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SPIN DEPENDENT STRUCTURE FUNCTION g_1 AT LOW x*

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Theoretical expectations concerning small x behaviour of the spin dependent structure function g_1 are summarised. This includes discussion of the Regge pole model predictions and of the double $\ln^2(1/x)$ effects implied by perturbative QCD. The quantitative implementation of the latter is described within the unified scheme incorporating both Altarelli–Parisi evolution and the double $\ln^2(1/x)$ resummation. The double $\ln^2(1/x)$ effects are found to be important in the region of x which can possibly be probed at HERA. Predictions for the polarized gluon distribution $\Delta G(x, Q^2)$ at low x are also given.

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Understanding of the small x behaviour of the spin dependent structure functions of the nucleon, where x is the Bjorken parameter is interesting both theoretically and phenomenologically. Present experimental measurements do not cover the low x values and so the knowledge of reliable extrapolation of the structure functions into this region is important for estimate of integrals which appear in the Bjorken and Ellis–Jaffe sum rules [1]. Theoretical description of the structure function $g_1^p(x, Q^2)$ at low x is also extremely relevant for the possible polarised HERA measurements [2]. The purpose of this talk is to summarize the theoretical QCD expectations concerning the small x behaviour of the spin dependent structure function g_1 of the nucleon. After brief reminder of the Regge pole expectations we shall discuss the effects of the double $\ln^2(1/x)$ resummation and its phenomenological implementation. Besides the structure function g_1 we shall also discuss the spin dependent gluon distribution ΔG.

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The small x behaviour of spin dependent structure functions for fixed Q^2 reflects the high energy behaviour of the virtual Compton scattering (spin dependent) total cross-section with increasing total CM energy squared W^2 since $W^2 = Q^2(1/x - 1)$. This is, by definition, the Regge limit and so the Regge pole exchange picture [3] is therefore quite appropriate for the theoretical description of this behaviour. Here as usual $Q^2 = -q^2$, where q is the four momentum transfer between the scattered leptons. The relevant Reggeons which describe the small x behaviour of the spin dependent structure functions are those which correspond to the axial vector mesons [4, 5].

The Regge pole model gives the following small x behaviour of the structure functions $g_1^i(x,Q^2)$, where $g_1^i(x,Q^2), i = s, ns$ denote either singlet ($g_1^{\mathrm{s}}(x,Q^2) = g_1^p(x,Q^2) + g_1^n(x,Q^2)$) or non-singlet ($g_1^{\mathrm{ns}}(x,Q^2) = g_1^p(x,Q^2) - g_1^n(x,Q^2)$) combination of structure functions:

$$g_1^i(x,Q^2) = \gamma_i(Q^2) x^{-\alpha_i(0)}\,, \tag{1}$$

where $g_1^i(x,Q^2)$ denote either singlet ($g_1^{\mathrm{s}}(x,Q^2) = g_1^p(x,Q^2) + g_1^n(x,Q^2)$) or non-singlet ($g_1^{\mathrm{ns}}(x,Q^2) = g_1^p(x,Q^2) - g_1^n(x,Q^2)$) combination of structure functions.

In Eq. (1) $\alpha_{\mathrm{s,ns}}(0)$ denote the intercept of the Regge pole trajectory corresponding to the axial vector mesons with $I = 0$ or $I = 1$, respectively. It is expected that $\alpha_{\mathrm{s,ns}}(0) \leq 0$ and that $\alpha_{\mathrm{s}}(0) \approx \alpha_{\mathrm{ns}}(0)$ *i.e.* the singlet spin dependent structure function is expected to have similar low x behaviour as the non-singlet one.

It may be instructive to confront this behaviour with the Regge pole expectations for the spin independent structure function $F_1(x,Q^2)$:

$$F_1^i(x,Q^2) = \beta_i(Q^2) x^{-\alpha_i(0)}. \tag{2}$$

The singlet part $F_1^{\mathrm{S}} = F_1^p + F_1^n$ of the structure function F_1 is controlled at small x by pomeron exchange, while the non-singlet part $F_1^{\mathrm{NS}} = F_1^p - F_1^n$ by A_2 reggeon. The pomeron intercept is significantly different from that of the A_2 Reggeon *i.e.* $\alpha_P(0) = 1 + \varepsilon$ with $\varepsilon \approx 0.08$ while $\alpha_{A_2}(0) \approx 0.5$ as determined from the fits to the total hadronic and photoproduction data [6]. This implies that in the spin independent case the singlet part $F_1^{\mathrm{S}}(x,Q^2)$ of the structure function $F_1(x,Q^2)$ dominates at low x over the non-singlet component.

Several of the Regge pole model expectations for both spin dependent and spin independent structure functions are modified by perturbative QCD effects. In particular as far as the spin dependent structure functions are concernced the Regge behaviour (1) becomes unstable against the QCD evolution which generates more singular behaviour than that given by Eq. (1)

for $\alpha_i(0) \le 0$. In the LO approximation one gets:

$$g_1^{\rm NS}(x,Q^2) \sim \exp\left[2\sqrt{\Delta P_{qq}(0)\xi(Q^2)\ln(1/x)}\right] ,$$

$$g_1^{\rm S}(x,Q^2) \sim \exp\left[2\sqrt{\gamma^+\xi(Q^2)\ln(1/x)}\right] , \tag{3}$$

where

$$\xi(Q^2) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2}\frac{\alpha_{\rm s}(q^2)}{2\pi} \tag{4}$$

and

$$\gamma^+ = \frac{\Delta P_{qq}(0) + \Delta P_{gg}(0) + \sqrt{(\Delta P_{qq}(0) - \Delta P_{gg}(0))^2 + 4\Delta P_{qg}(0)\Delta P_{gq}(0)}}{2} \tag{5}$$

with $\Delta P_{ij}(0) = \Delta P_{ij}(z=0)$ where $\Delta P_{ij}(z)$ denote the LO splitting functions describing evolution of spin dependent parton densities. To be precise we have:

$$\begin{aligned} \Delta P_{qq}(0) &= \frac{4}{3}\,, \\ \Delta P_{qg}(0) &= -N_F\,, \\ \Delta P_{gq}(0) &= \frac{8}{3}\,, \\ \Delta P_{gg}(0) &= 12. \end{aligned} \tag{6}$$

We recall for comparison the analogous small x behaviour of the structure function $F_1(x,Q^2)$ in the LO approximation :

$$F_1^{\rm S}(x,Q^2) \sim \frac{1}{x}\exp[2\sqrt{6\xi(Q^2)\ln(1/x)}]. \tag{7}$$

The Regge behaviour of the non-singlet structure function $F_1^{\rm NS}(x,Q^2)$ remains stable against the QCD evolution.

The LO (and NLO) QCD evolution which sums the leading (and next-to-leading) powers of $\ln Q^2/Q_0^2)$ is however incomplete at low x. In this region one should worry about another "large" logarithm which is $\ln(1/x)$ and resum its leading powers. In the spin independent case this is provided by the Balitzkij, Fadin, Kuraev, Lipatov (BFKL) equation [7] which gives in the leading $\ln(1/x)$ approximation the following small x behaviour of $F_1^{\rm S}(x,Q^2)$

$$F_1^{\rm S}(x,Q^2) \sim x^{-\lambda_{\rm BFKL}} , \tag{8}$$

where the intercept of the BFKL pomeron $\lambda_{\rm BFKL}$ is given in the leading order by the following formula:

$$\lambda_{\rm BFKL} = 1 + \frac{3\alpha_{\rm s}}{\pi} 4\ln(2)\,. \tag{9}$$

It has recently been pointed out that the spin dependent structure function g_1 at low x is dominated by the *double* logarithmic $\ln^2(1/x)$ contributions *i.e.* by those terms of the perturbative expansion which correspond to the powers of $\ln^2(1/x)$ at each order of the expansion [8, 9]. Those contributions go *beyond* the LO or NLO order QCD evolution of polarised parton densities [10] and in order to take them into account one has to include the resummed double $\ln^2(1/x)$ terms in the coefficient and splitting functions [11]. In this talk we will present discussion of the double $\ln^2(1/x)$ resuumation following alternative approach based on unitegrated distributions [12, 13].

The dominant contribution to the double $\ln^2(1/x)$ resummation comes from the ladder diagrams with quark and gluon exchanges along the ladder (*cf.* Fig. 1). In what follows we shall neglect for simplicity possible non-ladder bremsstrahlung terms which are relatively unimportant [8, 9].

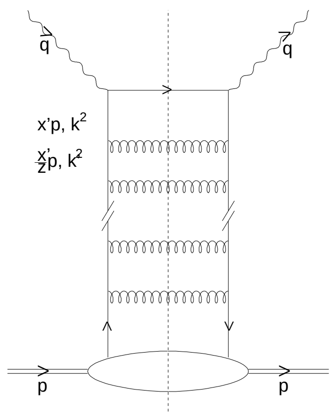

Fig. 1. An example of a ladder diagram generating double logarithmic terms in the spin structure function g_1.

It is convenient to introduce the unintegrated (spin dependent) parton distributions $f_i(x', k^2)$ $(i = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g)$, where k^2 is the transverse momentum squared of the parton i and x' the longitudinal momentum fraction of the parent nucleon carried by a parton. The conventional (integrated) distributions $\Delta p_i(x, Q^2)$ are related in the following way to the unintegrated distributions $f_i(x', k^2)$:

$$\Delta p_i(x, Q^2) = \Delta p_i^{(0)}(x) + \int\limits_{k_0^2}^{W^2} \frac{dk^2}{k^2} f_i\left(x' = x\left(1 + \frac{k^2}{Q^2}\right), k^2\right), \tag{10}$$

where $\Delta p_i^{(0)}(x)$ is the nonperturbative part of the of the distributions. The parameter k_0^2 is the infrared cut-off which will be set equal to 1 GeV^2. The origin of the nonperturbative part $\Delta p_i^{(0)}(x)$ can be viewed upon as originating from the non-perturbative region $k^2 < k_0^2$, *i.e.*

$$\Delta p_i^{(0)}(x) = \int\limits_0^{k_0^2} \frac{dk^2}{k^2} f_i(x, k^2)\,. \tag{11}$$

The spin dependent structure function $g_1^p(x, Q^2)$ of the proton is related in a standard way to the (integrated) parton distributions:

$$\begin{aligned} g_1^p(x, Q^2) \;=\; & \tfrac{1}{2}\Big[\tfrac{4}{9}(\Delta u_v(x, Q^2) + 2\Delta\bar{u}(x, Q^2)) \\ & + \tfrac{1}{9}(\Delta d_v(x, Q^2) + 2\Delta\bar{u}(x, Q^2) + 2\Delta\bar{s}(x, Q^2))\Big]\,, \end{aligned} \tag{12}$$

where $\Delta u_v(x, Q^2) = \Delta p_{u_v}(x, Q^2)$ *etc.* We assume $\Delta\bar{u} = \Delta\bar{d}$ and confine ourselves to the number of flavours N_F equal to three.

The valence quarks distributions and asymmetric part of the sea

$$f_{us}(x', k^2) = f_{\bar{u}}(x', k^2) - f_{\bar{s}}(x', k^2) \tag{13}$$

will correspond to ladder diagrams with quark exchange along the ladder. The singlet distributions

$$f_{\rm S}(x', k^2) = f_{u_v}(x', k^2) + f_{d_v}(x', k^2) + 4f_{\bar{u}}(x', k^2) + 2f_{\bar{s}}(x', k^2) \tag{14}$$

and the gluon distributions $f_g(x', k^2)$ will correspond to ladder diagrams with both quark (antiquark) and gluon exchanges along the ladder.

The sum of double logarithmic $\ln^2(1/x)$ terms corresponding to ladder diagrams is generated by the following integral equations (see Fig. 1):

$$f_k(x', k^2) = f_k^{(0)}(x', k^2) + \frac{\alpha_{\rm s}}{2\pi}\Delta P_{qq}(0)\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2} f_k\left(\frac{x'}{z}, k'^2\right) \tag{15}$$

$(k = u_v, d_v, us)$

$$
\begin{aligned}
&f_{\rm S}(x',k^2) = f_{\rm S}^{(0)}(x',k^2) \\
&+\frac{\alpha_{\rm s}}{2\pi}\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\left[\Delta P_{qq}(0)f_{\rm S}\left(\frac{x'}{z},k'^2\right)+\Delta P_{qg}(0)f_g\left(\frac{x'}{z},k'^2\right)\right], \\
&f_g(x',k^2) = f_g^{(0)}(x',k^2) \\
&+\frac{\alpha_{\rm s}}{2\pi}\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\left[\Delta P_{gq}(0)f_{\rm S}\left(\frac{x'}{z},k'^2\right)+\Delta P_{gg}(0)f_g\left(\frac{x'}{z},k'^2\right)\right] \qquad (16)
\end{aligned}
$$

with $\Delta P_{ij}(0) = \Delta P_{ij}(z=0)$ given by Eq. (6).

The variables $k^2(k'^2)$ denote the transverse momenta squared of the quarks (gluons) exchanged along the ladder, k_0^2 is the infrared cut-off and the inhomogeneous terms $f_i^{(0)}(x',k^2)$ will be specified later. The integration limit k^2/z follows from the requirement that the virtuality of the quarks (gluons) exchanged along the ladder is controlled by the tranverse momentum squared.

The origin of the double logarithmic $\ln^2(1/x)$ terms in $g_1(x,Q^2)$ can be traced to the fact that the conventional single logarithmic terms coming from the logarithmic integration over the longitudinal momentum fraction z are enhanced by the logarithmic integration over the transverse momentum up to the z dependent limit k^2/z in equations (15), (16) and up to the x dependent limit $W^2 = Q^2(1/x-1)$ in Eq. (10).

Equation (15) is similar to the corresponding equation in QED describing the double logarithmic resummation generated by ladder diagrams with fermion exchange [14]. The problem of double logarithmic asymptotics in QCD in the non-singlet channels was also discussed in Ref. [15].

Equations (15), (16) generate singular power behaviour of the spin dependent parton distributions and of the spin dependent structure functions g_1 at small x *i.e.*

$$
\begin{aligned}
g_1^{\rm NS}(x,Q^2) &\sim x^{-\lambda_{\rm NS}}, \\
g_1^{\rm S}(x,Q^2) &\sim x^{-\lambda_{\rm S}}, \\
\Delta G(x,Q^2) &\sim x^{-\lambda_{\rm S}}, \qquad (17)
\end{aligned}
$$

where $g_1^{\rm NS} = g_1^p - g_1^n$ and $g_1^{\rm S} = g_1^p + g_1^n$, respectively, and ΔG is the spin dependent gluon distribution. This behaviour reflects similar small x' behaviour of the unintegrated distributions. Exponents $\lambda_{\rm NS,S}$ are given by the following formulas:

$$\lambda_{\rm NS} = 2\sqrt{\left[\frac{\alpha_{\rm s}}{2\pi}\Delta P_{qq}(0)\right]}\,,$$
$$\lambda_{\rm S} = 2\sqrt{\left[\frac{\alpha_{\rm s}}{2\pi}\gamma^{+}\right]}\,, \tag{18}$$

where γ^+ is given by Eq. (5). The power-like behaviour (17) with the exponents $\lambda_{\rm NS,S}$ given by Eq. (18) remains the leading small x behaviour of the structure functions provided that their non-perturbative parts are less singular. This takes place if the latter are assumed to have the Regge pole like behaviour with the corresponding intercept(s) being near 0.

In order to understand origin of the power-like behaviour (17) it is useful to go to the moment space and inspect the singularities of the moment functions $\bar{f}_i(\omega,k^2)$ and $\Delta\bar{p}_i(\omega,Q^2)$

$$\begin{aligned}\bar{f}_i(\omega,k^2) &= \int\limits_0^1 dx' x'^{\omega-1} f_i(x',k^2)\,,\\ \Delta\bar{p}_i(\omega,Q^2) &= \int\limits_0^1 dx x^{\omega-1}\Delta p_i(x,Q^2)\end{aligned} \tag{19}$$

in the ω plane. It follows from Eq. (10) that the moment functions $\Delta\bar{p}_i(\omega,Q^2)$ are related in the following way to $\bar{f}_i(\omega,k^2)$:

$$\Delta\bar{p}_i(\omega,Q^2) = \Delta\bar{p}_i^{(0)}(\omega) + \int\limits_{k_0^2}^{\infty}\frac{dk^2}{k^2}\left(1+\frac{k^2}{Q^2}\right)^{-\omega}\bar{f}_i(\omega,k^2)\,, \tag{20}$$

where $\Delta\bar{p}_i^{(0)}(\omega)$ denote the moment functions of the nonperturbative distributions $\Delta p_i^{(0)}(x)$. Let us first consider the case $i=u_v,d_v,us$. Equation (15) implies the following equation for the moment functions $\bar{f}_i(\omega,k^2)$

$$\bar{f}_i(\omega,k^2) = \bar{f}_i^{(0)}(\omega,k^2) + \frac{\bar{\alpha}_{\rm s}}{\omega}\left[\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\bar{f}_i(\omega,k'^2) + \int\limits_{k^2}^{\infty}\frac{dk'^2}{k'^2}\left(\frac{k^2}{k'^2}\right)^{\omega}\bar{f}_i(\omega,k'^2)\right]. \tag{21}$$

In this equation $\bar{f}_i^{(0)}(\omega,k^2)$ denote the moment functions of $f_i^{(0)}(x',k^2)$ and $\bar{\alpha}_{\rm s}$ is defined by :

$$\bar{\alpha}_{\rm s} = \Delta P_{qq}(0)\frac{\alpha_{\rm s}}{2\pi}\,. \tag{22}$$

Equation (21) follows from (15) after taking into account the following relation:

$$\int_0^1 \frac{dz}{z} z^\omega \Theta\left(\frac{k^2}{k'^2} - z\right) = \frac{1}{\omega}\left[\Theta(k^2 - k'^2) + \left(\frac{k^2}{k'^2}\right)^\omega \Theta(k'^2 - k^2)\right]. \qquad (23)$$

For fixed coupling $\bar{\alpha}_s$ equation (21) can be solved analytically. Assuming for simplicity that the inhomogeneous term is independent of k^2 (*i.e.* that $\bar{f}_i^{(0)}(\omega, k^2) = C_i(\omega)$) we get the following solution of Eq. (21):

$$\bar{f}_i(\omega, k^2) = R_i(\bar{\alpha}_s, \omega)\left(\frac{k^2}{k_0^2}\right)^{\tilde{\gamma}_{NS}(\alpha_s,\omega)}, \qquad (24)$$

where

$$\tilde{\gamma}_{NS}(\alpha_s, \omega) = \frac{\omega - \sqrt{\omega^2 - 4\bar{\alpha}_s}}{2}, \qquad (25)$$

and

$$R_i(\bar{\alpha}_s, \omega) = C_i(\omega)\frac{\tilde{\gamma}_{NS}(\alpha_s, \omega)\omega}{\bar{\alpha}_s}. \qquad (26)$$

Equation (26) defines the non-singlet anomalous dimension in which the double logarithmic $\ln^2(1/x)$ terms *i.e.* the powers of $\frac{\alpha_s}{\omega^2}$ have been resummed to all orders. It can be seen from (26) that this anomalous dimension has a (square root) branch point singularity at $\omega = \lambda_{NS}$

$$\lambda_{NS} = 2\sqrt{\bar{\alpha}_s}. \qquad (27)$$

This singularity will of course be also present in the moment functions $\bar{f}_i(\omega, k^2)$ and $\Delta\bar{p}_i(\omega, Q^2)$. It generates singular power-like behaviour of the non-singlet structure function $g_1^{NS}(x, Q^2)$ (*cf.* Eq. (17)). For $C_i(\omega) = \frac{\bar{\alpha}_s}{\omega}\Delta\bar{p}_i^{(0)}(\omega)$ the moment functions $\Delta\bar{p}_i(\omega, Q^2)$ can be shown to have a familiar RG form

$$\Delta\bar{p}_i(\omega, Q^2) = \bar{R}_i(\omega, \alpha_s)\left(\frac{Q^2}{k_0^2}\right)^{\tilde{\gamma}_{NS}(\alpha_s,\omega)} + O\left(\frac{k_0^2}{Q^2}\right), \qquad (28)$$

where

$$\bar{R}_i(\omega, \alpha_s) = \Delta\bar{p}_i^{(0)}(\omega)\frac{\Gamma(\tilde{\gamma}_{NS}(\alpha_s, \omega) + 1))\Gamma(\omega - \tilde{\gamma}_{NS}(\alpha_s, \omega))}{\Gamma(\omega)}. \qquad (29)$$

It may be seen from Eq. (29) that the singularity at $\omega = \lambda_{NS}$ which is present in the anomalous dimension $\tilde{\gamma}_{NS}(\alpha_s, \omega)$ does also appear in the functions $\bar{R}_i$. It is therefore present in the moment functions $\Delta\bar{p}_i(\omega, Q^2)$ for arbitrary values of the scale Q^2 including $Q^2 = k_0^2$. The situation in this case is similar to that in the solution of the BFKL Eq. (16).

The formula for the exponent λ_S which controls the small x behaviour of $g_1^{\mathrm{S}}(x,Q^2)$ and of $\Delta G(x,Q^2)$ can be obtained by inspecting the singularities in the ω plane of the moment functions $\bar{f}_S(\omega,k^2)$ and $\bar{f}_g(\omega,k^2)$ which are generated by the corresponding equations for those functions. They can be obtained from Eqs. (16) and have the following form:

$$\begin{aligned}
\bar{f}_{\mathrm{S}}(\omega,k^2) &= \bar{f}_{\mathrm{S}}^{(0)}(\omega,k^2) + \frac{\alpha_{\mathrm{s}}}{2\pi\omega}\int_{k_0^2}^{\infty}\frac{dk'^2}{k'^2}\left[\Theta(k^2-k'^2)+\Theta(k'^2-k^2)\left(\frac{k^2}{k'^2}\right)^{\omega}\right] \\
&\quad \times\left[\Delta P_{qq}(0)\bar{f}_{\mathrm{S}}(\omega,k'^2)+\Delta P_{qg}(0)\bar{f}_g(\omega,k'^2)\right] , \\
\bar{f}_g(\omega,k^2) &= \bar{f}_g^{(0)}\omega,k^2) + \frac{\alpha_{\mathrm{s}}}{2\pi\omega}\int_{k_0^2}^{\infty}\frac{dk'^2}{k'^2}\left[\Theta(k^2-k'^2)+\Theta(k'^2-k^2)\left(\frac{k^2}{k'^2}\right)^{\omega}\right] \\
&\quad \times\left[\Delta P_{gq}(0)\bar{f}_{\mathrm{S}}(\omega,k'^2)+\Delta P_{gg}(0)\bar{f}_g(\omega,k'^2)\right] .
\end{aligned} \tag{30}$$

For k^2 independent inhomogeneous terms ($\bar{f}_{S,g}^{(0)}(\omega,k^2) = C_{S,g}(\omega)$) these equations have the following solution:

$$\bar{f}_{S,g}(\omega,k^2) = R_{S,g}^{+}(\omega,\alpha_{\mathrm{s}})\left(\frac{k^2}{k_0^2}\right)^{\tilde{\gamma}^{+}(\omega,\alpha_{\mathrm{s}})} + R_{S,g}^{-}(\omega,\alpha_{\mathrm{s}})\left(\frac{k^2}{k_0^2}\right)^{\tilde{\gamma}^{-}(\omega,\alpha_{\mathrm{s}})} . \tag{31}$$

The (singlet) anomalous dimensions $\tilde{\gamma}^{\pm}(\omega,\alpha_{\mathrm{s}})$ are given by the following equations:

$$\tilde{\gamma}^{\pm}(\omega,\alpha_{\mathrm{s}}) = \frac{\omega-\sqrt{\omega^2-4\frac{\alpha_{\mathrm{s}}}{2\pi}\gamma^{\pm}}}{2} , \tag{32}$$

where γ^{+} is defined by equation (5) while γ^{-} is:

$$\gamma^{-} = \frac{\Delta P_{qq}(0)+\Delta P_{gg}(0)-\sqrt{(\Delta P_{qq}(0)-\Delta P_{gg}(0))^2+4\Delta P_{qg}(0)\Delta P_{gq}(0)}}{2} . \tag{33}$$

The functions $R_{\mathrm{S},g}^{\pm}(\omega,\alpha_{\mathrm{s}})$ can be expressed in terms of $C_{\mathrm{S},g}(\omega)$ and the anomalous dimensions $\tilde{\gamma}^{\pm}(\omega,\alpha_{\mathrm{s}})$. The moment functions $\Delta\bar{p}_{\mathrm{S},g}(\omega,Q^2)$ can be shown to have the RG form:

$$\Delta\bar{p}_{\mathrm{S},g}(\omega,Q^2) = \bar{R}_{\mathrm{S},g}^{+}(\omega,\alpha_{\mathrm{s}})\left(\frac{Q^2}{k_0^2}\right)^{\tilde{\gamma}^{+}(\omega,\alpha_{\mathrm{s}})} + \bar{R}_{\mathrm{S},g}^{-}(\omega,\alpha_{\mathrm{s}})\left(\frac{Q^2}{k_0^2}\right)^{\tilde{\gamma}^{-}(\omega,\alpha_{\mathrm{s}})} + O\left(\frac{k_0^2}{Q^2}\right). \tag{34}$$

It may be seen from Eq. (32) that the anomalous dimensions $\bar{\gamma}^{\pm}(\omega,\alpha_{\rm s})$ have (branch point) singularities at $\bar{\omega}^{\pm}$

$$\bar{\omega}^{\pm}=2\sqrt{\frac{\alpha_{\rm s}}{2\pi}\gamma^{\pm}} \tag{35}$$

with $\bar{\omega}^{+}>\bar{\omega}^{-}$ *i.e.* $\bar{\omega}^{+}=\lambda_{\rm S}$ (*cf.* Eqs. (17) and (18)) is the leading singularity. We also have $\lambda_{\rm S}>>\lambda_{\rm NS}$ since $\gamma^{+}>>\Delta P_{qq}(0)$ ($\gamma^{+}\approx 1.18$ and $\Delta P_{qq}(0)=4/3$). This means that singlet distributions and singlet structure functions dominate over the non-singlet ones at low x. Since this singularity is also present in the functions $\bar{R}^{\pm}_{{\rm S},g}(\omega,\alpha_{\rm s})$ it appears in $\Delta\bar{p}_{{\rm S},g}(\omega,Q^2)$ for arbitrary value of the scale Q^2. It should be noticed that the effect of the double $\ln^2(1/x)$ resummation can be quite strong and, in particular the exponent λ_S can easily become of the order of unity. This will be seen more explicitely in the quantitative implementation of the double $\ln^2(1/x)$ contributions which we are going to discuss below. In the exact leading double $\ln^2(1/x)$ approximation the anomalous dimensions $\tilde{\gamma}^{\rm NS}$ and $\tilde{\gamma}^{\pm}$ and the exponents $\lambda_{\rm NS}$ and $\lambda_{\rm S}$ acquire additional contributions due to bremsstrahlung diagrams. Their effect on $\lambda_{\rm NS,S}$ was estimated in Refs. [8, 9] where it was found that the bremmstrahlung terms can enhance $\lambda_{\rm NS}$ by about 4% and reduce $\lambda_{\rm S}$ by about 10%.

In order to make the quantitative predictions one has to constrain the structure functions by the existing data at large and moderately small values of x. For such values of x however the equations (15) and (16) are inaccurate. In this region one should use the conventional Altarelli–Parisi equations with complete splitting functions $\Delta P_{ij}(z)$ and not restrict oneself to the effect generated only by their $z\to 0$ part. Following Refs. [12, 13] we do therefore extend equations (15), (16) and add to their right hand side the contributions coming from the remaining parts of the splitting functions $\Delta P_{ij}(z)$. We also allow the coupling $\alpha_{\rm s}$ to run setting k^2 as the relevant scale. In this way we obtain unified system of equations which contain both the complete LO Altarelli–Parisi evolution and the double logarithmic $\ln^2(1/x)$ effects at low x. The corresponding system of equations reads:

$$\begin{aligned} f_k(x',k^2) &= f_k^{(0)}(x',k^2)+\frac{\alpha_{\rm s}(k^2)}{2\pi}\frac{4}{3}\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}f_k\left(\frac{x'}{z},k'^2\right) \\ &+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\frac{4}{3}\int\limits_{x'}^{1}\frac{dz}{z}\frac{(z+z^2)f_k(\frac{x'}{z},k'^2)-2zf_k(x',k'^2)}{1-z} \end{aligned}$$

$$+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[2+\frac{8}{3}\ln(1-x')\right]f_k(x',k'^2) \tag{36}$$

$(k=u_v,d_v,us)$,

$$f_{\rm S}(x',k^2)=f_{\rm S}^{(0)}(x',k^2)+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\frac{4}{3}f_{\rm S}\left(\frac{x'}{z},k'^2\right)$$

$$+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\frac{4}{3}\int\limits_{x'}^{1}\frac{dz}{z}\frac{(z+z^2)f_S(\frac{x'}{z},k'^2)-2zf_S(x',k'^2)}{1-z}$$

$$+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[2+\frac{8}{3}\ln(1-x')\right]f_{\rm S}(x',k'^2)$$

$$+\frac{\alpha_{\rm s}(k^2)}{2\pi}N_F\left[-\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}f_g\left(\frac{x'}{z},k'^2\right)+\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\int\limits_{x'}^{1}\frac{dz}{z}2zf_g\left(\frac{x'}{z},k'^2\right)\right],$$

$$f_g(x',k^2)=f_g^{(0)}(x',k^2)+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}\frac{8}{3}f_{\rm S}\left(\frac{x'}{z},k'^2\right)$$

$$+\frac{\alpha_{\rm s}(k^2)}{2\pi}\left[\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\int\limits_{x'}^{1}\frac{dz}{z}(-\frac{4}{3})zf_{\rm S}\left(\frac{x'}{z},k'^2\right)+12\int\limits_{x'}^{1}\frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2}f_g\left(\frac{x'}{z},k'^2\right)\right]$$

$$+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\int\limits_{x'}^{1}\frac{dz}{z}6z\left[\frac{f_g\left(\frac{x'}{z},k'^2\right)-f_g(x',k'^2)}{1-z}-2f_g\left(\frac{x'}{z},k'^2\right)\right]$$

$$+\frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[\frac{11}{2}-\frac{N_F}{3}+6\ln(1-x')\right]f_g(x',k'^2)\,. \tag{37}$$

The inhomogeneous terms $f_i^{(0)}(x',k^2)$ are expressed in terms of the input (integrated) parton distributions and are the same as in the case of the LO Altarelli–Parisi evolution [12]:

$$f_k^{(0)}(x',k^2) = \frac{\alpha_s(k^2)}{2\pi}\frac{4}{3}\int_{x'}^{1}\frac{dz}{z}\frac{(1+z^2)\Delta p_k^{(0)}(\frac{x'}{z}) - 2z\Delta p_k^{(0)}(x')}{1-z} + \frac{\alpha_s(k^2)}{2\pi}\left[2+\frac{8}{3}\ln(1-x')\right]\Delta p_k^{(0)}(x') \tag{38}$$

$(k = u_v, d_v, us)$,

$$f_S^{(0)}(x',k^2) = \frac{\alpha_s(k^2)}{2\pi}\frac{4}{3}\int_{x'}^{1}\frac{dz}{z}\frac{(1+z^2)\Delta p_S^{(0)}(\frac{x'}{z}) - 2z\Delta p_S^{(0)}(x')}{1-z}$$
$$+\frac{\alpha_s(k^2)}{2\pi}\left[(2+\frac{8}{3}\ln(1-x'))\Delta p_S^{(0)}(x') + N_F\int_{x'}^{1}\frac{dz}{z}(1-2z)\Delta p_g^{(0)}\left(\frac{x'}{z}\right)\right],$$
$$f_g^{(0)}(x',k^2) = \frac{\alpha_s(k^2)}{2\pi}$$
$$\times\left[\frac{4}{3}\int_{x'}^{1}\frac{dz}{z}(2-z)\Delta p_S^{(0)}(\frac{x'}{z}) + (\frac{11}{2}-\frac{N_F}{3}+6\ln(1-x'))\Delta p_g^{(0)}(x')\right]$$
$$+\frac{\alpha_s(k^2)}{2\pi}6\int_{x'}^{1}\frac{dz}{z}\left[\frac{\Delta p_g^{(0)}(\frac{x'}{z}) - z\Delta p_g^{(0)}(x')}{1-z} + (1-2z)\Delta p_g^{(0)}\left(\frac{x'}{z}\right)\right]. \tag{39}$$

Equations (36), (37) together with (38), (39) and (10) reduce to the LO Altarelli–Parisi evolution equations with the starting (integrated) distributions $\Delta p_i^0(x)$ after we set the upper integration limit over dk'^2 equal to k^2 in all terms in equations (36), (37) and if we set Q^2 in place of W^2 as the upper integration limit in the integral in Eq. (10).

Equations (36), (37) were solved in Refs. [12,13] assuming the following simple parametrisation of the input distributions:

$$\Delta p_i^{(0)}(x) = N_i(1-x)^{\eta_i} \tag{40}$$

with $\eta_{u_v} = \eta_{d_v} = 3$, $\eta_{\bar{u}} = \eta_{\bar{s}} = 7$ and $\eta_g = 5$. The normalisation constants N_i were determined by imposing the Bjorken sum-rule for $\Delta u_v^{(0)} - \Delta d_v^{(0)}$ and requiring that the first moments of all other distributions are the same as those determined from the recent QCD analysis [17]. All distributions $\Delta p_i^{(0)}(x)$ behave as x^0 in the limit $x \to 0$ that corresponds to the implicit assumption that the Regge poles which correspond to axial vector mesons, which should control the small x behaviour of g_1 [4,5] have their intercept equal to 0. It was checked that the parametrisation (40) combined with equations (10), (12), (36), (37) gives reasonable description of the recent SMC data on $g_1^{\rm NS}(x,Q^2)$ and on $g_1^p(x,Q^2)$ [18]. In Fig. 2 we show the nonsinglet part of $g_1(x,Q^2)$ for $Q^2 = 10{\rm GeV}^2$ in the small x region [12]. We show predictions based on equations (36), (10) and confront them with the expectations which follow from solving the LO Altarelli–Parisi

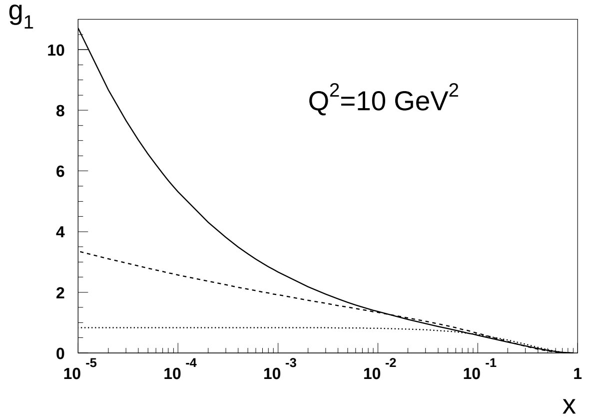

Fig. 2. Non-singlet part of the proton spin structure function $g_1(x,Q^2)$ as a function of x for Q^2 =10 GeV^2. Continuous line corresponds to the calculations which contain the leading $\ln^2(1/x)$ resummation, broken line is a leading order Altarelli–Parisi prediction, and a dotted one shows the non-perturbative part $g_1^{\rm NS(0)} = g_{\rm A}/6(1-x)^3$, where $g_{\rm A}$ denotes the axial vector coupling. The Figure is taken from Ref. [12].

evolution equations with the input distributions at $Q_0^2 = 1{\rm GeV}^2$ given by equation (40). We also show the nonperturbative part of the non-singlet distribution $g_1^{\rm NS(0)}(x) = g_{\rm A}/6(1-x)^3$, where $g_{\rm A}$ is the axial vector coupling. In Fig. 3 we show $g_1^p(x,Q^2)$ for $Q^2 = 10\,{\rm GeV}^2$, where we again confront predictions based on equations (36), (37), (10) with those based on the LO Altarelli–Parisi evolution equations. We also show in this Figure the "experimental" points which were obtained from the extrapolations based on the NLO QCD analysis together with estimated statistical errors of possible polarised HERA measurements [2]. We see that the structure function $g_1^p(x,Q^2)$ which contains effects of the double $\ln^2(1/x)$ resummation begins to differ from that calculated within the LO Altarelli–Parisi equations already for $x \sim 10^{-3}$. It is however comparable to the structure function obtained from the NLO analysis for $x > 10^{-4}$ which is indicated by the "experimental" points. This is presumably partially an artifact of the difference in the input distributions but it also reflects the fact that the NLO approx-

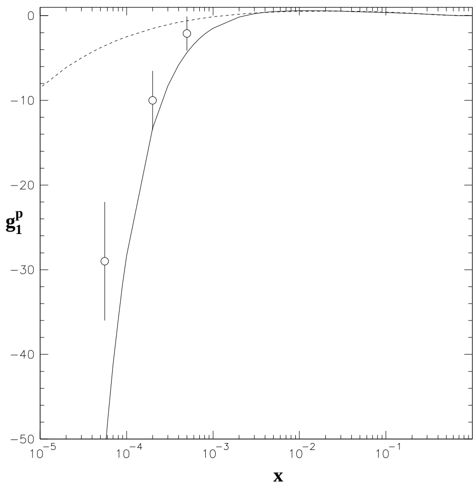

Fig. 3. The structure function $g_1^p(x,Q^2)$ for $Q^2 = 10\mathrm{GeV}^2$ plotted as the function of x. Solid line represents this structure function with the double $\ln^2(1/x)$ terms included and the dashed line corresponds to g_1^p obtained from the LO Altarelli-Parisi equations The "experimental" points are based on the NLO QCD predictions with the statistical errors expected at HERA [2]. The Figure is taken from Ref. [13].

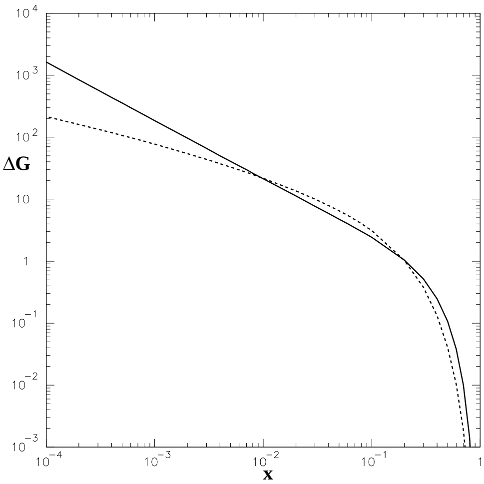

Fig. 4. The spin dependent gluon distribution $\Delta G(x,Q^2)$ for $Q^2 = 10\mathrm{GeV}^2$ plotted as the function of x. Solid line represents $\Delta G(x,Q^2)$ with the double $\ln^2(1/x)$ terms included and the dashed line corresponds to the $\Delta G(x,Q^2)$ obtained from the LO Altarelli–Parisi equations. The Figure is taken from Ref. [13].

imation contains the first two terms of the double $\ln^2(1/x)$ resummation in the corresponding splitting and coefficient functions. It can also be seen from Fig. 3 that the (complete) double $\ln^2(1/x)$ resummation generates the structure function which is significantly steeper than that obtained from the NLO QCD analysis and the difference between those two extrapolations becomes significant for $x < 10^{-4}$. In Fig. 4 we show the spin dependent gluon distribution which contains effects of the double $\ln^2(1/x)$ resummation and confront it with that which was obtained from the LO Altarelli–Parisi equations. It can be seen that the former exhibits characteristic $x^{-\lambda_S}$ behaviour with $\lambda_{\rm S} \sim 1$. Similar behaviour is also exhibited by the structure function $g_1^p(x,Q^2)$ itself.

To sum up we have presented theoretical expectations for the low x behaviour of the spin dependent structure function $g_1(x,Q^2)$ which follows from the resummation of the double $\ln^2(1/x)$ terms. We have also presented results of the analysis of the "unified" equations which contain the LO Altarelli–Parisi evolution and the double $\ln^2(1/x)$ effects at low x. As the first approximation we considered those double $\ln^2(1/x)$ effects which are generated by ladder diagrams. The double logarithmic effects were found to be very important and they should in principle be visible in possible HERA measurements (*cf.* Fig. 3).

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