# SINGLE SPIN ASYMMETRIES IN SEMI-INCLUSIVE DEEP INELASTIC SCATTERING* 

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In this talk I want to illustrate the many possibilities for studying the structure of hadrons in hard scattering processes by giving a number of examples involving increasing complexity in the demands for particle polarization, particle identification or polarimetry. In particular the single spin asymmetries will be discussed. The measurements discussed in this talk are restricted to lepton-hadron scattering, but can be found in various other hard processes such as Drell-Yan scattering or $e^{+} e^{-}$-annihilation.

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## 1. Introduction

In inclusive deep inelastic lepton-hadron scattering (DIS) one is familiar with the factorization of the cross section, schematically

$$
\begin{equation*}
\sigma^{e H \rightarrow e X}=\sum_{q} f^{H \rightarrow q} \otimes \sigma^{e q \rightarrow e q} \tag{1}
\end{equation*}
$$

which can be justified via the operator product expansion. Restricting ourselves to quarks one finds local operators of the form $\bar{\psi} D \ldots D \psi$ to be important, which can be resummed into nonlocal operators $\bar{\psi}(0) \psi(x)$, in which the nonlocality is restricted along the lightcone. In the case of inclusive scattering transverse momenta are irrelevant. In semi-inclusive deep inelastic lepton-hadron scattering (SIDIS) the factorization

$$
\begin{equation*}
\sigma^{e H \rightarrow e h X}=\sum_{q} f^{H \rightarrow q} \otimes \sigma^{e q \rightarrow e q} \otimes D^{q \rightarrow h} \tag{2}
\end{equation*}
$$

[^0]

Fig. 1. The leading order diagrams for inclusive lepton-hadron scattering (left) and for semi-inclusive lepton-hadron scattering (right).
is much less well founded. There is no operator product expansion for the process, but one starts with a hard scattering approach. Furthermore, transverse momenta do matter in this process. The soft parts, the distribution function $f$ and the fragmentation function $D$ involve not only operators $\bar{\psi} D \ldots D \psi$, but also operators of the form $\partial(\bar{\psi} D \ldots D \psi)$, which implies, when organized into nonlocal operators $\bar{\psi}(0) \psi(x)$, that the transverse separation becomes important, although the separation remains lightlike. In the hard scattering approach the cross section for lepton-hadron scattering is for DIS in leading order given by the left diagram in figure 1 representing the squared amplitude ( + a similar antiquark contribution), while for SIDIS the cross section is given by the right diagram in Fig. 1 (again + similar antiquark distribution). It is these contributions that will be analyzed in a number of cases.

## 2. Quark distribution functions

The first soft part to consider is the one that defines the quark distribution functions. In a hard process such as leptoproduction one can introduce two lightlike vectors, $n_{+}$and $n_{-}$, satisfying $n_{+}^{2}=n_{-}^{2}=0$ and $n_{+} \cdot n_{-}=1$. The hadron momenta in the hard scattering process can be taken proportional to one of the lightlike vectors up to mass terms that are small compared to the hard scale in leptoproduction, the four momentum squared of the virtual photon, $q^{2}=-Q^{2}$. We will assume $P \propto Q n_{+}$and $P_{h} \propto Q n_{-}$. The lightlike vectors are used to define lightcone coordinates $a^{ \pm} \equiv a \cdot n_{\mp}$. The connection of hadron momenta with the momenta of quarks and gluons is made via a soft part in which all invariants, $p^{2} \sim P \cdot p \sim P^{2}=M^{2} \ll Q^{2}$. This implies that all - components of momenta in the soft distribution part are $\mathcal{O}(1 / Q)$. The momenta in the hard part have large + and large components.


Fig. 2. The soft parts describing the quark distribution (left) and quark fragmentation (right).

In DIS the only relevant component of the quark momentum in the soft distribution part (Fig. 2) is then the component $p^{+} \equiv x P^{+}$. Integrating over the other components of the quark momentum, the soft part is (in the lightcone gauge $A^{+}=0$ ) given by

$$
\begin{equation*}
\Phi_{i j}(x)=\left.\int \frac{d \xi^{-}}{2 \pi} e^{i p \cdot \xi}\langle P, S| \bar{\psi}_{j}(0) \psi_{i}(\xi)|P, S\rangle\right|_{\xi^{+}=\xi_{\mathrm{T}}=0} \tag{3}
\end{equation*}
$$

Using Lorentz invariance, hermiticity, parity ( P ) and time-reversal ( T ) one finds that in leading order in $1 / Q$ it can be expanded as

$$
\begin{equation*}
\Phi(x)=\frac{1}{2}\left\{f_{1}(x) \not h_{+}+\lambda g_{1}(x) \gamma_{5} h_{+}+h_{1}(x) \gamma_{5} \frac{\left[S_{\mathrm{T}}, n_{+}\right]}{2}\right\}+\mathcal{O}\left(\frac{M}{P^{+}}\right) \tag{4}
\end{equation*}
$$

where $\lambda=M S^{+} / P^{+}$is the (lightcone) helicity and $S_{\mathrm{T}}$ is the transverse spin vector of the (spin $1 / 2$ ) target (satisfying $\lambda^{2}+\boldsymbol{S}_{\mathrm{T}}^{2}=1$ ). The quantities $f_{1}, g_{1}$ and $h_{1}$ can be readily interpreted as densities (or differences thereof) for unpolarized quarks, longitudinally polarized or transversely polarized quarks in a polarized target. Using $\Phi$ to calculate the cross sections for lepton-hadron scattering one obtains the familiar cross sections,

$$
\begin{align*}
& \frac{d \sigma_{\mathrm{OO}}}{d x_{B} d y}=\frac{2 \pi \alpha^{2} s}{Q^{4}} x_{B}\left(1+(1-y)^{2}\right) \underbrace{\sum_{a, \bar{a}} e_{a}^{2} f_{1}^{a}\left(x_{B}\right)}_{2 F_{1}\left(x_{B}\right)=F_{2}\left(x_{B}\right) / x_{B}}  \tag{5}\\
& \frac{d \sigma_{\mathrm{LL}}}{d x_{B} d y}=\frac{2 \pi \alpha^{2} s}{Q^{4}} x_{B} y(2-y) \lambda_{e} \lambda \underbrace{\sum_{a, \bar{a}} e_{a}^{2} g_{1}^{a}\left(x_{B}\right)}_{2 g_{1}\left(x_{B}\right)} \tag{6}
\end{align*}
$$

where $x_{B}=Q^{2} / 2 P \cdot q$ and $y=P \cdot q / P \cdot k$ are the usual invariants. From the result one reads off, as indicated, the expressions for the structure functions $F_{1}, F_{2}$ in the case of unpolarized leptons scattering off an unpolarized target (OO). They are given by the quark distributions $f_{1}^{a}$ weighted with the quark
charges squared and summed over quarks and antiquarks. Similarly one reads of the result for $g_{1}$ in the case of longitudinally polarized leptons scattering off a longitudinally polarized spin $1 / 2$ target (LL).

## 3. Fragmentation functions

Next we turn to SIDIS. In the simplest case in which one detects one hadron belonging to the current jet and determines in essence only its longitudinal momentum, i.e. measures in addition to $x_{B}$ and $y$ the variable $z_{h}$ $=P \cdot P_{h} / P \cdot q$, one needs only to consider the dependence on the component $k^{-} \equiv P_{h}^{-} / z$ in the soft fragmentation part (Fig. 2). Integrating over the other components of $k$ the soft part is then (in lightcone gauge $A^{-}=0$ ) given by

$$
\begin{equation*}
\Delta(z)=\left.\sum_{X} z \int \frac{d \xi^{+}}{4 \pi} e^{i k \cdot \xi}\langle 0| \psi(\xi)\left|X ; P_{h}, S_{h}\right\rangle\left\langle X ; P_{h}, S_{h}\right| \bar{\psi}(0)|0\rangle\right|_{\xi^{-}=\xi_{\mathrm{T}}=0} \tag{7}
\end{equation*}
$$

The part relevant in $\Delta$ at leading order in $1 / Q$ is

$$
\begin{equation*}
\Delta(z)=z\left\{D_{1}(z) h_{-}+\lambda_{h} G_{1}(z) \gamma_{5} h_{-}+H_{1}(z) \gamma_{5} \frac{\left[S_{h \mathrm{~T}}, h_{-}\right]}{2}\right\}+\mathcal{O}\left(\frac{M_{h}}{P_{h}^{-}}\right) \tag{8}
\end{equation*}
$$

where $\lambda_{h}=M_{h} S_{h}^{-} / P_{h}^{-}$and $S_{h \mathrm{~T}}$ determine the polarization of the detected hadron. The fragmentation functions $D_{1}, G_{1}$ and $H_{1}$ can be directly interpreted as quark decay functions describing the decay for unpolarized, longitudinally polarized or transversely polarized quarks into a polarized hadron. Using $\Phi$ and $\Delta$ in Fig. 1 to calculate the cross section for SIDIS one obtains

$$
\begin{align*}
& \frac{d \sigma_{\mathrm{OO}}}{d x_{B} d y d z_{h}}=\frac{2 \pi \alpha^{2} s}{Q^{4}} x_{B}\left(1+(1-y)^{2}\right) \sum_{a, \bar{a}} e_{a}^{2} f_{1}^{a}\left(x_{B}\right) D_{1}^{a}\left(z_{h}\right),  \tag{9}\\
& \frac{d \sigma_{\mathrm{LL}}}{d x_{B} d y d z_{h}}=\frac{2 \pi \alpha^{2} s}{Q^{4}} x_{B} y(2-y) \lambda_{e} \lambda \sum_{a, \bar{a}} e_{a}^{2} g_{1}^{a}\left(x_{B}\right) D_{1}^{a}\left(z_{h}\right), \tag{10}
\end{align*}
$$

where the structure functions are given by products of distribution and fragmentation functions.

We note that the cross section for SIDIS in principle can depend in additional to the variables $x_{B}, y$ and $z_{h}$ on the transverse momentum of the produced hadron, denoted $P_{h \perp}$ in the frame where $P$ and $q$ do not have transverse components. Theoretically it is convenient to work with

$$
\begin{equation*}
q_{\mathrm{T}}^{\mu}=q^{\mu}+x_{B} P^{\mu}-\frac{P_{h}^{\mu}}{z_{h}}=-\frac{P_{h \perp}}{z_{h}} \equiv-Q_{\mathrm{T}} \hat{h}^{\mu}, \tag{11}
\end{equation*}
$$

which is the transverse component of $q$ in a frame in which $P$ and $P_{h}$ do not have transverse momenta. This vector is orthogonal to $n_{+}$and $n_{-}$. We will consider in the remainder cross sections dependending on $x_{B}, y$ and $z_{h}$, but obtained after weighting the full cross section with some function that may depend on azimuthal angles as defined in Fig. 3,

$$
\begin{equation*}
\langle W\rangle_{A B C}=\int d \phi^{\ell} d^{2} \boldsymbol{q}_{\mathrm{T}} W \frac{d \sigma_{A B C}^{[\vec{e} \vec{H} \rightarrow e \vec{h} X]}}{d x_{B} d y d z_{h} d \phi^{\ell} d^{2} \boldsymbol{q}_{\mathrm{T}}} \tag{12}
\end{equation*}
$$

where $W=W\left(Q_{\mathrm{T}}, \phi_{h}^{\ell}, \phi_{S}^{\ell}, \phi_{S_{h}}^{\ell}\right)$. In order to see in a glance which polarizations are involved, we have added the subscripts ABC for polarizations of lepton, target hadron and produced hadron, respectively. The cross section in Eq. (9) is then denoted $\langle 1\rangle_{\mathrm{OOO}}$, that in Eq. 10 as $\langle 1\rangle_{\text {LLO }}$.


Fig. 3. Kinematics for 1-particle inclusive leptoproduction. The lepton scattering plane is determined by lepton momenta and the hadron momentum $P$.

## 4. Polarimetry in SIDIS

As an example of a weighted cross section consider the process $\ell+H^{\dagger} \rightarrow$ $\ell+h^{\uparrow}+X\left(e . g . e p^{\uparrow} \rightarrow e \Lambda^{\uparrow} X\right)$ in which a spin $1 / 2$ target is transversely polarized and one looks for transversely polarized spin $1 / 2$ hadrons in the final state. The cross section using the expressions for $\Phi$ and $\Delta$ in the diagram in Fig. 1 gives

$$
\begin{align*}
& \left\langle\cos \left(\phi_{S}^{\ell}+\phi_{S_{h}}^{\ell}\right)\right\rangle_{\text {OTT }} \\
& \quad=\frac{2 \pi \alpha^{2} s}{Q^{4}}\left|\boldsymbol{S}_{\mathrm{T}}\right|\left|\boldsymbol{S}_{h \mathrm{~T}}\right|(1-y) \sum_{a, \bar{a}} e_{a}^{2} x_{B} h_{1}^{a}\left(x_{B}\right) H_{1}^{a}\left(z_{h}\right) . \tag{13}
\end{align*}
$$

This weighted cross section [5] is the nonvanishing transverse spin correlation between target hadron and produced hadron, probed via the scattering
off a transversely polarized quark, schematically target $^{\dagger} \Longrightarrow$ quark $^{\dagger} \Longrightarrow$ $h^{2} d r o{ }^{\uparrow}$. Both the (transversely polarized) quark distribution and the quark fragmentation function are chirally odd [6].

## 5. Transverse momentum dependent quark distributions

In the previous example, the transverse momentum of the outgoing hadron, did not play a role. If measured, it requires consideration of transverse momenta in the soft part. Instead of Eq. (3) one needs

$$
\begin{equation*}
\Phi\left(x, \boldsymbol{p}_{\mathrm{T}}\right)=\left.\int \frac{d \xi^{-} d^{2} \boldsymbol{\xi}_{\mathrm{T}}}{(2 \pi)^{3}} e^{i p \cdot \xi}\langle P, S| \bar{\psi}(0) \psi(\xi)|P, S\rangle\right|_{\xi^{+}=0}, \tag{14}
\end{equation*}
$$

for which the relevant part in leading order is given by [3, 7]

$$
\begin{align*}
\Phi\left(x, \boldsymbol{p}_{\mathrm{T}}\right)=\frac{1}{2}\{ & f_{1} h_{+}+g_{1 s} \gamma_{5} h_{+} \\
& \left.+h_{1 \mathrm{~T}} \gamma_{5} \frac{\left[\phi_{\mathrm{T}}, h_{+}\right]}{2}+h_{1 s}^{\perp} \gamma_{5} \frac{\left[p_{\mathrm{T}}, h_{+}\right]}{2 M}\right\} \tag{15}
\end{align*}
$$

with arguments $f_{1}=f_{1}\left(x, \boldsymbol{p}_{\mathrm{T}}^{2}\right)$ etc. The quantity $g_{1 s}$ (and similarly $\left.h_{1 s}^{\perp}\right)$ is shorthand for

$$
\begin{equation*}
g_{1 s}\left(x, \boldsymbol{p}_{\mathrm{T}}\right)=\lambda g_{1 \mathrm{~L}}\left(x, \boldsymbol{p}_{\mathrm{T}}^{2}\right)+\frac{\boldsymbol{p}_{\mathrm{T}} \cdot \boldsymbol{S}_{\mathrm{T}}}{M} g_{1 \mathrm{~T}}\left(x, \boldsymbol{p}_{\mathrm{T}}^{2}\right) . \tag{16}
\end{equation*}
$$

The two functions $g_{1 \mathrm{~L}}$ and $g_{1 \mathrm{~T}}$ are interpreted as the quark helicity distribution in a longitudinally and transversely polarized target, respectively. Integrating over $\boldsymbol{p}_{\mathrm{T}}$ only $g_{1}(x)=\int d^{2} p_{\mathrm{T}} g_{\mathrm{IL}}\left(x, \boldsymbol{p}_{\mathrm{T}}^{2}\right)$ survives. In the $p_{\mathrm{T}}-$ weighted result, $\Phi_{\partial}^{\alpha}=\int d^{2} p_{\mathrm{T}} p_{\mathrm{T}}^{\alpha} \Phi$ only the $\boldsymbol{p}_{\mathrm{T}}^{2}$-moment defined as $g_{1 \mathrm{~T}}^{(1)}(x)$ $=\int d^{2} p_{\mathrm{T}}\left(\boldsymbol{p}_{\mathrm{T}}^{2} / 2 M^{2}\right) g_{1 \mathrm{~T}}\left(x, \boldsymbol{p}_{\mathrm{T}}^{2}\right)$ survives. This function appears for instance in the following weighted cross section $[8,9]$ in $\vec{\ell}+H^{\uparrow} \rightarrow \ell+h+X$ (e.g. $\left.\vec{e} p^{\uparrow} \rightarrow e \pi^{+} X\right)$,

$$
\begin{align*}
& \left\langle\frac{Q_{\mathrm{T}}}{M} \cos \left(\phi_{h}^{\ell}-\phi_{S}^{\ell}\right)\right\rangle_{\mathrm{LTO}} \\
& \quad=\frac{4 \pi \alpha^{2} s}{Q^{4}} \lambda_{e}\left|\boldsymbol{S}_{\mathrm{T}}\right| y\left(1-\frac{y}{2}\right) \sum_{a, \bar{a}} e_{a}^{2} x_{B} g_{1 \mathrm{~T}}^{(1) a}\left(x_{B}\right) D_{1}^{a}\left(z_{h}\right) . \tag{17}
\end{align*}
$$

This weighted cross section correlates the transverse polarization of the target with the azimuthal distribution of unpolarized hadrons via the scattering of a longitudinally polarized quark, target $\uparrow \Longrightarrow$ quark $\rightarrow$ unpolarized hadron.

## 6. Extension to subleading order

When going to subleading order, i.e. $1 / Q$ contributions in the cross section, it is necessary to use for $\Phi$ the parametrization up to $1 / P^{+}$,

$$
\begin{align*}
\Phi(x)= & \frac{1}{2}\left\{f_{1}(x) n_{+}+\lambda g_{1}(x) \gamma_{5} h_{+}+h_{1}(x) \gamma_{5} \frac{\left[S_{\mathrm{T}}, h_{+}\right]}{2}\right\} \\
& +\frac{M}{2 P^{+}}\left\{e(x)+g_{\mathrm{T}}(x) \gamma_{5} S_{\mathrm{T}}+\lambda h_{\mathrm{L}}(x) \gamma_{5} \frac{\left[h_{+}, h_{-}\right]}{2}\right\} \\
& +\mathcal{O}\left(\frac{M^{2}}{\left(P^{+}\right)^{2}}\right) \tag{18}
\end{align*}
$$

The twist-three functions $e, g_{\mathrm{T}}$ and $h_{\mathrm{L}}$ do not have a simple partonic interpretation. From the Lorentz structure of $\Phi$ and the constraints imposed on it by Hermiticity, P and T one obtains (at tree-level) relations $[1,10]$ with the transverse momentum dependent functions discussed in the previous section,

$$
\begin{align*}
& \underbrace{g_{\mathrm{T}}-g_{1}}_{g_{2}}=\frac{d}{d x} g_{1 \mathrm{~T}}^{(1)},  \tag{19}\\
& \underbrace{h_{\mathrm{L}}-h_{1}}_{\frac{1}{2} h_{2}}=-\frac{d}{d x} h_{1 \mathrm{~L}}^{\perp(1)} . \tag{20}
\end{align*}
$$

The first relation has been used to get an estimate of $g_{1 \mathrm{~T}}$ from the $g_{2}$-data [11].


Fig. 4. Examples of diagrams for DIS (left) and for SIDIS (right) needed at subleading order.

At subleading order one needs to include soft parts containing gluon fields [6] as shown in Fig. 4. These, however, can be dealt with via the

QCD equations of motion and they do not introduce new functions. Their contribution is important to obtain a gauge invariant result. The most wellknown example is the structure function $g_{2}$ in DIS, or given as a weighted cross section,

$$
\begin{equation*}
\left\langle\cos \left(\phi_{S}^{\ell}\right)\right\rangle_{\mathrm{LT}}=-\frac{4 \pi \alpha^{2} s}{Q^{4}} \lambda_{e}\left|\boldsymbol{S}_{\mathrm{T}}\right| y \sqrt{1-y} \sum_{a, \bar{a}} e_{a}^{2} \frac{M x_{B}^{2}}{Q} g_{\mathrm{T}}^{a}\left(x_{B}\right) . \tag{21}
\end{equation*}
$$

## 7. T-odd fragmentation functions

In the soft part discussed for the quark fragmentation, no constraints arise from T invariance, because the states $\left|P_{h}, X\right\rangle$ are out-states, which change into in-states under T. This allows additional fragmentation functions in the case that transverse momentum is taken into account. Restricting ourselves to unpolarized hadrons, we can see that in the $\boldsymbol{p}_{\mathrm{T}}$-dependent distribution part only one function $f_{1}\left(x, \boldsymbol{p}_{\mathrm{T}}^{2}\right)$ remains. The relevant part in $\Delta$ in leading order for unpolarized final states contains two functions,

$$
\begin{equation*}
z \Delta\left(z, \boldsymbol{k}_{\mathrm{T}}\right)=D_{1} n_{-}+H_{1}^{\perp} \frac{i\left[k_{\mathrm{T}}, n_{-}\right]}{2 M_{h}}+\mathcal{O}\left(\frac{M_{h}}{P_{h}^{-}}\right) \tag{22}
\end{equation*}
$$

with arguments $D_{1}=D_{1}\left(z, z^{2} \boldsymbol{k}_{\mathrm{T}}^{2}\right)$ etc. Note that $\boldsymbol{k}_{\mathrm{T}}^{\prime}=-z \boldsymbol{k}_{\mathrm{T}}$ is the transverse momentum of the produced hadron with respect to the quark.

It turns out that the so-called T-odd functions (in this case $H_{1}^{\perp}$ ) lead to single spin asymmetries $[2,8,9,12]$. For example the above function appears in the production of unpolarized hadrons in leptoproduction in the case of unpolarized leptons and a (longitudinally or transversely) polarized target, e.g. ep ${ }^{\uparrow} \rightarrow e \pi^{+} X$,

$$
\begin{align*}
& \left\langle\frac{Q_{\mathrm{T}}}{M_{h}} \sin \left(\phi_{h}^{\ell}+\phi_{S}^{\ell}\right)\right\rangle_{\text {отО }} \\
& \quad=\frac{4 \pi \alpha^{2} s}{Q^{4}}\left|\boldsymbol{S}_{\mathrm{T}}\right|(1-y) \sum_{a, \bar{a}} e_{a}^{2} x_{B} h_{1}^{a}\left(x_{B}\right) H_{1}^{\perp(1) a}\left(z_{h}\right),  \tag{23}\\
& \left\langle\frac{Q_{\mathrm{T}}^{2}}{4 M M_{h}} \sin \left(2 \phi_{h}^{\ell}\right)\right\rangle_{\text {OLO }} \\
& \quad=-\frac{4 \pi \alpha^{2} s}{Q^{4}} \lambda(1-y) \sum_{a, \bar{a}} e_{a}^{2} x_{B} h_{\mathrm{LL}}^{\perp(1) a}\left(x_{B}\right) H_{1}^{\perp(1) a}\left(z_{h}\right) . \tag{24}
\end{align*}
$$

The interpretation is a correlation between the (transverse or longitudinal) polarization of the target and the azimuthal distribution of the produced unpolarized hadrons, probed via scattering off a transversely polarized quark,
target ${ }^{\uparrow / \rightarrow} \Longrightarrow$ quark $^{\uparrow} \stackrel{\text { T-odd }}{\Longrightarrow}$ unpolarized hadron. The same fragmentation function actually also appears in the scattering of a polarized lepton from an unpolarized target [14], but in that case appears in a subleading $\sin \phi_{h}^{\ell}$ asymmetry proportional to $e\left(x_{B}\right) H_{1}^{\perp(1)}\left(z_{h}\right)$.

## 8. T-odd distribution functions

For the distribution functions, it has been conjectured that T-odd quantities also might appear without violating time-reversal invariance [15-18]. This might be due to soft initial state interactions or, as suggested recently [18], be a consequence of chiral symmetry breaking. Within QCD a possible description of the effects may come from gluonic poles [4]. Here, let's simply assume the functions exist [2] in which case Eq. (15) is extended with

$$
\begin{equation*}
\Phi\left(x, \boldsymbol{p}_{\mathrm{T}}\right)=\ldots+\frac{1}{2}\left\{f_{1 \mathrm{~T}}^{\perp} \frac{\varepsilon_{\mu \nu \rho \sigma} \gamma^{\mu} n_{+}^{\nu} p_{\mathrm{T}}^{\rho} S_{\mathrm{T}}^{\sigma}}{M}+h_{1}^{\perp} \frac{i\left[p_{\mathrm{T}}, n_{+}\right]}{2 M}\right\} \tag{25}
\end{equation*}
$$

A single spin asymmetry in which the function $f_{1 \mathrm{~T}}^{\perp}$ appears is in the process $\ell+H^{\dagger} \rightarrow \ell+h+X\left(\right.$ e.g. $\left.e p^{\uparrow} \rightarrow e \pi^{+} X\right)$. One finds

$$
\begin{align*}
& \left\langle\frac{Q_{\mathrm{T}}}{M_{h}} \sin \left(\phi_{h}^{\ell}-\phi_{S}^{\ell}\right)\right\rangle_{\text {ото }} \\
& \quad=\frac{2 \pi \alpha^{2} s}{Q^{4}}\left|\boldsymbol{S}_{\mathrm{T}}\right|\left(1-y-\frac{1}{2} y^{2}\right) \sum_{a, \bar{a}} e_{a}^{2} x_{B} f_{1 \mathrm{~T}}^{\perp(1) a}\left(x_{B}\right) D_{1}^{a}\left(z_{h}\right) . \tag{26}
\end{align*}
$$

This asymmetry is interpreted as a correlation between the transverse polarization of the target and the azimuthal distribution of produced hadrons via scattering off an unpolarized quark, target $\stackrel{\text { T-odd }}{\Longrightarrow}$ unpolarized quark $\Longrightarrow$ unpolarized hadron. Note that this is not the only single spin asymmetry for the OTO case (see Eq. (23)).

Finally, we note the interesting possibility that a combination of T-odd distribution and fragmentation functions appears in unpolarized scattering. This is the case for the $\cos \left(2 \phi_{h}^{\ell}\right)$ asymmetry in leptoproduction,

$$
\begin{align*}
& \left\langle\frac{Q_{\mathrm{T}}^{2}}{4 M M_{h}} \cos \left(2 \phi_{h}^{\ell}\right)\right\rangle_{\text {ooo }} \\
& \quad=\frac{4 \pi \alpha^{2} s}{Q^{4}}(1-y) \sum_{a, \bar{a}} e_{a}^{2} x_{B} h_{1}^{\perp(1) a}\left(x_{B}\right) H_{1}^{\perp(1) a}\left(z_{h}\right), \tag{27}
\end{align*}
$$

interpreted as unpolarized target $\stackrel{\text { T-odd }}{\Longrightarrow}$ quark ${ }^{\uparrow} \xrightarrow{\text { T-odd }}$ unpolarized hadron. This is a leading asymmetry, which in the absence of T-odd distribution functions would start at order $1 / Q^{2}$ (twist 4).

## 9. Summary

In this talk many new possibilities have been outlined to probe the quark and gluon structure of hadrons. The emphasis was on the transverse momentum dependence in distribution and fragmentation functions that appear in semi-inclusive deep inelastic scattering at leading order. Some of these functions or to be more precise their $\boldsymbol{p}_{\mathrm{T}}^{2}$-moments (e.g. $g_{1 \mathrm{~T}}^{(1)}$ and $h_{1 \mathrm{~L}}^{\perp(1)}$ ) are at tree-level simply related to twist-three functions (such as $g_{\mathrm{T}}$ and $h_{\mathrm{L}}$ ). Finally, the systematic investigation of the soft parts formalizes many effects, such as the Collins effect or final state interactions in leptoproduction.

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