{(n),(0)} + \Delta a_{ij}^{(n),(1)} + \varepsilon\Delta a_{ij}^{\varepsilon,(n),(1)}\Big]$$

$$
\begin{aligned}
&+\ \hat{a}_s^2 S_\varepsilon^2 (\frac{-p^2}{\mu^2})^\varepsilon \Big[\frac{1}{\varepsilon^2}\Big\{\frac{1}{2}\Delta\gamma_{ik}^{(n),(0)}\Delta\gamma_{kj}^{(n),(0)} - \beta_0 \Delta\gamma_{ij}^{(n),(0)}\Big\} \\
&+\ \frac{1}{\varepsilon}\Big\{\frac{1}{2}\Delta\gamma_{ij}^{(n),(1)} - 2\beta_0 \Delta a_{ij}^{(n),(1)} + \Delta\gamma_{ik}^{(n),(0)}\Delta a_{kj}^{(n),(1)}\Big\} \\
&+\ \Delta a_{ij}^{(n),(2)} - 2\beta_0 \Delta a_{ij}^{\varepsilon,(n),(1)} + \Delta\gamma_{ik}^{(n),(0)}\Delta a_{kj}^{\varepsilon,(n),(1)}\Big]\,, \qquad (1.16)
\end{aligned}
$$

where S_ε is the spherical factor characteristic of N-dimensional regularization. Here the hat indicates that all quantities are unrenormalized with respect to coupling constant and operator renormalization. The algebraic structure shown by the expression above follows from the renormalization group. In addition to the anomalous dimensions one also encounters the coefficients of the beta-function. For instance β_0 (see Eq. (1.10)) is the lowest order coefficient, which also appears in the coupling constant renormalization, given by

$$
\hat{a}_s = a_s(\mu^2)\Big[1 + a_s(\mu^2) S_\varepsilon \Big\{2\beta_0 \frac{1}{\varepsilon}\Big\}\Big]\,, \qquad a_s = \frac{\alpha_s}{4\pi}\,. \qquad (1.17)
$$

From the expression above one can in principle extract the first and second order anomalous dimension of the local operators in Eq. (1.14) which are given by $\Delta\gamma_{ij}^{(n),(0)}$ and $\Delta\gamma_{ij}^{(n),(1)}$ respectively. However in the renormalization of the OME's one has to deal with two difficulties. The first one is caused by the fact that usually the external momentum p is taken off-shell ($p^2 < 0$). This means that the OME in Eq. (1.15) ceases to be a genuine S-matrix element and it becomes gauge dependent. Therefore one also has to carry out gauge parameter renormalization. The second problem, which is characteristic of spin operators, is the appearance of the γ_5-matrix and the Levi–Civita tensor $\epsilon_{\mu\nu\alpha\beta}$ in the operator vertices (see Appendix A in [1]). In the case of N-dimensional regularization one has to find a suitable prescription to define these essentially four dimensional quantities in N-dimensions. In [1] and [3] the vertex $\gamma_\mu\gamma_5$ is replaced by

$$
\gamma_\mu\gamma_5 = \frac{i}{6}\epsilon_{\mu\alpha\beta\sigma}\gamma^\alpha\gamma^\beta\gamma^\sigma\,, \qquad (1.18)
$$

so that only the Levi–Civita tensor appears in the OME's. This prescription is equivalent to the one given by 't Hooft and Veltman [4] which is worked out in more detail by Breitenlohner and Maison [5] (HVBM). For the replacement of the γ_5-matrix in Eq. (1.18) see [6, 7]. Although this prescription preserves the cyclicity of the traces it destroys the anticommutativity of the γ_5-matrix. This will mean that some Ward-identies or theorems will be violated. For example the non-singlet axial vector current, presented by the operator $O_q^{\rm NS,(1)}$ in Eq. (1.14), gets renormalized in second order although it is conserved. Furthermore the Adler–Bardeen theorem [8] concerning the nonrenormalization of the Adler-anomaly is violated. This will affect the renormalization of the singlet axial-vector operator $O_q^{\rm S,(1)}$ in order α_s^3. To undo these effects one has to introduce an additional renormalization constant in order to obtain the correct anomalous dimensions in the $\overline{\rm MS}$-scheme. The latter have now to be extracted from the renormalized rather than the unrenormalized OME's. After coupling constant renormalization the OME's are renormalized as follows

$$\begin{aligned} \bar{A}_{qq}^{\rm NS,(n)} &= \bar{Z}_{qq}^{5,{\rm NS},(n)}(\bar{Z}^{-1})_{qq}^{\rm NS,(n)}\hat{A}_{qq}^{\rm NS,(n)}\,, \\ \bar{A}_{ij}^{\rm S,(n)} &= Z_{qq}^{5,{\rm S},(n)}(Z^{-1})_{iq}^{\rm S,(n)}\hat{A}_{qj}^{\rm S,(n)} + (\bar{Z}^{-1})_{ig}^{\rm S,(n)}\hat{A}_{gj}^{\rm S,(n)}\,, \end{aligned} \tag{1.19}$$

where we have chosen the $\overline{\rm MS}$-scheme. In this scheme the constant for the HVBM-prescription can be written up to order α_s^2 as

$$Z_{qq}^{5,r,(n)}\left(\frac{1}{\varepsilon}\right) = 1 + a_s S_\varepsilon\left[z_{qq}^{(n),(1)}\right] + a_s^2 S_\varepsilon^2\left[\frac{1}{\varepsilon}\beta_0 z_{qq}^{(n),(1)} + z_{qq}^{r,(n),(2)}\right], \tag{1.20}$$

with $r = {\rm NS, S}$. Notice that the difference between the singlet (S) and the non-singlet (NS) expression for $Z_{qq}^{5,r}$ shows up for the first time in second order (see [7]). In this reference $Z_{qq}^{5,r}$ has been calculated for the first moment ($n = 1$) up to order α_s^3. Recently this constant has been computed up to second order in [3] but now for general moments. In the non-singlet case it can be computed from the ratio

$$Z_{qq}^{5,{\rm NS},(n)}\left(\frac{1}{\varepsilon}\right) = \frac{\hat{A}_{qq}^{\rm NS,(n)}(-p^2/\mu^2, 1/\varepsilon)\mid_{\rm naive}}{\hat{A}_{qq}^{\rm NS,(n)}(-p^2/\mu^2, 1/\varepsilon)\mid_{\rm HVBM}}\Bigg|_{p^2=-\mu^2}\,, \tag{1.21}$$

where in the numerator one has used the so called naive prescription in which the γ_5-matrix anticommutes with all other γ-matrices irrespective of the value for the dimension N. The use of the naive method implies that the numerator can be replaced by the spin averaged OME $\hat{A}_{qq}^{\rm NS,(n)}$ in which the γ_5-matrix does not appear. A similar derivation exists for $Z_{qq}^{5,{\rm S},(n)}$ where one also makes a comparison between the naive γ_5 and the HVBM prescription. From the considerations presented above one could have asked the question why it is preferable to choose the HVBM instead of the naive prescription since in the latter case $Z_{qq}^{5,r,(n)} = 1$? The reason is that the Levi–Civita tensor appears in the OME $\hat{A}_{gq}$ which induces in the subgraphs containing quark lines the HVBM prescription. Therefore the naive method is inconsistent and it is better to use a consistent procedure like HVBM where all constants are fixed once and for all. The operator renormalization constants in Eq. (1.19), presented in the $\overline{\rm MS}$-scheme, read as follows

$$(\bar{Z}^{-1})^{r,(n)}_{ij}\left(\frac{1}{\varepsilon}\right) = \delta_{ij} + a_s S_\varepsilon\Big[-\frac{1}{\varepsilon}\Delta\gamma^{(n),(0)}_{ij}\Big] + a_s^2 S_\varepsilon^2\Big[\frac{1}{\varepsilon^2}\Big\{\frac{1}{2}\Delta\gamma^{(n),(0)}_{ik}\Delta\gamma^{(n),(0)}_{kj} - \beta_0\Delta\gamma^{(n),(0)}_{ij}\Big\} + \frac{1}{2\varepsilon}\Big\{\Delta\bar{\gamma}^{(n),(1)}_{ij} \pm \Delta\gamma^{(n),(0)}_{ik}\Delta z^{(n),(1)}_{kj}\Big\}\Big]\,. \tag{1.22}$$

The above expression differs from the usual one by the appearance of the term $z^{(n),(1)}_{kj}$ with $k=j=q$ which only contributes to $(\bar{Z}^{-1})_{qg}$ (plus sign) and $(\bar{Z}^{-1})_{gq}$ (minus sign) up to order α_s^2. If this term is omitted then the anomalous dimensions will equal those present in Eq. (1.16), which differ by a finite renormalization from the ones presented in the $\overline{\rm MS}$-scheme. Notice that the lowest order coefficients $\Delta\gamma^{(n),(0)}_{ij}$ are not affected by any γ_5 prescription. Finally we want to emphasize that due to the pole term in Eq. (1.20) $Z^{5,r}_{qq}$ does not represent a finite renormalization constant in the usual sense. Using the above procedure one can write the following expression for the renormalized OME in the $\overline{\rm MS}$-scheme

$$\bar{A}_{ij}\left(\frac{-p^2}{\mu^2}\right) = \delta_{ij} + a_s\Big[\frac{1}{2}\Delta\gamma^{(n),(0)}_{ij}\ln\left(\frac{-p^2}{\mu^2}\right) + \Delta\bar{a}^{(n),(1)}_{ij}\Big] + a_s^2\Big[\Big\{\frac{1}{8}\Delta\gamma^{(n),(0)}_{ik}\Delta\gamma^{(n),(0)}_{kj} - \frac{1}{4}\beta_0\Delta\gamma^{(n),(0)}_{ij}\Big\}\ln^2\left(\frac{-p^2}{\mu^2}\right) + \Big\{\frac{1}{2}\Delta\bar{\gamma}^{(n),(1)}_{ij} - \beta_0\Delta\bar{a}^{(n),(1)}_{ij} + \frac{1}{2}\Delta\gamma^{(n),(0)}_{ik}\Delta\bar{a}^{(n),(1)}_{kj}\Big\}\ln\left(\frac{-p^2}{\mu^2}\right) + \Delta\bar{a}^{(n),(2)}_{ij}\Big]\,. \tag{1.23}$$

Notice that the coefficients $\Delta\bar{a}^{(n),(k)}_{ij}$ in this expression differ from the $\Delta a^{(n),(k)}_{ij}$ present in Eq. (1.16). From the relation

$$\Delta f^{(n)}_i(\mu^2) = \bar{A}^{(n)}_{ij}\left(\frac{-p^2}{\mu^2}\right)\Delta f^{(n)}_i(-p^2)\,, \tag{1.24}$$

and Eq. (1.11) one concludes that the renormalized $\bar{A}_{ij}^{(n)}$ satisfy the renormalization group equation given by

$$D\bar{A}_{ij}^{(n)} = -\Delta\bar{\gamma}_{ik}^{(n)}\bar{A}_{kj}^{(n)} \,. \tag{1.25}$$

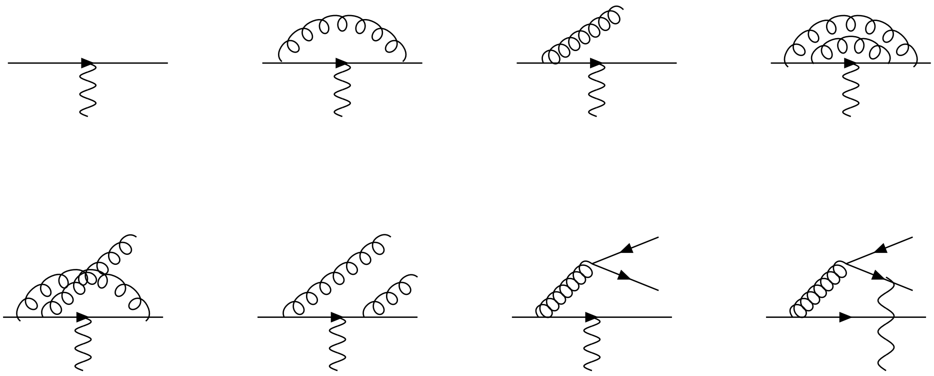

Fig. 2. Contributions to the process $\gamma^* + q \to$ 'X' contributing to the partonic structure function $\hat{g}_{1,q}$.

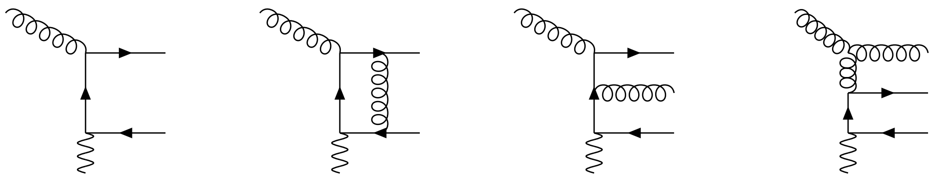

Fig. 3. Contributions to the process $\gamma^* + g \to$ 'X' contributing to the partonic structure function $\hat{g}_{1,g}$.

After having discussed the renormalization of the OME's we now explain the procedure to compute the coefficient functions appearing in the spin structure function of Eq. (1.6). They are obtained from the partonic subprocesses denoted by

$$\gamma^* + i \to \text{'X'} \,, \tag{1.26}$$

where i stands either for a quark (q) or a gluon (g) and 'X' represents an inclusive multi-partonic state. The above processes have been calculated up to order α_s^2 in [9]. Some Feynman graphs are depicted in Fig. 2 ($i = q$) and in Fig. 3 ($i = g$). The computation of the QCD corrections reveal ultraviolet, infrared and colinear divergences which appear in the Feynman integrals and in the phase space integrals. Like in the case of the operator matrix elements we regularize them by means of N-dimensional regularization. Further we use the HVBM prescription for the γ_5-matrix as discussed below Eq. (1.18). Adding all contributions one observes that the infrared singularities cancel and the radiative corrections are described by the parton structure functions which in general can be presented by the expression

$$
\begin{aligned}
&\hat{g}_{1,i}^{r,(n)}\left(\frac{Q^2}{\mu^2},\frac{1}{\varepsilon}\right) = \delta_{qi} \\
&+\hat{a}_s S_\varepsilon \left(\frac{Q^2}{\mu^2}\right)^{\varepsilon/2}\Big[-\frac{1}{\varepsilon}\Delta\gamma_{qi}^{(n),(0)} + \Delta c_{1,i}^{(n),(1)} + \varepsilon \Delta c_{1,i}^{\varepsilon,(n),(1)}\Big] \\
&+\hat{a}_s^2 S_\varepsilon^2 \left(\frac{Q^2}{\mu^2}\right)^{\varepsilon}\Big[\frac{1}{\varepsilon^2}\Big\{\frac{1}{2}\Delta\gamma_{qj}^{(n),(0)}\Delta\gamma_{ji}^{(n),(0)} + \beta_0 \Delta\gamma_{qi}^{(n),(0)}\Big\} \\
&+\frac{1}{\varepsilon}\Big\{-\frac{1}{2}\Delta\gamma_{qi}^{(n),(1)} - 2\beta_0\Delta c_{1,i}^{(n),(1)} - \Delta c_{1,j}^{(n),(1)}\Delta\gamma_{ji}^{(n),(0)}\Big\} \\
&+\Delta c_{1,i}^{(n),(2)} - 2\beta_0 \Delta c_{1,i}^{\varepsilon,(n),(1)} - \Delta c_{1,j}^{\varepsilon,(n),(1)}\Delta\gamma_{ji}^{(n),(0)}\Big]\,, \qquad (1.27)
\end{aligned}
$$

with $r = \mathrm{NS}, \mathrm{S}$. The above expression still contains ultraviolet and colinear divergences both represented by the pole terms $(1/\varepsilon)^k$. The former are removed by coupling constant renormalization (see Eq. (1.17)). The residues of the colinear divergences are usually denoted by the DGLAP spin splitting functions ΔP_{ij} which are related to the anomalous dimensions via a Mellin transform *i.e.*

$$
\Delta\gamma_{ij}^{(n)} = -\int_0^1 dz\, z^{n-1}\,\Delta P_{ij}(z)\,. \qquad (1.28)
$$

The colinear divergences are removed by applying mass factorization which proceeds as follows

$$
\begin{aligned}
&\Delta\bar{\mathcal{C}}_{1,q}^{\mathrm{NS,(n)}} = (Z_{qq}^{5,\mathrm{NS},(n)})^{-1}(\bar{\Gamma}^{-1})_{qq}^{\mathrm{NS},(n)}\hat{g}_{1,q}^{\mathrm{NS},(n)} \\
&\Delta\bar{\mathcal{C}}_{1,q}^{\mathrm{S},(n)} = (Z_{qq}^{5,\mathrm{S},(n)})^{-1}\Big[(\bar{\Gamma}^{-1})_{qq}^{\mathrm{S},(n)}\hat{g}_{1,q}^{\mathrm{S},(n)} + (\bar{\Gamma}^{-1})_{gq}^{\mathrm{S},(n)}\hat{g}_{1,g}^{\mathrm{S},(n)}\Big] \\
&\Delta\bar{\mathcal{C}}_{1,g}^{\mathrm{S},(n)} = (\bar{\Gamma}^{-1})_{qg}^{\mathrm{S},(n)}\hat{g}_{1,q}^{\mathrm{S},(n)} + (\bar{\Gamma}^{-1})_{gg}^{\mathrm{S},(n)}\hat{g}_{1,g}^{\mathrm{S},(n)}\,, \qquad (1.29)
\end{aligned}
$$

and the transition functions $(\bar{\Gamma}^{-1})_{ij}^{r,(n)}$ are given by (see Eq. (1.22))

$$(\bar{\Gamma}^{-1})_{qq}^{\mathrm{NS},(n)} = \bar{Z}_{qq}^{\mathrm{NS},(n)}\,, \qquad (\bar{\Gamma}^{-1})_{ij}^{\mathrm{S},(n)} = \bar{Z}_{ij}^{\mathrm{S},(n)}\,. \tag{1.30}$$

Using these expressions the longitudinal spin coefficient functions can be written as

$$\Delta\bar{\mathcal{C}}_{1,i}^{r,(n)}\left(\frac{Q^2}{\mu^2}\right) = \delta_{qi} + a_s\Big[-\frac{1}{2}\Delta\gamma_{qi}^{(n),(0)}\ln\left(\frac{Q^2}{\mu^2}\right) + \Delta\bar{c}_{1,i}^{(n),(1)}\Big]$$
$$+a_s^2\Big[\Big\{\frac{1}{8}\Delta\gamma_{qj}^{(n),(0)}\Delta\gamma_{ji}^{(n),(0)} + \frac{1}{4}\beta_0\Delta\gamma_{qi}^{(n),(0)}\Big\}\ln^2\left(\frac{Q^2}{\mu^2}\right)$$
$$+\Big\{-\frac{1}{2}\Delta\bar{\gamma}_{qi}^{(n),(1)} - \beta_0\Delta\bar{c}_{1,i}^{(n),(1)} - \frac{1}{2}\Delta\gamma_{ji}^{(n),(0)}\Delta\bar{c}_{1,j}^{(n),(1)}\Big\}\ln\left(\frac{Q^2}{\mu^2}\right)$$
$$+\Delta\bar{c}_{1,i}^{(n),(2)}\Big]\,, \tag{1.31}$$

so that they satisfy the renormalization group equations in Eq. (1.12). Notice that in the above expression the coefficients $\Delta\bar{c}_{1,i}^{(n),(k)}$ differ from $\Delta c_{1,i}^{(n),(k)}$ in Eq. (1.27). From the discussion above one infers that the renormalization of the operator matrix elements determine the way one has to perform the mass factorization on $\hat{g}_{1,i}^{r,(n)}$ and not vice versa. The reason is that the renormalization of the former (but not of the latter) is ruled by the Ward-identities and some theorems which are violated by the HVBM-prescription. This has forced us to introduce the additional constant $Z_{qq}^{5,r,(n)}$ in Eq. (1.20) to restore the wanted properties on the level of the renormalized operator matrix elements presented in Eq. (1.23). If we would have accidentally put $Z_{qq}^{5,r,(n)} = 1$ the coefficient functions and the renormalized operator matrix elements get different anomalous dimensions and the relations in Eq. (1.30) would be violated. From Eqs (1.24), (1.25) it also follows that the parton densities and the coefficient functions would have different anomalous dimensions. Hence the structure function $g_1(x,Q^2)$ (1.6) would not satisfy Eq. (1.13) anymore so that it is no longer a physical quantity. Therefore this choice for $Z_{qq}^{5,r}$ is unacceptable.

2. Fragmentation into polarized hadrons in electron–positron annihilation

Single hadron (denoted by H) inclusive production in electron-positron annihilation is given by the reaction

$$e^-(l_1,\sigma_1) + e^+(l_2,\sigma_2) \to V(q) \to H(p,s) + \text{‘X’}\,. \tag{2.1}$$

Here we have introduced a similar notation to the one in reaction (1.1). However the Bjørken scaling variable is defined by

$$x = \frac{2p\,q}{Q^2}\,, \quad q^2 = Q^2 > 0\,, \quad 0 < x \leq 1\,, \tag{2.2}$$

for timelike momenta of the vector boson V. The annihilation process can be depicted as in Fig. 1 where now the incoming hadron is outgoing and the electron in the final state becomes a positron in the initial state. In the case the incoming electron in reaction (2.1) is longitudinally polarized downwards, *i.e.* $\sigma_1 =\downarrow$, and the positron is unpolarized then one can simplify the cross section when the process becomes purely electromagnetic. In this case $V = \gamma$ and we get

$$\frac{d\sigma^{H(\downarrow)}(\downarrow)}{dx\,d\cos\theta} - \frac{d\sigma^{H(\uparrow)}(\downarrow)}{dx\,d\cos\theta} = N_{\rm C}\frac{\pi\alpha^2}{Q^2}\cos\,\theta\, g_1^H(x,Q^2)\,. \tag{2.3}$$

The above expression represents the difference between the cross sections where the detected hadron H is polarized parallel $s =\downarrow$ or anti-parallel $s =\uparrow$ with respect to the electron. Further $N_{\rm C}$ denotes the colour factor in SU($N_{\rm C}$) and θ is the polar angle describing the direction of the momentum of H in the C.M. frame with respect to the incoming electron. Notice that the hadron fragmentation function $g_1^H(x,Q^2)$ can be also measured in unpolarized electron–positron scattering provided reaction (2.1) is mediated by the Z-boson. Here it appears via the axial-vector coupling of this boson to the lepton-pair. The above hadron fragmentation function shows up in the anti-symmetric part of the hadronic structure tensor

$$W_{\mu\nu}(p,q,s) =$$

$$\frac{1}{4\pi}\int d^4z\, {\rm e}^{iq\cdot z}\langle 0 \mid J_\mu(z) \mid H(p,s), X\rangle\langle H(p,s), X \mid J_\nu(0) \mid 0\rangle\,, \tag{2.4}$$

which can be decomposed in the same way as shown for Eq. (1.5) in deep-inelastic scattering. Since H is exclusive the above expression is not a Fourier transform of a current–current correlation function which implies that we cannot insert the lightcone expansion of Eq. (1.15). Therefore we can only work in the QCD improved parton model picture in which the fragmentation function has the following form

$$
\begin{aligned}
g_1^H(x,Q^2) \;=\; & \frac{1}{n_f}\sum_{k=1}^{n_f} e_k^2 \int_x^1 \frac{dz}{z}\Big[\Delta D_q^{\rm H,S}\left(\frac{x}{z},\mu^2\right)\Delta\mathcal{C}_{1,q}^{\rm S}\Big(z,\frac{Q^2}{\mu^2}\Big) \\
& +\Delta D_g^{\rm H,S}\left(\frac{x}{z},\mu^2\right)\Delta\mathcal{C}_{1,g}^{\rm S}\left(z,\frac{Q^2}{\mu^2}\right) \\
& +n_f\Delta D_k^{\rm H,NS}\left(\frac{x}{z},\mu^2\right)\Delta\mathcal{C}_{1,q}^{\rm NS}\left(z,\frac{Q^2}{\mu^2}\right)\Big]\,. \qquad (2.5)
\end{aligned}
$$

The spin parton fragmentation densities denoted by $\Delta D_i^H(z,\mu^2)$ are the analogues of the parton densities in Eq. (1.6) and they satisfy the same renormalization group equations. However beyond lowest order the anomalous dimensions are different for these two densities (see [10–12]). The anomalous dimensions ruling the evolution of the spin fragmentation densities have recently been calculated up to second order in [12]. To obtain the timelike spin coefficient functions one has to calculate the timelike photon analogues of the graphs in Figs 2, 3. Here the incoming quark and gluon become outgoing and they now fragment into the hadron H. Furthermore the spacelike photon turns into a timelike one. The calculation of these coefficient functions was recently done up to second order in [13]. It proceeds in the same way as for deep inelastic scattering in Section 1 where again the HVBM prescription for the γ_5-matrix is chosen. After having calculated the parton fragmentation functions denoted by $\hat{g}_{1,i}^{H,r,(n)}$ one has to perform mass factorization analogously to Eq. (1.29). However our calculation reveals that the renormalization constant $Z_{qq}^{5,r}$ is different for timelike (fragmentation function) and spacelike (structure function) quantities. In [13] one has obtained the non-singlet part of this constant for the timelike (T) process (2.1) by computing the ratio

$$
Z_{qq}^{5,{\rm NS,T},(n)} = \left.\frac{\hat{g}_{1,q}^{\rm H,NS,(n)}(Q^2/\mu^2,1/\varepsilon)}{\hat{\mathcal{F}}_{3,q}^{\rm H,NS,(n)}(Q^2/\mu^2,1/\varepsilon)}\right|_{\mu^2=Q^2}, \qquad (2.6)
$$

where $\hat{\mathcal{F}}_{3,q}^{\rm H,NS,(n)}$ is the parton fragmentation function in unpolarized electron-positron annihilation. It arises due to the axial-vector coupling of the Z-boson to the outgoing quark anti-quark pair (see Fig. 1). If the γ_5 anticommutes with the other γ-matrices one can show that the coefficient functions

$\Delta\mathcal{C}_{1,q}^{\rm NS}$ and $\mathcal{C}_{3,q}^{\rm NS}$ are equal up to order α_s^2. Since the HVBM prescription destroys this property for $\hat{g}_{1,q}^{{\rm H,NS},(n)}$ we have to multiply the latter by $Z_{qq}^{5,{\rm NS,T}}$ in order to obtain the correct coefficient functions. If we assign to the constant in Eq. (1.20) the superscript S (here spacelike), the following relation holds for the inverse Mellin transforms

$$Z_{qq}^{5,{\rm NS,T}}(z) = -zZ_{qq}^{5,{\rm NS,S}}\left(\frac{1}{z}\right) + a_s^2\Big[\beta_0 z_{qq}^{{\rm NS},(1)}(z)\ln z\Big]\,. \tag{2.7}$$

The above equality demonstrates the breakdown of the Gribov–Lipatov relation [14] in order α_s^2. The above relation is also found for the timelike and spacelike non-singlet splitting functions $P_{qq}^{\rm NS}$ in unpolarized scattering in Eqs. (6.37) and (6.38) of [10] where $z_{qq}^{(1)}$ is replaced by the lowest order DGLAP splitting function $P_{qq}^{{\rm NS},(0)}$. Notice that the same relation holds for the spin splitting functions because $\Delta P_{qq}^{\rm NS} = P_{qq}^{\rm NS}$ (see [12]). The dependence of $Z_{qq}^{5,{\rm NS}}$ on the quantity under consideration reveals that it is not an universal constant. Therefore aside from the pole term already mentioned above Eq. (1.23), it does not represent a genuine renormalization constant in the usual sense.

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