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A NLO QCD ANALYSIS OF THE POLARIZED STRUCTURE FUNCTION g_1*

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We present a Next-to-Leading (NLO) QCD analysis of the presently available world data on the spin structure function g_1, including the data on g_1^p taken by the SMC in 1996. The fit result is used to determine g_1^p at a constant Q^2 and such the first moment $\Gamma_1^p = \int g_1^p(x)dx$. Its value is found to depend on the approach used to describe the behaviour of $g_1^{\rm p}$ at low-x. Independent of the approach we find that the Ellis–Jaffe sum rule is violated while the validity of the Bjorken sum rule is confirmed.

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Polarized deep inelastic lepton-nucleon scattering is an important tool to study the internal spin structure of the nucleon. Measurements on proton, deuteron and neutron targets allow verification of the Bjorken sum rule [1], which is a relation between the first moments of the polarized structure functions for the proton g_1^p and the neutron g_1^n, and is a fundamental relation of QCD. All experiments up to now have confirmed the validity of this sum rule but at the same time agree on the fact that they observe a smaller than expected value of the individual first moments of $g_1^{p,d,n}$. This means the violation of the Ellis–Jaffe spin sum rule [2].

To test the sum rules experimentally, *i.e.* to determine the first moments $\Gamma_1(Q_0^2) = \int_0^1 g_1(x)dx$ it is necessary to evaluate the structure functions at a common $Q^2 = Q_0^2$ for different x-values. For the present experiments this poses a practical problem, because x and Q^2 of the data are strongly correlated. To transport the g_1 values to a common Q_0^2 an assumption on the Q^2 behaviour of g_1 has to be made. The traditional assumption was

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that the asymmetry $A_1^{p/n}(\sim g_1/F_1)$ is Q^2 independent. Although within the accuracy of the present data and the limited available Q^2 range this assumption can not be proven wrong, different Q^2 behaviours of g_1 and F_1 are expected from perturbative QCD. The accuracy of data collected by experiments at CERN and SLAC in the past few years has motivated and allowed perturbative QCD analyses of the nucleon spin-dependent structure function $g_1(x, Q^2)$ at next-to-leading-order (NLO) [3–8].

SMC has taken data on the polarized structure function g_1 from 1992 through 1996 on proton and deuteron targets [9, 10]. For an overview of the experiment see Ref. [11] in these proceedings. A detailed description of the experiment and the analysis can be found in [12]. A new systematic NLO QCD analysis where the final, more precise, data of SMC is used is in progress and will be published soon. That analysis will include several systematic studies, *e.g.* the dependence on the factorization scheme, different programs will be compared and the Bjorken sum rule will be tested in a fully consistent way using perturbative QCD.

The existing world data on g_1 consists mainly of two groups of data. Data taken in an electron beam at SLAC [13–16] and at DESY [17] and the data from the SMC at CERN taken in a muon beam of $E_\mu \approx 190$ GeV. The kinematic range covered by these groups of experiments is very different. Due to the high energy of the muon beam, the SMC data reaches to lower values of Bjorken-x: $0.003 < x, 0.7$ and higher values of Q^2, $\langle Q^2 \rangle = 10\,\mathrm{GeV}^2$, compared to the SLAC/DESY experiments ($\langle Q^2 \rangle \approx 5$ GeV$^2, x > 0.01$).

In our NLO-QCD analysis, described here, all available data with $Q^2 > 1$ GeV^2 on the proton [9, 13, 18, 19], on the deuteron [10, 14, 19], and on the neutron [15–17] is included. We use the program, developed by Ball, Forte and Ridolfi [3] and work in the Adler–Bardeen scheme. The analysis corresponds to the one in Ref. [9], where for the first time the latest SMC data, taken in 1996, on g_1^p was included. The combination of these data with the older SMC g_1^p data reduced the statistical error by a factor of ≈ 2.

g_1 is related to the polarized quark and gluon distributions by

$$\begin{aligned} g_1(x,t) \;=\; \tfrac{1}{2}\langle e^2\rangle \int\limits_x^1 \frac{\mathrm{d}y}{y} \Big[& C_{\mathrm{S}}^q(\tfrac{x}{y}, \alpha_s(t))\Delta\Sigma(y,t) \\ & + 2n_f C^g(\tfrac{x}{y}, \alpha_s(t))\Delta g(y,t) + C_{\mathrm{NS}}^q(\tfrac{x}{y}, \alpha_s(t))\Delta q_{\mathrm{NS}}(y,t)\Big], \quad (1) \end{aligned}$$

where $\langle e^2\rangle = n_f^{-1}\sum_{k=1}^{n_f} e_k^2$ is the average squared quark charge, $t = \ln(Q^2/\Lambda^2)$ where Λ is the QCD scale parameter, $\Delta\Sigma$ and Δq_{NS} are the singlet and non-singlet polarized quark distributions and $C_{\mathrm{S,NS}}^q(\alpha_s(Q^2))$ and $C^g(\alpha_s(Q^2))$ are the quark and gluon coefficient functions. The Q^2 dependence of the polarized quark and gluon distributions is given by the DGLAP equations [20].

The full set of coefficient functions [21] and splitting functions [22] has been computed up to next-to-leading order in α_s. To extract the polarized parton distributions from experimental data we parameterize the initial polarized parton distribution functions at a starting value $Q^2 = Q_i^2$ as

$$\Delta f(x, Q^2) = N\eta_f x^{\alpha_f}(1-x)^{\beta_f}(1+a_f x), \tag{2}$$

where N is fixed by the normalization $\eta_f = \int_0^1 \Delta f_j(x)dx$ and Δf denotes $\Delta\Sigma$, $\Delta q_{\rm NS}^{p/n}$, or Δg. With this normalization the parameters η_g, $\eta_{\rm NS}^{p/n}$ and $\eta_{\rm S}$ are the first moments of the gluon, the non-singlet quark and the singlet quark distributions at the starting scale, respectively.

The parameterizations have to be flexible enough to be able to describe the low-x as well as the high-x behaviour of the data with the minimal number of free parameters. We find that the parameter a_f is needed only for the parameterization of $\Delta\Sigma$, and β_g was fixed to 4 as suggested by QCD counting rules [23]. The normalizations $\eta_{\rm NS}^p$ and $\eta_{\rm NS}^n$ are constrained by relating the moments of $\Delta q_{\rm NS}^p$ and $\Delta q_{\rm NS}^n$ to the flavour–SU(3) coupling constants F and D. For these coupling constants we used $F+D = g_A/g_V = 1.2601 \pm 0.0025$ [24] and $F/D = 0.575 \pm 0.016$ [25]. This means that we assume the Bjorken sum rule to be valid, and is the reason we do not test this sum rule directly in this QCD analysis.

We evolve these initial parton distributions to the x and Q^2 of the data points using the DGLAP evolution equations [20], evaluate g_1 with Eq. (1) and fit the calculated $g_1^{\rm calc}(x,Q^2)$ to the measured $g_1^{\rm data}(x,Q^2)$ of the data sets mentioned above by minimizing the χ^2. Only statistical errors of the data were used in the fit. The starting scale was $Q_i^2 = 1$ GeV^2 and the value for the strong coupling constant $\alpha_s(M_Z^2) = 0.118 \pm 0.003$ [24].

The parton distributions resulting from the best fit are shown in Fig. 1 at the initial $Q_i^2 = 1$ GeV^2 and evolved to 3, 5, and 10 GeV^2.

The result for g_1^p, g_1^d and g_1^n is shown in Fig. 2. It can be seen that the data is well described by the NLO QCD fit. This is quantified by the obtained value for the χ^2/d.o.f $= 312/323$.

To determine the first moment of g_1 from the SMC data we obtain g_1 at a fixed Q_0^2 as follows:

$$g_1(x, Q_0^2) = g_1(x, Q^2) + \left[g_1^{\rm fit}(x, Q_0^2) - g_1^{\rm fit}(x, Q^2)\right], \tag{3}$$

where $g_1^{\rm fit}(x,Q_0^2)$ and $g_1^{\rm fit}(x,Q^2)$ are the values of g_1 evaluated at Q_0^2 and at the Q^2 of the experiment, using the fit parameters. We choose $Q_0^2 = 10\,{\rm GeV}^2$ which is close to the average Q^2 of our data. We obtain:

$$\int\limits_{0.003}^{0.7} g_1^{\rm p}(x, Q_0^2)dx = 0.139 \pm 0.006 \pm 0.008 \pm 0.006 \quad (Q_0^2 = 10\ {\rm GeV}^2), \tag{4}$$

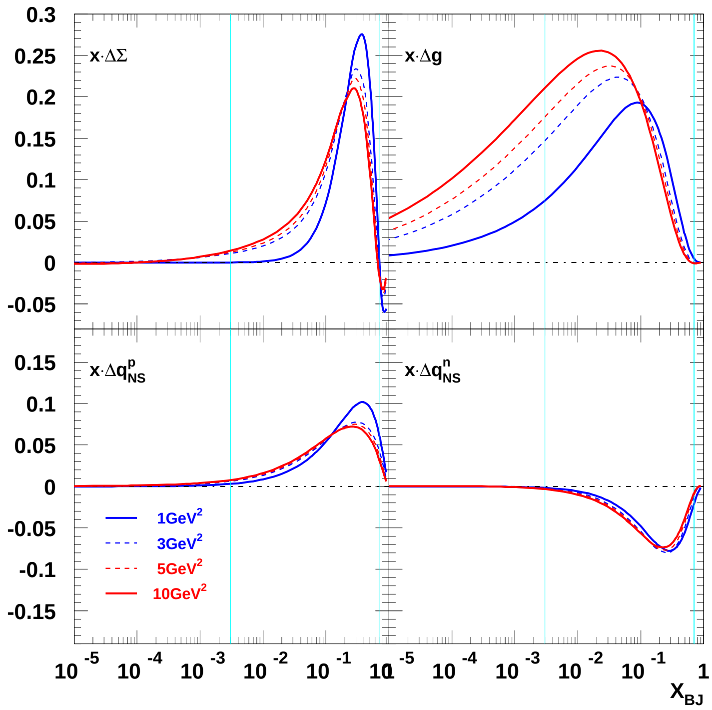

Fig. 1. The polarized parton distribution resulting from the best fit at the initial $Q_i^2 = 1$ GeV^2 and evolved to 3, 5, and 10 GeV^2.

where the first uncertainty is statistical, the second is systematic and the third is due to the uncertainty in the Q^2 evolution. This last uncertainty origins from uncertainties in various input parameters to the QCD analysis. The main contributions are due to the factorization and renormalisation scales, the value of α_s and the functional form of the initial parton distributions. Less influence have the values of the quark mass thresholds and the values of g_A/g_V. We evaluated all contributions individually by varying each of the parameters by their known errors. The scales were both varied by factors of 2 in either direction. The error due to functional form was estimated by comparing the best fit to fits with different input parameterizations giving comparable χ^2's. The influence of the experimental systematic errors on the fit was also included, taking into account bin-to-bin correlations. Details on the error evaluation are given in [12]. Fig. 3 shows $xg_1^{\rm p}$ as a function of x. In this figure the area under the data points represents the integral given in Eq. (4).

To estimate the contribution to the first moment from the unmeasured high x region $0.7 < x < 1.0$, we assume $A_1^p = 0.7 \pm 0.3$ which is consistent with

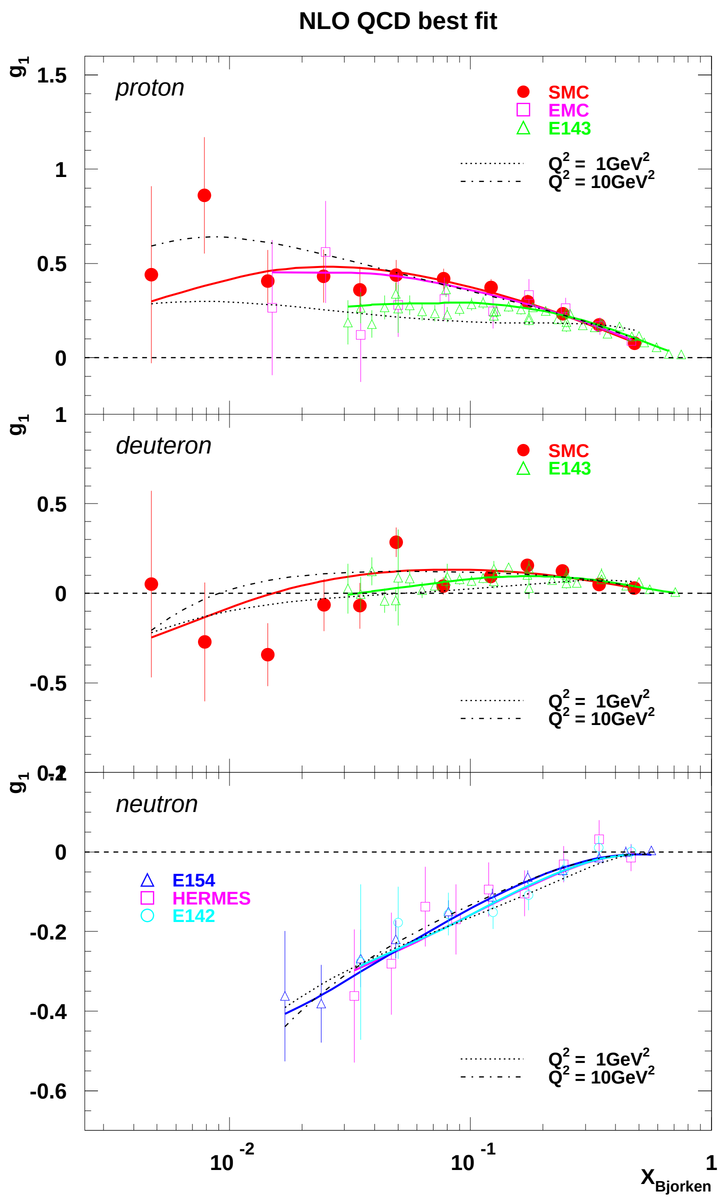

Fig. 2. The result for $g_1^{p/d/n}$ from the best fit compared to the data of the different experiments. The errors bars correspond to statistical errors. The full curves represent the fitted g_1 at the measured Q^2 of the data. The dotted/dash-dotted curve corresponds to $Q^2 = 1/10$ GeV^2.

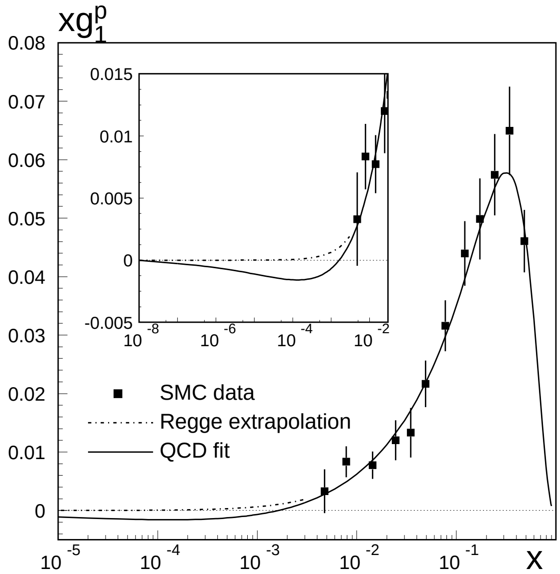

Fig. 3. xg_1^p as a function of x; SMC data points (squares) with the total error are shown together with the result of the QCD fit (continuous line), both at $Q^2 = 10$ GeV^2. For $x < 0.003$ the extrapolation assuming Regge behaviour is indicated by the dot-dashed line. The inset is a close-up extending to lower x.

the data and covers the upper bound $A_1 \leq 1$. We obtain $\int_{0.7}^{1} g_1^{\mathrm{p}}(x, Q_0^2)dx = 0.0015 \pm 0.0006$. To estimate the contribution from the unmeasured low-x region we consider two approaches :

1. Consistent with a Regge behaviour $g_1^{\mathrm{p}} \propto x^{-\alpha}$ $(-0.5 \leq \alpha \leq 0.0)$ [26], we assume g_1^{p} = constant at 10 GeV^2. This constant, 0.69 ± 0.14, obtained from the three lowest x data points evolved to 10 GeV^2, leads to

$$\int_{0.0}^{0.003} g_1^{\mathrm{p}}(x, Q_0^2)dx = 0.002 \pm 0.002 \quad \text{(Regge assumption)}, \tag{5}$$

where we assign a 100% error to this extrapolation, as was done in our previous publication [12]. The area under the dot-dashed curve in Fig. 3 and its inset corresponds to this low-x contribution.

2. Alternatively, we calculate the low-x integral from the QCD fit. Integrating this fit in the low-x region gives

$$\int_{0.0}^{0.003} g_1^{\mathrm{p}}(x, Q_0^2)dx = -0.011 \pm 0.011 \quad \text{(QCD analysis)}, \tag{6}$$

which is clearly different from the previous result using the Regge approach 5. The area under the QCD fit for $x < 0.003$ in Fig. 3 and its inset corresponds to this low-x contribution. The uncertainty in the low x integral is obtained using the same procedure as mentioned above and described in [10, 12]. For the low-x region, it is dominated by the uncertainties in factorization and renormalisation scales. The resulting values for the first moment $\Gamma_1^{\mathrm{p}}(Q^2) = \int_0^1 g_1^{\mathrm{p}}(x, Q^2)dx$ of g_1^{p} over the entire range in x are,

$$\Gamma_1^{\mathrm{p}}(Q_0^2 = 10\mathrm{GeV}^2) = \left.\begin{array}{c} 0.142 \text{ (Regge)} \\ 0.130 \text{ (QCD)} \end{array}\right\} \pm 0.006 \pm 0.008 \pm 0.014\,, \tag{7}$$

where the first uncertainty is statistical and the second is systematic. The third uncertainty is due to the low x extrapolation and the Q^2 evolution, both of which have theoretical origins, and due to the high x extrapolation. The value given here was determined for the case of the QCD extrapolation (for the Regge case it was estimated to 0.006). As the data do not allow us to exclude either approach we keep the two numbers using the larger value for the third uncertainty. The Ellis–Jaffe sum rule [2] predicts $\Gamma_1^p = 0.170 \pm 0.04$, for values of the coupling constants $(g_A/g_V, F/D)$ as given above. This means irrespective of which low-x extrapolation is used our result for Γ_1^p is significantly smaller than the prediction.

The result with the Regge extrapolation from Eq. (7) can be combined with the SMC result on $\Gamma_1^{\mathrm{d}} = 0.041 \pm 0.008$ at $Q_0^2 = 10$ GeV^2 [10], which was evaluated in a similar way. We obtain for the Bjorken sum

$$\Gamma_1^p - \Gamma_1^n = 0.195 \pm 0.029 \quad (Q_0^2 = 10 \text{ GeV}^2). \tag{8}$$

This agrees well with the theoretical prediction: $\Gamma_1^p - \Gamma_1^n(Q_0^2 = 10 \text{ GeV}^2) = 0.86 \pm 0.003$. (For consistency we have to use here 0.006 for the theoretical error on Γ_1^p, as determined with the Regge approach.) We observe that a large contribution to the uncertainty on Γ_1, is due to the unknown low-x behaviour of g_1 which can only be reduced significantly by future measurements [27] of the structure function in the very low x region.

Another interesting result of this QCD analysis is the value of the first moment of the polarized gluon distribution

$$\Delta g = 0.9 \pm 0.3(\exp) \pm 1.0\,(\text{theory})$$

at $Q^2 = 1$ GeV^2. The corresponding value of Δg at $Q^2 = 10$ GeV^2 is 1.7. The large theoretical uncertainties in the estimation of Δg point to the need of direct measurements [28] of Δg through processes in which the gluon polarization contributes at leading order.

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