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POLARIZATION IN SEMILEPTONIC $B \to X\tau$ DECAYS * ,**

P. Urban

Department of Field Theory and Particle Physics, Institute of Physics
Uniwersytecka 4, 40-007 Katowice, Poland

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Recently the one-loop perturbative QCD correction to the longitudinal polarization of tau lepton in semileptonic B decays has been found. Its smallness suggests that the higher order perturbative corrections are generally insignificant as far as the polarization is concerned. Prompted by the experimental need, I give here the polarization with respect to the W boson direction in Born approximation.

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1. Introduction

The interest in semileptonic B decays is currently increasing as the B factory in KEK is scheduled to begin collecting data later this year. This domain of physics is likely to upgrade our knowledge on the Standard Model parameters as well as to provide tests on its validity. The semileptonic B decays can contribute to the former as their theoretical description is now far more successful than that of the hadronic processes [1–5].

The polarization of the charged lepton does not depend on the Cabibbo–Kobayashi–Maskawa matrix element and so it can be instrumental in finding the quark masses. The longitudinal polarization of the tauon has been found analytically by taking the analytical decay width for the unpolarized case [6] and calculating the width for a negative polarization. Then the result can be integrated to give tau energy distribution. The procedure can easily be modified to give various kinds of polarization. This fact matters insomuch that experimentally it is the polarization along the intermediating W boson direction that is easier to measure [7]. The present calculation includes tree

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level only, but strong evidence suggests that higher-order perturbative QCD corrections are negligible. In particular, the above-mentioned longitudinal polarization does not change visibly after the first-order correction has been included either in the rest frame of the W boson [8] or that of the decaying quark [9]. The non-perturbative HQET corrections are presently being studied.

The paper is broken down into four sections. In Sec. 2, kinematics is discussed. Sec. 3 is to explain the method used in the calculation and then the results are shown is Sec. 4.

2. Kinematics

In this section we define the kinematical variables used throughout the article as well as the constraints on those. The calculation is performed in the rest frame of the decaying b quark. The four-momenta of the particles are denoted as following: Q for the b quark, q for the c quark, W for the virtual W boson, τ for the charged lepton, and ν for the corresponding antineutrino. All the real particles assumed to be on-shell, their squared four-momenta equal their masses:

$$Q^2 = m_b^2\,, \quad q^2 = m_c^2\,, \quad \tau^2 = m_\tau^2\,, \quad \nu^2 = G^2 = 0\,. \tag{1}$$

The employed variables are scaled in the units of the decaying quark mass m_b:

$$\begin{aligned} \rho &= \frac{m_c^2}{m_b^2}\,, \quad \eta = \frac{m_\tau^2}{m_b^2}\,, \\ x &= \frac{2E_\tau}{m_b}\,, \quad t = \frac{W^2}{m_b^2}\,, \quad z = \frac{P^2}{m_b^2}\,. \end{aligned} \tag{2}$$

Henceforth we scale all quantities so that $m_b^2 = Q^2 = 1$. The charged lepton is described by the light-cone variables:

$$\tau_\pm = \frac{1}{2}\left(x \pm \sqrt{x^2 - 4\eta}\right). \tag{3}$$

The W boson is characterized likewise:

$$w_0 = \frac{1}{2}(1 + t - \rho)\,, \tag{4}$$

$$w_3 = \sqrt{w_0^2 - t}\,, \tag{5}$$

$$w_\pm = w_0 \pm w_3\,. \tag{6}$$

The phase space is defined by the ranges of the kinematical variables:

$$2\sqrt{\eta} \le x \le 1 + \eta - \rho \;=\; x_m\,, \tag{7}$$

$$t_1 = \tau_-\left(1 - \frac{\rho}{1-\tau_-}\right) \le t \le \tau_+\left(1 - \frac{\rho}{1-\tau_+}\right) \;=\; t_2\,. \tag{8}$$

3. Polarization evaluation

The polarization is found by evaluating the unpolarized decay width and any of those corresponding to a definite polarization, according to the definition,

$$P = \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-} = 1 - 2\frac{\Gamma^-}{\Gamma}, \tag{9}$$

where $\Gamma = \Gamma^+ + \Gamma^-$. The calculation of the polarized width is structured after the manner of that which has yielded the longitudinal polarization [9]. Thus in the rest frame of the decaying quark, one can decompose:

$$s = \mathcal{A}Q + \mathcal{B}W\,. \tag{10}$$

The coefficients $\mathcal{A}, \mathcal{B}$ can be evaluated using the conditions defining the polarization four-vector s to read,

$$\mathcal{A}^\pm \;=\; \mp\frac{t+\eta}{\sqrt{t(x-x_-)(x_+-x)}}\,, \tag{11}$$

$$\mathcal{B}^\pm \;=\; \pm\frac{x}{\sqrt{t(x-x_-)(x_+-x)}}\,. \tag{12}$$

where the superscripts at $\mathcal{A}, \mathcal{B}$ denote the polarization of the lepton, while

$$x_\pm = (1 + \eta/t)w_\pm\,. \tag{13}$$

This observation is made relevant by the fact that the decay width for a definite polarization of the charged lepton is gotten from the analogous expression for the unpolarized case,

$$d\Gamma_0 = G_F^2 M_b^5 |V_{CKM}|^2 \mathcal{M}_{0,3}^{un} d\mathcal{R}_3(Q; q, \tau, \nu)/\pi^5\,, \tag{14}$$

where

$$\mathcal{M}_{0,3}^{un}(\tau) = q\,\tau\;Q\,\nu\,, \tag{15}$$

by formally replacing the lepton four–momentum by the four–vector $\frac{1}{2}K$, with K defined as follows:

$$K = \tau - m_\tau s\,. \tag{16}$$

Then we obtain,

$$\mathcal{M}_{0,3}^{\rm pol} = \frac{1}{2}\mathcal{M}_{0,3}^{un}(K = \tau - m_\tau s) = \frac{1}{2}(q\cdot K)(Q\cdot\nu)\,, \tag{17}$$

Applying now the representation (10) of the polarization s we readily obtain the following useful formula for the matrix element with the lepton polarized:

$$\mathcal{M}_{0,3}^{\pm} = \frac{1}{2}\mathcal{M}_{0,3}^{un}(\tau) \pm \frac{\sqrt{\eta}}{t(x-x_-)(x_+-x)}\left[x\mathcal{M}_{0,3}^{un}(W) - (t+\eta)\mathcal{M}_{0,3}^{un}(Q)\right]\,. \tag{18}$$

The first term on the right hand side of (18) can be calculated immediately once we know the result for the unpolarized case. Then the other terms require the formal replacement of the four–momenta $l \to W$ and $l \to Q$ in the argument, respectively.

4. Results

The lepton energy distribution can be cast in the form,

$$\frac{d\Gamma^\pm}{dx} = 12\Gamma_0\left[\frac{1}{2}f(x) \pm \Delta f(x)\right]\,, \tag{19}$$

where

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3}|V_{CKM}|^2\,, \tag{20}$$

and the function $f(x)$ represents the unpolarized case.

$$f(x) = \frac{1}{6}\zeta^2\tau_3\left\{\zeta[x^2 - 3x(1+\eta) + 8\eta] + (3x - 6\eta)(2-x)\right\}\,. \tag{21}$$

The function $\Delta f(x)$ reads,

$$\begin{aligned}\Delta f(x) \;=\; & a^{-3}\sqrt{\eta}\,\{\Psi[15b^3/8 + 9ab(-2c + 2b\rho + 2by - b\eta)/4\\
&+3a^2(b(\rho y - 2\rho\eta + \rho^2 - y\eta + y + y^2 - 1)\\
&+c(\eta - 2\rho - 2y)) + 6a^3(-\rho y\eta + 2\rho y - \rho y^2 - \rho^2 y - \rho^2\eta\\
&-y\eta - y + y^2 + \eta)] + \Phi_0[-15b^2/4 + a(-9b\rho - 9by + 9b\eta/2 + 4c)\\
&+6a^2(1 - \rho y + 2\rho\eta - \rho^2 + y\eta - y - y^2)]\\
&+\Phi_1[5ab/2 + 3a^2(2\rho + 2y - \eta)] - 2a^2\Phi_2\}\,.\end{aligned} \tag{22}$$

In the formulae above,

$$\tau_3 = \sqrt{x^2 - 4\eta}\,, \qquad \zeta = 1 - \frac{\rho}{1 - x + \eta}\,, \tag{23}$$

$$\Psi = \begin{cases} \frac{1}{\sqrt{|a|}}(\arcsin\omega_{\max} - \arcsin\omega_{\min}) & \text{for} \quad a < 0 \\ \frac{1}{\sqrt{|a|}}(\operatorname{arcosh}\omega_{\max} - \operatorname{arcosh}\omega_{\min}) & \text{for} \quad a > 0 \end{cases} \tag{24}$$

$$\Phi_n = \frac{t_{\max}}{\sqrt{at_{\max}^2 + bt_{\max} + c}^{\,n}} - \frac{t_{\min}}{\sqrt{at_{\min}^2 + bt_{\min} + c}^{\,n}}, \tag{25}$$

$$\omega_{\min,\max} = \frac{2|a|}{\sqrt{b^2 - 4ac}}(t_{\min,\max} + \frac{b}{2a}), \tag{26}$$

$$a = x - 1, \quad b = -2\eta - x(x + \rho - \eta - 1), \quad c = -\eta(\eta - x + x\rho). \tag{27}$$

Although we are mostly concerned with the tau lepton polarization here, the formula may well be used in evaluating the polarization of the light leptons. Interestingly, in the limit of a vanishing mass of the charged lepton the polarization falls to zero except for the endpoints. This is due to the fact that, according to Eq. (17), the polarization is linear in $m_\tau s$, which can be decomposed as follows:

$$s^\mu = \left(\frac{p}{m_\tau}\sqrt{1 - (\vec{s}_\perp)^2}, \vec{s}_\perp, \frac{E}{m_\tau}\sqrt{1 - (\vec{s}_\perp)^2}\right). \tag{28}$$

The vector $\vec{s}_\perp$ is understood to mean the part of $\vec{s}$ perpendicular to the lepton direction. The quantities E and p denote, respectively, the energy and the three-momentum value of the charged lepton. This form can easily be seen to meet the definition of the polarization vector. As the lepton mass approaches zero, the above expansion gives

$$m_\tau s^\mu = \sqrt{1 - (\vec{s}_\perp)^2}\ \tau^\mu, \tag{29}$$

which, however, is zero if we want to keep the angle subtended by the polarization vector and the lepton momentum constant. For this requirement implies that the parallel part of the polarization should be proportional to the perpendicular one, thereby forcing the factor of $\sqrt{1 - (\vec{s}_\perp)^2}$ to be proportional to the vanishing mass. The polarization can then be non-zero only where the virtual W boson is collinear with the charged lepton. In general, this happens for the maximal energy of the lepton, while in the massless case also for the minimum of energy.

We show both the longitudinal polarization and the one in the direction of W in Fig. 1. The question arises whether perturbative QCD corrections can change this picture significantly. As already suggested in the introduction, evidence exists that no such thing happens.

On integration over the whole range of the charged lepton energy one arrives at the total polarization along the direction of the virtual W boson at the tree level. For $m_b = 4.75$ GeV and $m_c = 1.35$ GeV, we obtain

$$P_0 = -0.7235. \tag{30}$$

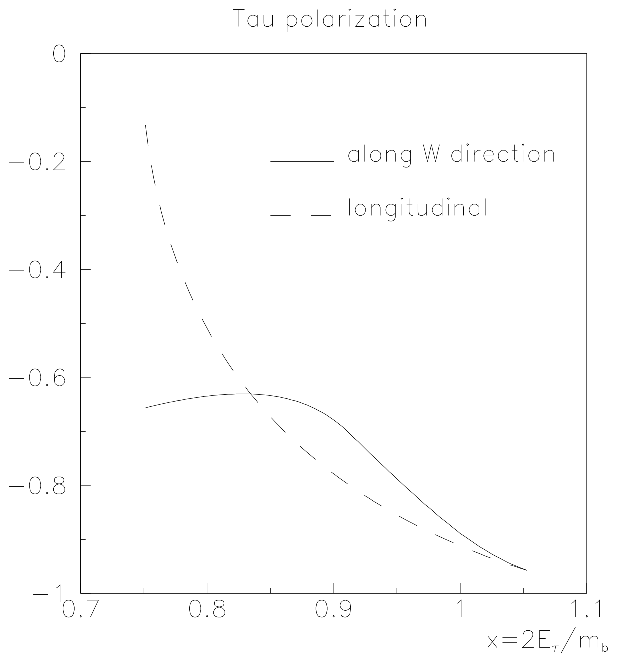

Fig. 1. Polarization of τ lepton along the direction of virtual W (solid) and the longitudinal one (dashed) in the Born approximation as functions of the scaled τ energy x. The mass of the b quark taken to be 4.75 GeV, c quark 1.35 GeV.

5. Summary

The polarization of the tau lepton along the W boson direction in semileptonic B decays has been found at the tree level in perturbative QCD. The quantity is of experimental interest and the fact that it does but slowly vary in the regime of low energies of the charged lepton is rather favorable in this context [7]. The QCD one-loop corrections are unknown but their irrelevance for the longitudinal polarization both in the rest frame of the decaying quark and that of the W boson indicate that no great change is to be expected once they are incorporated. Non-perturbative HQET corrections are under study.

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