

Q^2 –EVOLUTION OF SPIN–DEPENDENT PARTON DENSITIES*

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We discuss the NLO evolution of quark transversity densities and of the parton distribution function for linearly polarized gluons in a linearly polarized hadron. A supersymmetric relation between the NLO evolution kernels for transversity and for linear polarization is found. We also study the implications of NLO evolution for Soffer’s inequality and the prospects of measuring transversity densities in polarized Drell–Yan at RHIC.

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1. Introduction

Experimentally, the vast majority of data sensitive to parton densities have been taken without fixing the polarization of the initial beams or the target. The densities extracted in this way are usually referred to as the ‘unpolarized’ parton distributions $f(x, Q^2)$ ($f = q, \bar{q}, g$).

Within roughly the last decade, also more and more data have become available that are sensitive to the ‘longitudinally’ polarized (‘helicity-weighted’) parton densities of the nucleon. The tool to obtain such information has (almost exclusively) been deep–inelastic scattering (DIS) of longitudinally polarized leptons and nucleons. The spin–asymmetry measured in such reactions is related to the probability for finding a certain parton–type with positive helicity in a nucleon of positive helicity *minus* the probability for finding it with negative helicity. These densities, denoted as $\Delta f(x, Q^2)$ ($f = q, \bar{q}, g$), contain information different from that contained in the more familiar unpolarized ones.

For a *transversely* polarized spin–1/2 hadron one can define a further quark density [1–3] in very much the same way as the longitudinally polarized quark distributions, by taking differences of probabilities for finding quarks

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with transverse spin aligned and anti-aligned with the transverse hadron spin. These densities are called ‘transversity’ densities and are denoted by $\Delta_T f(x, Q^2)$. It turns out that – unlike the situation for unpolarized and longitudinally polarized densities — there is *no* gluonic analogue of quark transversity [2]. This is due to angular momentum conservation: transversity densities are related to helicity–flip amplitudes. A gluonic helicity–flip amplitude would require the hadron to absorb two units of helicity, which no spin-1/2 target can do.

The transversity distributions are completely unmeasured so far since they cannot be directly accessed in DIS. It seems certain, however, that measurements of the $\Delta_T f$ will be attempted at the future polarized proton-proton collider RHIC at Brookhaven [4]. The most suitable candidate for such measurements is believed to be Drell–Yan dimuon production [1].

The $\Delta_T f$ complete the twist-2 sector of parton densities of spin-1/2 hadrons. Nevertheless, we have not yet depleted the full set of parton densities that can be defined if spin is taken into account. There finally is a further spin-dependent *gluon* distribution that to some extent can be regarded as the gluonic counterpart of quark transversity. Unlike the ‘helicity’ density Δg that describes *circular* polarization of the gluon, it is encountered if the gluon is *linearly* polarized [3,5,6]. The density is denoted by $\Delta_L g(x, Q^2)$ and exists only in a linearly polarized hadron (or photon [6]), which therefore has to have spin ≥ 1 . There is no quark distribution in this case [5]. Even though a measurement of $\Delta_L g$ does not seem very realistic at the moment, it possesses some interesting theoretical aspects which justify its analysis.

Table I summarizes the parton densities we have defined.

TABLE I

List of twist-2 quark and gluon densities including spin-dependence. We have suppressed the ubiquitous argument (x, Q^2) of the densities. Note that ‘ q ’ always runs over quarks as well as over antiquarks. Labels $+$, $-$ denote helicities, $\uparrow, \downarrow$ transverse polarizations, and $\hat{x}$ ($\hat{y}$) stands for linear polarization along the x (y) axis, where the particle is moving along the z -direction. Subscripts refer to partons and superscripts to the parent hadron.

polarization	quarks	gluons
unpolarized	$q \equiv q_+^+ + q_+^- \equiv q_{\uparrow}^{\uparrow} + q_{\uparrow}^{\downarrow}$	$g \equiv g_+^+ + g_+^- \equiv g_x^x + g_y^x$
long. polarized	$\Delta q = q_+^+ - q_+^-$	$\Delta g = g_+^+ - g_+^-$
transversity	$\Delta_T q = q_{\uparrow}^{\uparrow} - q_{\uparrow}^{\downarrow}$	—
linearly pol. glue	—	$\Delta_L g = g_{\hat{x}}^{\hat{x}} - g_{\hat{y}}^{\hat{x}}$

It is important to realize that each set of parton densities in Table I (*i.e.*, each of the rows of Table I) is subject to its own set of evolution equations.

For instance, the evolution of the longitudinally polarized densities proceeds independently from that of the unpolarized partons, and so forth. In this way, we are led to introducing separate sets of evolution kernels (splitting functions) for each type of polarization.

The Q^2 -evolution of the unpolarized densities has been worked out up to NLO accuracy of QCD already a long time ago [7–9], and it has become standard since about ten years to analyse the unpolarized data within the NLO framework. The LO evolution of the $\Delta f(x, Q^2)$, the $\Delta_T f(x, Q^2)$, and of $\Delta_L g(x, Q^2)$ has also been known for a long time [3, 7, 10], whereas the derivation of the NLO evolution kernels for longitudinally polarized partons has been a more recent development [11, 12]. Very recently, the NLO splitting functions for transversity were derived within three independent calculations [13, 14]. In this paper, we will discuss the NLO evolution of the $\Delta_T f$, and its implications for an inequality between the f , Δf and $\Delta_T f$ derived by Soffer [15]. We will also for the first time present the NLO evolution kernel for $\Delta_L g$.

2. NLO evolution equations

Since there are no gluons involved in the case of the transversity distributions, their evolution equations reduce to simple non-singlet type equations. Introducing

$$\Delta_T q_{\pm}^n(Q^2) \equiv \Delta_T q^n(Q^2) \pm \Delta_T \bar{q}^n(Q^2) , \quad (1)$$

where the Mellin moments are defined by

$$\Delta_T q^n(Q^2) \equiv \int_0^1 x^{n-1} \Delta_T q^n(x, Q^2) , \quad (2)$$

one has the evolution equations (see, for example, [16])

$$\frac{d}{d \ln Q^2} \Delta_T q_{\pm}^n(Q^2) = \Delta_T P_{qq, \pm}^n(\alpha_s(Q^2)) \Delta_T q_{\pm}^n(Q^2) \quad (3)$$

for all flavours. The splitting functions $\Delta_T P_{qq, \pm}^n(\alpha_s(Q^2))$ are taken to have the following perturbative expansion:

$$\Delta_T P_{qq, \pm}^n(\alpha_s) = \left(\frac{\alpha_s}{2\pi} \right) \Delta_T P_{qq}^{(0), n} + \left(\frac{\alpha_s}{2\pi} \right)^2 \Delta_T P_{qq, \pm}^{(1), n} + \dots . \quad (4)$$

As indicated, $\Delta_T P_{qq, +}$ and $\Delta_T P_{qq, -}$ are equal at LO.

Similarly to (3) one has for linearly polarized gluons:

$$\frac{d}{d \ln Q^2} \Delta_L g^n(Q^2) = \Delta_L P_{gg}^n(\alpha_s(Q^2)) \Delta_L g^n(Q^2) , \quad (5)$$

where $\Delta_L P_{gg}^n$ has a perturbative expansion analogous to (4). These NLO evolution equations can be easily solved; for details see, for instance, [13].

3. Calculation of splitting functions

Historically, the evolution kernels for parton densities have been calculated within two rather different methods. On the one hand, the very powerful, yet quite formal, technique of the ‘Operator–Product–Expansion’ (OPE) has been applied. Here the evolution kernels are derived as anomalous dimensions of matrix elements of local operators. The other method is more intuitive and relies on parton model ideas and on the factorization of mass singularities in a physical (ghost-free) gauge [9, 17]. This is the method we will use. Very roughly, the strategy goes as follows: The NLO splitting functions are related to the residues of the collinear singularities in the *next-to*-NLO (NNLO) calculation of a partonic subprocess cross section for a ‘physical’ process. Being the pole part of this cross section, the singular terms are manifestly gauge-independent. This means that we can employ a gauge in their calculation that simplifies the extraction of the mass singularities as much as possible. As was shown in [17], working in a physical gauge, like the light-cone gauge, reduces the number of singular graphs at a given order significantly. Thus, a full NNLO calculation is clearly not required. In particular, only ladder-like diagrams, corresponding to a parton cascade, contribute to the pole part in such gauges, so that the cross section $\hat{\sigma}$ for any physical partonic process can be diagrammatically expanded into a sum of ladders of ‘two-particle-irreducible’ kernels that are individually finite. Mass singularities only occur on the lines connecting the kernels (*i.e.*, on the ‘sides’ of the ladders) and can therefore be projected out easily using a projector $\mathcal{P}$ that converts these lines into on-shell physical states. Thus, the factorization of mass singularities can be achieved: the cross section $\hat{\sigma}$ decomposes into a finite part $\hat{\sigma}_F$, and a function Γ that contains all the collinear singularities and is universal since no process-dependence is left over in it. The function Γ is then directly related to the splitting functions one is looking for [9, 17].

For the case of transversity, the projector $\Delta_T \mathcal{P}$ is found to be

$$\Delta_T \mathcal{P} \sim \frac{1}{4n \cdot k} \not{n} \not{s} \gamma_5, \quad (6)$$

where k is the momentum of the particle emerging from the top of a kernel, n is the gauge vector with $n \cdot A = 0$, $n^2 = 0$ for the light-cone gauge, and s is the transverse spin vector. More details on the calculation in the transversity case can be found in [13].

For linearly polarized gluons one has

$$\Delta_L \mathcal{P}_{\hat{x}\hat{y}}^{\alpha\beta} \equiv \frac{1}{2} \left(\epsilon_{\hat{x}}^\alpha \epsilon_{\hat{x}}^\beta - \epsilon_{\hat{y}}^\alpha \epsilon_{\hat{y}}^\beta \right), \quad (7)$$

where the polarization vectors

$$\epsilon_{\hat{x}} \equiv (0, 1, 0, \dots, 0), \quad \epsilon_{\hat{y}} \equiv (0, 0, 1, 0, \dots, 0) \quad (8)$$

have non-vanishing entries only in the x and y components, respectively. A more detailed account of our calculation of $\Delta_L P_{gg}^{(1)}$ will be given in a future publication [18].

4. Results

Our results for $\Delta_T P_{qq,\pm}^{(1)}(x)$ can be found in [13] and need not be repeated here. The one for $\Delta_L P_{gg}^{(1)}$ is new and reads in the $\overline{\text{MS}}$ scheme

$$\begin{aligned} \Delta_L P_{gg}^{(1)}(x) = & N_C^2 \left[\left(\frac{67}{18} + \frac{1}{2} \ln^2 x - 2 \ln x \ln(1-x) - \frac{\pi^2}{6} \right) \delta_L P_{gg}^{(0)}(x) \right. \\ & \left. + \frac{1-x^3}{6x} + S_2(x) \delta_L P_{gg}^{(0)}(-x) + \left(\frac{8}{3} + 3\zeta(3) \right) \delta(1-x) \right] \\ & + N_C T_f \left[-\frac{10}{9} \delta_L P_{gg}^{(0)}(x) + \frac{1-x^3}{3x} - \frac{4}{3} \delta(1-x) \right] \\ & - C_F T_f \left[2 \frac{1-x^3}{3x} + \delta(1-x) \right], \end{aligned} \quad (9)$$

where

$$\delta_L P_{gg}^{(0)}(x) = \frac{2x}{(1-x)_+}, \quad (10)$$

$$\begin{aligned} S_2(x) &= \int_{\frac{x}{1+x}}^{\frac{1}{1+x}} \frac{dz}{z} \ln \left(\frac{1-z}{z} \right) \\ &= -2\text{Li}_2(-x) - 2 \ln x \ln(1+x) + \frac{1}{2} \ln^2 x - \frac{\pi^2}{6}, \end{aligned} \quad (11)$$

with $\text{Li}_2(x)$ being the dilogarithm. Furthermore, we have as usual $C_F = 4/3$, $N_C = 3$, and $T_f = n_f/2$, n_f being the number of active flavours. Finally, $\zeta(3) \approx 1.202057$. We note that in [6] a result for the part $\sim C_F T_f$ in

(9) at $x < 1$ was presented, which corresponds to the two-loop splitting function for transitions between linearly polarized gluons and *photons*. The result of [6] disagrees with ours by a an additional factor $1/x$ in [6]. The calculation of [6] therefore has to be in error, since a behaviour $\sim 1/x^2$ of the splitting function – more singular than the unpolarized one – cannot be correct.

There are two aspects of the result (9) that deserve further attention. Firstly, the small- x behaviour of the splitting function for linear polarization changes quite dramatically when going from LO to NLO. At LO, the splitting function behaves $\sim x$ as $x \rightarrow 0$. At NLO, there are terms $\sim 1/x$ in the splitting function (as one also encounters in the unpolarized case); we have

$$\Delta_L P_{gg}^{(1)}(x) \approx \frac{1}{3x} \left[\frac{1}{2} N_C^2 + N_C T_f - 2 C_F T_f \right] + \mathcal{O}(x) \quad (x \rightarrow 0). \quad (12)$$

We note that all logarithmic terms $\sim x \ln^2 x$ cancel out in this limit.

The other interesting point concerns the ‘supersymmetric’ limit $C_F = N_C = 2T_f \equiv N$ [19] which has already been investigated for the unpolarized and the longitudinally polarized NLO splitting functions in [11, 12, 20] and for the ‘time-like’ ones in [21]. One first of all finds that in the supersymmetric limit the LO splitting functions for quark transversity and for linearly polarized gluons become equal [3, 6]:

$$\Delta_T P_{qq}^{(0)}(x) = N \left[\frac{2x}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right] = \Delta_L P_{gg}^{(0)}(x). \quad (13)$$

Thus, we have found the supersymmetric ‘counterpart’ of transversity: linear polarization (see also [19]). This nicely completes the supersymmetry relations found in the case of unpolarized and longitudinally polarized parton densities and fragmentation functions. The fact that two rather different polarizations are linked here is not too surprising as both of them are transverse in a certain sense.

To see whether the supersymmetric relation also holds at NLO, we have to transform the splitting functions to a regularization scheme that respects supersymmetry, dimensional reduction. The procedure here follows closely that discussed in [12, 21]. The task is simplified by the fact that there are no parts $\sim \epsilon$ in the d -dimensional LO splitting functions for transversity or linearly polarized gluons at $x < 1$. Such terms – which are always absent in dimensional reduction, but can be present for dimensional *regularization* – are the reason why, for instance, the longitudinally polarized NLO splitting functions do not directly satisfy the supersymmetric relation in dimensional regularization, but have to be transformed to dimensional reduction first (see [12] for a more detailed discussion). The fact that the

LO ϵ -terms are absent at $x < 1$ in our case, means that our final results for the NLO transversity splitting function $\Delta_T P_{qq,+}^{(1)}$ of [13] and for the NLO splitting function for linearly polarized gluons, Eq. (9), already coincide for $x < 1$ with their respective $\overline{\text{MS}}$ expressions in dimensional *reduction*. We can therefore immediately compare these expressions in the supersymmetric limit. Indeed, we find for $C_F = N_C = 2T_f \equiv N$:

$$\Delta_T P_{qq,+}^{(1)}(x) \equiv \Delta_L P_{gg}^{(1)}(x) \quad (x < 1) . \quad (14)$$

The satisfaction of the supersymmetric relation at $x = 1$ is trivial; see [12], where the appropriate factorization scheme transformation to dimensional reduction at $x = 1$ is given.

We thus find that indeed the supersymmetric relation between the transversity splitting function and the evolution kernel for linearly polarized gluons is also verified at NLO. We note that the relation involves as expected the ‘+–combination’ $\Delta_T P_{qq,+}^{(1)} \equiv \Delta_T P_{qq}^{(1)} + \Delta_T P_{q\bar{q}}^{(1)}$ which corresponds to ‘singlet’ evolution. Finally, also note the miraculous cancellation of the terms $\sim 1/x$ in $\Delta_L P_{gg}^{(1)}$ in the supersymmetric limit (see Eq. (12)) – after all, such terms are not present in $\Delta_T P_{qq,+}^{(1)}$.

5. Soffer’s inequality and the transversely polarized Drell–Yan process

The unpolarized, longitudinally and transversely polarized quark distributions (q , Δq , $\Delta_T q$) of the nucleon are expected to obey the rather interesting relation

$$2|\Delta_T q(x)| \leq q(x) + \Delta q(x) \quad (15)$$

derived by Soffer [15]. It has recently been clarified that Soffer’s inequation is preserved by leading order (LO) QCD evolution, *i.e.* if (15) is valid at some scale Q_0 , it will also be valid at $Q > Q_0$ [22]. To NLO the situation is not as simple. The parton distributions are now subject to the choice of the factorization scheme which one may fix independently for q , Δq and $\Delta_T q$. One can therefore always find ‘sufficiently incompatible’ schemes in which a violation of (15) occurs. However, it was shown in [13] with analytical methods that the inequality for valence densities is preserved by NLO QCD evolution in a certain ‘Drell-Yan scheme’ in which the NLO cross sections for dimuon production maintain their LO forms, and also in the $\overline{\text{MS}}$ scheme. In this section, which is taken from [23], we shall show numerically that Soffer’s inequation for sea quarks is also preserved under NLO ($\overline{\text{MS}}$) evolution. We note that there is also a recent analytical proof of the preservation of Soffer’s inequality [24] at NLO.

For our study, we will assume saturation of Soffer's inequality at the input scale for parton evolution. Our choice for the r.h.s. of (15) will then be the NLO $\overline{\text{MS}}$ radiative parton model inputs for $q(x, Q_0^2)$ of [25] and for the longitudinally polarized $\Delta q(x, Q_0^2)$ of the 'standard' scenario of [26] at $Q_0^2 = \mu_{NLO}^2 = 0.34 \text{ GeV}^2$. For simplicity we will slightly deviate from the actual $q(x, Q_0^2)$ of [25] in so far as we will neglect the breaking of SU(2) in the input sea quark distributions originally present in this set. This seems reasonable as SU(2) symmetry was also assumed for the $\Delta \bar{q}(x, Q_0^2)$ of [26], which in that case was due to the fact that in the longitudinally polarized case there are no data yet that could discriminate between $\Delta \bar{u}$ and $\Delta \bar{d}$. We therefore prefer to assume $\Delta_T \bar{u}(x, Q_0^2) = \Delta_T \bar{d}(x, Q_0^2)$ also for the transversity input.

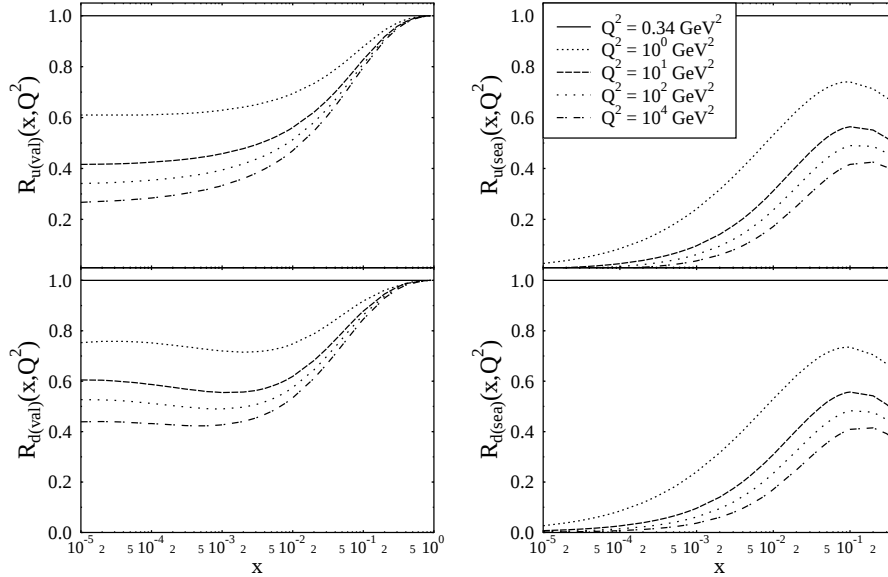


Fig. 1. The ratio $R_q(x, Q^2)$ as defined in (16) for $q = u_v, \bar{u}, d_v, \bar{d}$ and several fixed values of Q^2 .

In order to numerically check the preservation of (15), Fig. 1 shows the ratio

$$R_q(x, Q^2) = \frac{2|\Delta_T q(x, Q^2)|}{q(x, Q^2) + \Delta q(x, Q^2)} \quad (16)$$

as a function of x for several different Q^2 values for $q = u_v = u_-, \bar{u} = (u_+ - u_-)/2, d_v = d_-, \bar{d} = (d_+ - d_-)/2$ (cf. Eq. (1)). If NLO evolution preserves Soffer's inequality, then $R_q(x, Q^2)$ should not become larger than 1 for any $Q^2 \geq Q_0^2$. As we already know from [13] this is the case for

the two valence distributions. Figure 1 confirms that Soffer's inequality is indeed also preserved for sea distributions. Furthermore, we see in Fig. 1 that evolution leads to a strong suppression of $R_q(x, Q^2)$ at small values of x , in particular for the sea quarks.

As is obvious from (15), Soffer's inequality only restricts the absolute value of the transversity distribution. Therefore, we are free to choose the sign when saturating (15), and we have to check the results for the two distinct possibilities $\Delta_T q_v(x, Q_0^2) > 0$, $\Delta_T \bar{q}(x, Q_0^2) > 0$ and $\Delta_T q_v(x, Q_0^2) > 0$, $\Delta_T \bar{q}(x, Q_0^2) < 0$. Our results do not noticeably depend on the actual choice. Neither does it matter whether we decide to saturate Soffer's inequality for the *valence* densities q_v or for the full quark ($q \equiv q_v + \bar{q}$) distributions at the input scale.

One can utilise a saturated Soffer inequality to derive upper bounds on the transverse double-spin asymmetry A_{TT} to be measured in transversely polarized Drell-Yan muon pair production at RHIC. A_{TT} is defined as $A_{TT} = d\delta\sigma/d\sigma$ where the polarized and unpolarized hadronic cross sections are

$$d\delta\sigma \equiv \frac{1}{2} (d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}) \quad , \quad d\sigma \equiv \frac{1}{2} (d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}) \quad . \quad (17)$$

We employ the same input distributions as before, along with the same value for the initial scale Q_0 . We choose $Q_F = Q_R = M$ for the factorization and renormalization scales, where M is the invariant mass of the muon pair. Further details of the calculation of the Drell-Yan cross section to LO and NLO can be found in [23].

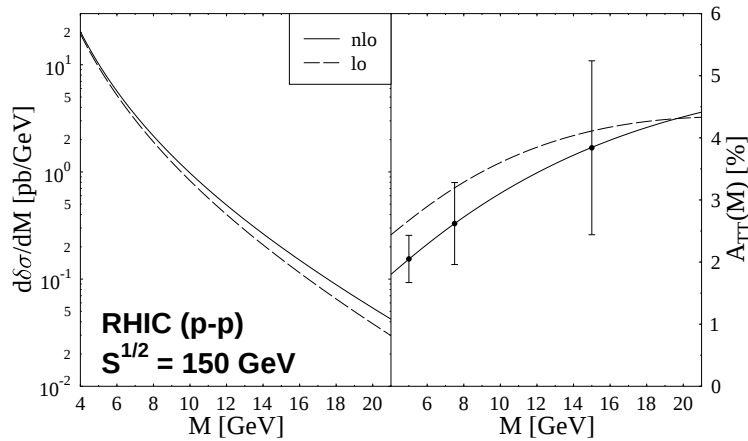


Fig. 2. NLO and LO maximal polarized Drell-Yan cross sections and asymmetries for RHIC at $\sqrt{S} = 150$ GeV. The error bars have been calculated assuming $\mathcal{L} = 240 \text{ pb}^{-1}$, 70% beam polarization and 100% detection efficiency.

Figure 2 shows the transversely polarized pp cross section and the ‘maximal’ double-spin asymmetry A_{TT} at LO and NLO for $\sqrt{S} = 150$ GeV, corresponding to the RHIC pp collider. We note that the region $9 \text{ GeV} \lesssim M \lesssim 11 \text{ GeV}$ will presumably not be accessible experimentally since it will be dominated by muon pairs from bottomonium decays. The predicted maximal asymmetry is of the order of a few per cent. The expected statistical error bars have been included in the plot; they have been calculated for 70% beam polarization, an integrated luminosity $\mathcal{L} = 240 \text{ pb}^{-1}$, and 100% detection efficiency. One concludes that asymmetries of this size should be well measurable at RHIC. Similar asymmetries are found [23] for another conceivable experimental situation, namely a proposed fixed-target experiment at HERA that would utilise the possibly forthcoming polarized 820 GeV HERA proton beam on a transversely polarized target, resulting in $\sqrt{S} = 40 \text{ GeV}$.

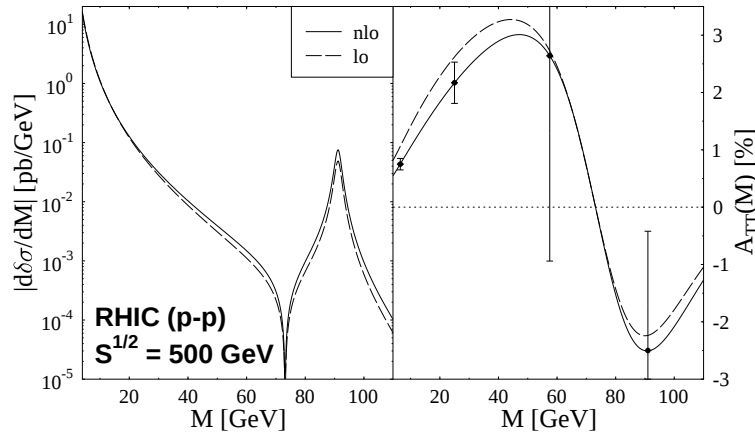


Fig. 3. As in Fig. 2, but for $\sqrt{S} = 500 \text{ GeV}$ and $\mathcal{L} = 800 \text{ pb}^{-1}$.

Figure 3 shows similar results for the high-energy end of RHIC, $\sqrt{S} = 500 \text{ GeV}$, where the integrated luminosity is expected to be $\mathcal{L} = 800 \text{ pb}^{-1}$. It turns out that the asymmetries become smaller as compared to the lower energies, but thanks to the higher luminosity the error bars become relatively smaller as well, at least in the region $5 \text{ GeV} \lesssim M \lesssim 25 \text{ GeV}$ where the errors are approximately 1/10 of the maximal asymmetry. One can also clearly see in Fig. 3 the effect of Z exchange and the Z resonance.

A comparison of the LO and NLO results in Figs. 2 and 3 answers one key question concerning the transversely polarized Drell-Yan process: our predictions for the maximal A_{TT} show good perturbative stability, *i.e.* the NLO corrections to the cross sections and A_{TT} are of moderate size, albeit not negligible.

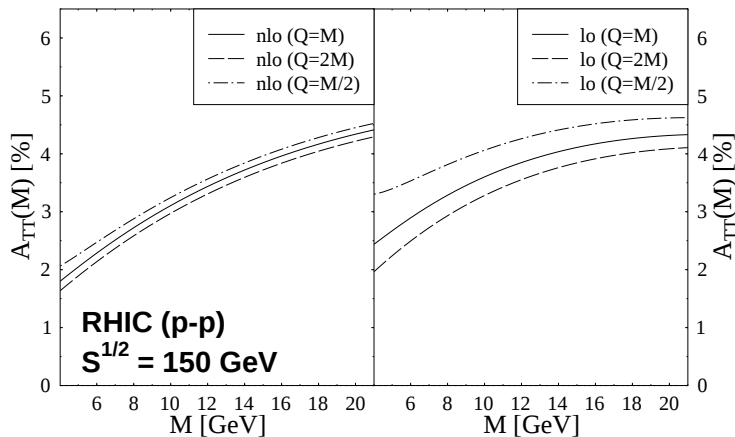


Fig. 4. Scale dependence of the LO and NLO asymmetries at $\sqrt{S} = 150$ GeV. The renormalization and factorization scales were chosen to be $Q_R = Q_F = M/2, M, 2M$. The solid line is as in Fig. 2.

Finally, we also find a significantly reduced scale dependence of the results when going from LO to NLO. This is shown in Fig. 4 for the case $\sqrt{S} = 150$ GeV. We plot here the maximal asymmetry in LO and NLO, varying the renormalization and factorization scales in the range $M/2 \leq Q_F = Q_R \leq 2M$. One can see that already the LO asymmetry is fairly stable with respect to scale changes, which is in accordance with the finding of generally moderate NLO corrections. The NLO asymmetry even shows a significant improvement, so that A_{TT} becomes largely insensitive to the choice of scale.

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