COHERENCE EFFECTS IN DEUTERON SPIN STRUCTURE FUNCTIONS* **

J. Edelmann, G. Piller and W. Weise

Physik Department, Institut für Theoretische Physik
Technische Universität München
D-85747 Garching, Germany

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We investigate coherence effects in the deuteron spin structure functions g_1^d and b_1. In the kinematic domain of current fixed target experiments we observe that shadowing effects in g_1^d are approximately twice as large as for the unpolarized structure function F_2^d. Furthermore, we find that b_1 is large at $x < 0.1$ and receives dominant contributions from coherent double scattering.

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1. Introduction

In recent unpolarized lepton-nucleus scattering experiments at CERN (NMC) and FNAL (E665) [1] nuclear shadowing at small values of the Bjorken scaling variable $x < 0.1$ has been established as a leading twist effect. It is driven by the diffractive excitation of the (virtual) photon into hadronic states which interact coherently with several nucleons in the target nucleus.

Considering the growing interest in spin-dependent structure functions, a study of shadowing effects in polarized deep-inelastic scattering is needed. In particular, the extraction of the neutron spin structure function g_1^n from deuteron and ^{3}He data requires a detailed knowledge of nuclear effects. Furthermore, planned experimental investigations of the yet unmeasured deuteron structure function b_1 [2] call for an analysis of its small-x behavior which is driven by coherent double scattering contributions. A more complete and extended exposition of the here presented material can be found in [3].

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2. Helicity amplitudes

In the following we focus on the deuteron structure functions $F^d_{1,2}$, g^d_1 and b_1 at small values of the Bjorken scaling variable $x < 0.1$ (for references see [3–7]). Based on the optical theorem which connects forward virtual Compton scattering and deep-inelastic scattering, these structure functions can be expressed in terms of (virtual) photon-deuteron helicity amplitudes $\mathcal{A}^d_{+h}$, where "+" denotes the helicity of the transversely polarized photon and $h = 0, +, -$ labels the deuteron helicity [8]:

$$\begin{aligned} F^d_1 &\sim \tfrac{1}{3}\mathrm{Im}\left(\mathcal{A}^d_{+-} + \mathcal{A}^d_{++} + \mathcal{A}^d_{+0}\right), && (1)\\ g^d_1 &\sim \tfrac{1}{2}\mathrm{Im}\left(\mathcal{A}^d_{+-} - \mathcal{A}^d_{++}\right), && (2)\\ b_1 &\sim \tfrac{1}{2}\mathrm{Im}\left(2\mathcal{A}^d_{+0} - \mathcal{A}^d_{++} - \mathcal{A}^d_{+-}\right). && (3) \end{aligned}$$

The deuteron helicity amplitudes can be split into contributions from the incoherent scattering off either the proton or neutron, and a term which accounts for the coherent scattering from both nucleons:

$$\mathcal{A}^d_{+h} = \mathcal{A}^p_{+h} + \mathcal{A}^n_{+h} + \delta\mathcal{A}_{+h}. \tag{4}$$

Since nuclear effects from binding and Fermi-motion are relevant only at moderate and large $x \gtrsim 0.2$ (*e.g.* see references in [9]) we neglect them in the following. Then the single scattering amplitudes $\mathcal{A}^p_{+h}$ and $\mathcal{A}^n_{+h}$ are directly related to the free proton and neutron structure functions, respectively. Note that in this approximation single scattering yields no contribution to b_1. The double scattering amplitude $\delta\mathcal{A}_{+h}$ is responsible for shadowing corrections in $F^d_{1,2}$ and g^d_1 and dominates b_1 at small x. Treating the deuteron target as a non-relativistic bound state, described by the helicity dependent wave function $\psi_h(\boldsymbol{r})$ we obtain:

$$\begin{aligned} \delta\mathcal{A}_{+h} \;\sim\; & \sum_X \int d^3r \int \frac{d^3k}{(2\pi)^3}\, \psi^\dagger_h(\boldsymbol{r})\, \mathcal{T}(\gamma^* p \to Xp) \\ \times\; & \frac{\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}}{(q_0-k_0)^2 - \boldsymbol{k}^2_\perp - (q_z-k_z)^2 - M^2_X + i\epsilon}\, \mathcal{T}(Xn \to \gamma^* n)\, \psi_h(\boldsymbol{r}) \\ +\; & [p \leftrightarrow n], && (5) \end{aligned}$$

where $q^\mu = (q_0, \mathbf{0}_\perp, q_z)$ is the four-momentum of the virtual photon. The sum is taken over all diffractively excited hadronic intermediate states with momentum $q - k$ and invariant mass M_X. The amplitudes

$$\begin{aligned} \mathcal{T}(\gamma^* p \to Xp) &= P^p_\uparrow t^{\gamma^* p \to Xp}_{+\uparrow} + P^p_\downarrow t^{\gamma^* p \to Xp}_{+\downarrow}, && (6)\\ \mathcal{T}(Xn \to \gamma^* n) &= P^n_\uparrow t^{Xn \to \gamma^* n}_{+\uparrow} + P^n_\downarrow t^{Xn \to \gamma^* n}_{+\downarrow} && (7) \end{aligned}$$

stand for proper combinations of projection operators $P^{p(n)}_{\uparrow(\downarrow)}$ onto proton (neutron) states with helicity $+1/2$ $(-1/2)$, and $t^{\gamma^* p\to Xp}_{+\uparrow}$ *etc.*, the corresponding photon-nucleon helicity amplitudes for the diffractive production of the state X. For the following discussion we approximate the dependence of the diffractive production amplitudes on the momentum transfer $t = k^2 \approx -\boldsymbol{k}_\perp^2$ by:

$$\mathcal{T}^{NX}(k) \approx e^{-B\,\boldsymbol{k}_\perp^2/2}\,\mathcal{T}^{NX} \tag{8}$$

with the forward amplitude $\mathcal{T}^{NX} \equiv \mathcal{T}^{NX}(\boldsymbol{k} = 0)$. We investigate our results for deuteron spin structure functions in the kinematic range of fixed target experiments at CERN (NMC, COMPASS), FNAL (E665) and DESY (HERMES). Here in average moderate momentum transfers, $Q^2 \lesssim 3\,\mathrm{GeV}^2$, are accessible at $x < 0.1$. Various data on diffractive leptoproduction in this kinematic region [10] suggest an average slope $B \simeq (6\ldots 10)\,\mathrm{GeV}^{-2}$.

The double scattering amplitudes $\delta\mathcal{A}$ can now be expressed in terms of the integrated form factors:

$$\mathcal{F}_h(1/\lambda, B) = \int \frac{d^2 k_\perp}{(2\pi)^2}\, S_h(\boldsymbol{k}_\perp, 1/\lambda)\, \mathrm{e}^{-B\,\boldsymbol{k}_\perp^2} \tag{9}$$

with the helicity dependent deuteron form factor $S_h(\boldsymbol{k}) = \int d^3 r\, |\psi_h(\boldsymbol{r})|^2 \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ for each deuteron helicity $h = 0, +, -$. These form factors are functions of the inverse propagation length λ^{-1}; a hadronic fluctuation of mass M_X can contribute to coherent double scattering only if its propagation length $\lambda = 2q_0/(Q^2 + M_X^2)$ exceeds the deuteron size: $\lambda \gtrsim \langle r^2\rangle_d^{1/2} \approx 4\,\mathrm{fm}$.

3. Unpolarized structure function

Neglecting any spin- and isospin-dependence of the diffractive photon-nucleon amplitudes yields the standard result for shadowing in the unpolarized structure function $F_1^d = F_1^p + F_1^n + \delta F_1$ (*e.g.* see references in [9]):

$$\delta F_1(x, Q^2) = -\frac{Q^2}{x\pi\alpha}\int dM_X^2\, \left.\frac{d^2\sigma_T^{\gamma^* N}}{dM_X^2\, dt}\right|_{t=0} \mathcal{F}(1/\lambda, B), \tag{10}$$

where $\alpha = 1/137$. Here $\mathcal{F} = (\mathcal{F}_+ + \mathcal{F}_- + \mathcal{F}_0)/3$ is the helicity averaged integrated deuteron form factor, and $d^2\sigma^{\gamma^* N}/dM_X^2 dt$ is the unpolarized forward cross section for the diffractive production of hadronic states X from nucleons. Corrections to Eq. (10) are discussed in [3] (see also [4]).

4. Polarized structure function $\mathbf{g_1^d}$

In $g_1^d = (1 - \frac{3}{2}\omega_D)(g_1^p + g_1^n) + \delta g_1$ the single scattering contribution is modified by the D-state probability ω_D. Assuming isospin invariance of the unpolarized diffractive photon-nucleon amplitudes leads to:

$$\delta g_1(x,Q^2) \;=\; -\frac{Q^2}{4x\pi\alpha}\int dM_X^2 \left[\left.\frac{d^2\sigma_{+\downarrow}^{\gamma^* p}}{dM_X^2 dt}\right|_{t=0} - \left.\frac{d^2\sigma_{+\uparrow}^{\gamma^* p}}{dM_X^2 dt}\right|_{t=0} \right] \mathcal{F}_+(1/\lambda, B)$$
$$+[p \leftrightarrow n]. \tag{11}$$

In a laboratory frame description at $x < 0.1$, the virtual photon fluctuates into a hadronic state which then interacts with one or several nucleons inside the nuclear target. In the kinematic range of currently available experimental data on shadowing in unpolarized lepton scattering, it has turned out to be a good approximation to consider the interaction of only one effective hadronic state with invariant mass $M_X^2 \sim Q^2$ and a coherence length $\lambda \sim 1/2Mx$. This "one-state" approximation has been recently applied to shadowing in $g_1^{^3\mathrm{He}}$ [5]. For deuterium it yields:

$$\frac{\delta g_1}{g_1^N} = \mathcal{R}_{g_1}\frac{\delta F_2}{F_2^N}\,, \tag{12}$$

with $\mathcal{R}_{g_1}{=}2\mathcal{F}_+(2Mx,B)/\mathcal{F}(2Mx,B)$. At $x \lesssim 0.01$ we obtain for$B{=}7\mathrm{GeV}^{-2}$ and realistic deuteron wave functions [11, 12] $\mathcal{R}_{g_1} = 2.2$. In Fig. 1(b) we show the shadowing correction $\delta g_1/2g_1^N$ using recent data on $\delta F_2/2F_2^N$ from the E665 collaboration [1]. It should be noted that for decreasing values of x the experimental data for the shadowing ratio $\delta F_2/2F_2^N$ are taken at decreasing values of the average momentum transfer $\overline{Q^2}$. Therefore our result for δg_1 shown in Fig. 1(b) corresponds, strictly speaking, to the fixed target kinematics of E665 [1] which is not far from the kinematics of SMC [13].

In the kinematic domain of recent experiments one has $|g_1^N| = |g_1^n + g_1^p|/2 < 0.5$ [13]. Returning to Eq. (12) one then observes that shadowing amounts to less then 5% of the experimental error on g_1^n for the SMC analysis [1].

5. Polarized structure function $\mathbf{b_1}$

With the same approximations as used in Eq. (10) we obtain for b_1:

$$b_1 = \frac{Q^2}{\pi\alpha x}\int dM_X^2 \left.\frac{d^2\sigma^{\gamma_T^* N}}{dM_X^2 dt}\right|_{t=0} (\mathcal{F}_+(1/\lambda,B) - \mathcal{F}_0(1/\lambda,B)). \tag{13}$$

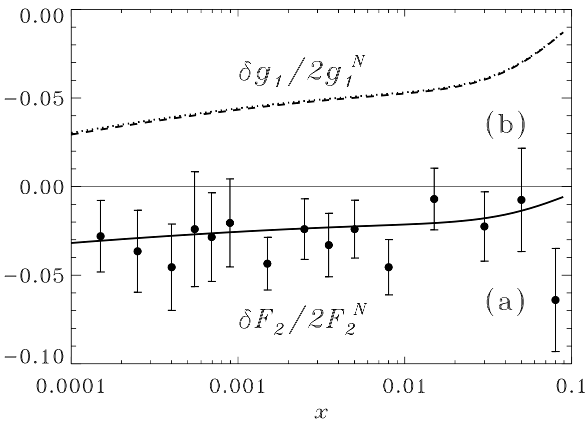

Fig. 1. (a) — shadowing correction $\delta F_2/2F_2^N$, data from E665 [1]. The full line represents a parametrization of the data used in (12) and (14). (b) — shadowing correction $\delta g_1/2g_1^N$ from (12). The dashed and dotted curves correspond to the Paris [11] and Bonn [12] potential respectively.

The shadowing correction δF_1 for the unpolarized structure function and the deuteron tensor structure function b_1 are directly related. At $x \ll 0.1$ the propagation lengths of diffractively excited hadrons which dominate double scattering exceed the deuteron size, $\lambda > \langle r^2 \rangle_d^{1/2} \approx 4\,\mathrm{fm}$. Here the deuteron form factors saturate, *i.e.* $\mathcal{F}_h(1/\lambda < 0.25\,\mathrm{fm}^{-1}, B) \approx \mathcal{F}_h(0, B)$. We then get:

$$b_1 = \mathcal{R}_{b_1}\,\delta F_1, \quad \text{with} \quad \mathcal{R}_{b_1} = -\frac{\mathcal{F}_+(0,B) - \mathcal{F}_0(0,B)}{\mathcal{F}(0,B)}\,. \tag{14}$$

Using $B = 7\,\mathrm{GeV}^{-2}$ we obtain from the Paris nucleon–nucleon potential [11] $\mathcal{R}_{b_1} = -0.33$, while the Bonn potential [12] gives $\mathcal{R}_{b_1} = -0.29$. A variation of the diffractive slope B by 30% leads to a change of the ratio $\mathcal{R}_{b_1}$ by maximally 20%. In Fig. (2) we present b_1 as obtained from Eq. (14), using the fit for $F_2^d/2F_2^N$ from Fig. (1), together with the empirical information on F_1^N [1].

For the ratio of structure functions b_1/F_1^d, which is given by an asymmetry of inclusive polarized deuteron cross sections, $\sigma_{+h}^{\gamma^* d} \sim \mathrm{Im}\,\mathcal{A}_{+h}^{\gamma^* d}$, we find:

$$\frac{b_1}{F_1^d} = -\frac{3}{2}\,\frac{\sigma_{++}^{\gamma^* d} + \sigma_{+-}^{\gamma^* d} - 2\sigma_{+0}^{\gamma^* d}}{\sigma_{++}^{\gamma^* d} + \sigma_{+-}^{\gamma^* d} + \sigma_{+0}^{\gamma^* d}} \approx \mathcal{R}_{b_1}\,\frac{\delta F_1}{2F_1^N} \approx 0.01, \tag{15}$$

i.e. b_1 amounts to around 1% of the unpolarized deuteron structure function F_1^d or, equivalently, to 2% of F_1^N. This result agrees with an early estimate in

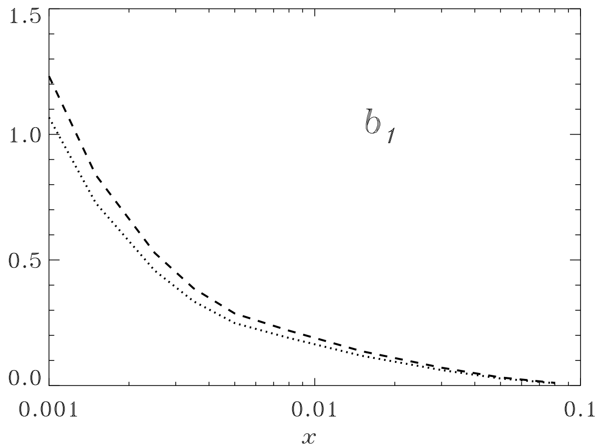

Fig. 2. Double scattering contribution to b_1 from (14). The dashed and dotted curves correspond to the Paris [11] and Bonn [12] potential respectively.

Ref. [7]. Note that the result shown here corresponds again to the kinematics of E665 [1]. It is therefore relevant with regard to possible future experiments at HERMES and eventual COMPASS.

We have studied nuclear effects in the polarized deuteron structure functions g_1^d and b_1 at small values of the Bjorken variable, $x < 0.1$, where the diffractive photo-excitation of hadronic states on a target nucleon and their subsequent interaction with the second nucleon becomes important. In the kinematic regime of current fixed target experiments, shadowing in g_1^d is found to be approximately twice as large as for the unpolarized structure function F_2^d. Nevertheless it plays a minor role for the extraction of the neutron structure function g_1^n. Furthermore we find dominant contributions to the deuteron structure function b_1 at $x < 0.1$ from coherent double scattering involving the deuteron D-state.

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