Vol. 29 (1998) ACTA PHYSICA POLONICA B No 5

SPIN EFFECTS IN HEAVY QUARKONIA*

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(Received March 19, 1998)

A model based on a nonrelativistic potential supplemented by spin dependent terms of the Breit–Fermi type is described. This model reproduces the known masses of the $\overline{b}b$ quarkonia within 1MeV and the known masses of the $\overline{c}c$ quarkonia within 5MeV. Many predictions are made, some of them, *e.g.* for the fine splittings of the high L states of the $\overline{b}b$ quarkonia and of the mass spectrum of the $\overline{b}c$ quarkonia, significantly different from previous predictions made by other authors.

PACS numbers: 13.20. Gd, 14.40. Gx

1. Introduction

For the hydrogen atom the nonrelativistic Schrödinger equation yields a good first approximation for the energy levels. The spin effects are responsible only for the fine and hyperfine structure of the levels. In heavy quarkonia, as well, the spin effects yield the fine and hyperfine splittings, but there are two important complications. In the hydrogen atom fine splittings are negligible compared to the energy differences between the nonrelativistic levels and hyperfine splittings are negligible compared to the fine splittings. In heavy quarkonia the fine and hyperfine splittings are of the same order. They are smaller than the energy intervals between the unsplit levels, but certainly not negligible. Therefore, the comparison of nonrelativistic calculations with experiment is ambiguous. In the hydrogen atom the velocity of the electron in the ground state is about $0.01c$, thus the nonrelativistic approximation is a well justified starting point. In heavy quarkonia the quarks are much faster. The root mean square velocity is, in the ground state of the $\overline{b}b$ system, about $0.3c$ and in the ground state of the $\overline{c}c$ system

* Presented at the Cracow Epiphany Conference on Spin Effects in Particle Physics and Tempus Workshop, Cracow, Poland, January 9–11, 1998.
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about $0.5c$. The c-quark in the $\overline{b}c$ system is probably even faster than that in the $\overline{c}c$ system. It may seem, therefore, that the nonrelativistic approximation is suitable only for crude estimates. Nevertheless, many authors have found that nonrelativistic models give amazingly successful predictions for the spin averaged energy levels of heavy quarkonia. Including perturbatively spin effects one can also reproduce very satisfactorily the fine and hyperfine structure. Many references to work of this kind can be found in the review paper [1]. A simple-minded interpretation is that most of the relativistic effects is somehow taken into account by fixing the parameters of a nonrelativistic model to fit the experimental data – they just renormalize the parameters of the nonrelativistic potential. It could be argued that instead of the poorly understood potential models it is better to use QCD sum rules (*cf. e.g.* [2]) or lattice methods (*cf. e.g.* [3]). At present, however, these methods have rather large uncertainties. In the present paper we describe a model [4], [5], which gives a remarkably good description of the mass spectra below the strong decay thresholds, *i.e.* below 10.558 GeV for the $\overline{b}b$ quarkonia and below 3.729GeV for the $\overline{c}c$ quarkonia. This makes it plausible that also its predictions for the $\overline{b}c$ quarkonia below 7.144 GeV (7.189 GeV) will not be far off the mark. The mass limit in brackets is applicable to quarkonia, which, because of angular momentum and parity conservation, cannot decay into two pseudoscalar mesons. In these ranges there are 34 mass states for the $\overline{b}b$ quarkonia (out of that nine are known), 16 mass states for the $\overline{b}c$ quarkonia and 8 states for the $\overline{c}c$ quarkonia (out of that six are known). We consider as known only the mass states included as firmly established in the 1996 Particle Data Group Tables [6].

2. Nonrelativistic potential

Our model contains six free parameters — the quark masses

$$m_b = 4.8030\,\mathrm{GeV}, \qquad m_c = 1.3959\,\mathrm{GeV}, \tag{1}$$

the coupling constant α_s at the m_c mass scale

$$\alpha_s(m_c) = 0.3376 \tag{2}$$

and the three parameters of the nonrelativistic potential

$$a = 0.32525, \quad b = 0.70638\,\mathrm{GeV}^{\frac{3}{2}}, \quad c = 0.78891\,\mathrm{GeV}. \tag{3}$$

Of course we do not claim to have fixed the quark masses with an accuracy of four digits *etc.* All these digits are necessary, however, in order to reproduce our results. The potential is spherically symmetric and reads

$$V(r) = m_Q + m_{\overline{Q}} - c + b\sqrt{r} - \frac{a}{r}. \tag{4}$$

The eigenvalues of the corresponding Schrödinger equation are interpreted as the centres of mass of the spin one multiplets. For the $\overline{b}c$ quarkonia this corresponds to the case, when the singlet-triplet mixing is switched off.

Comparison with experiment for the $\overline{b}b$ quarkonia can be made in five cases. The agreement is within 1 MeV. A rough estimate of the relativistic correction gives about 60 MeV. Thus, almost all this correction must reduce to a redefinition of the parameters of the potential. For the $\overline{c}c$ quarkonia comparison with experiment can be made in three cases. The dicrepancies are within 4 MeV. Since the relativistic corrections here should be much larger than in the previous case, the good agreement is even more surprising than before. It is instructive to compare our results with those of Gupta and Johnson [7]. They are using a very different model, but their fit is about as good as ours. In fact it is somewhat better for the $\overline{c}c$ system, though somewhat worse for the $\overline{b}b$ system. In spite of the agreement within a few MeV between the two models for the $\overline{b}b$ and $\overline{c}c$ systems, the predictions for the $\overline{b}c$ quarkonia differ significantly. We expect higher masses, by about 40 MeV for the S states and by 16 MeV for the P multiplet. Thus, a good fit for the $\overline{b}b$ and $\overline{c}c$ is compatible with a poor fit for the $\overline{b}c$ quarkonia. Of course, we do not know yet, whose fit is poor, but both fits cannot be good.

3. Hyperfine splittings

The hyperfine splittings are usually described using the Breit–Fermi interaction, which in the present case takes the form (*cf. e.g.* [8])

$$H_{HF} = \frac{32\pi\alpha_s}{9m_Q m_{\overline{Q}}} \left(\vec{S}_Q \cdot \vec{S}_{\overline{Q}} - \frac{1}{4} \right) \delta(\vec{r}). \tag{5}$$

In the first approximation of the Rayleigh–Schrödinger perturbation theory the corresponding energy splitting is

$$\Delta E_{HFS} = \frac{32\pi\alpha_s}{9m_Q m_{\overline{Q}}} |\psi(0)|^2. \tag{6}$$

In this approximation, the splitting occurs only for the S states, because for the states with $L \neq 0$ the wave function ψ vanishes, when the distance between the two quarks goes to zero. The measured splitting of the $1S$ state of the $\overline{c}c$ system yields the value of α_s (2). Once this is chosen, it is possible to predict any other hyperfine splitting, but there is no data to compare the predictions with. Some indirect support is obtained by calculating the coupling constant α_s at the mass of the Z^0 — the result $\alpha_s(M_{Z^0}) = 0.115$ is very acceptable — and from the calculation of the leptonic decay widths of heavy quarkonia [5]. It is instructive to compare

our prediction for the hyperfine splitting of the $1S$ state in the $\overline{b}b$ system (56.7 MeV) with the predictions of the sum rules [2] — 63 ± 30 MeV — and of the lattice approach [3] — 60 MeV with a large, but unspecified, error. Our agreement with the mean values is good, but in view of the large uncertainties of the more rigorous approaches, it is premature to draw any conclusions.

4. Fine splittings and mixing

The interaction terms responsible for the fine splittings are (*cf. e.g.* [9, 10])

$$V_A(r) = \frac{m_Q^{-2} - m_{\overline{Q}}^{-2}}{4}\left(-\frac{1}{r}\frac{\partial V}{\partial r} + \frac{8\alpha_s}{3r^3}\right)\vec{L}\cdot\left(\vec{S}_Q - \vec{S}_{\overline{Q}}\right), \tag{7}$$

$$V_{LS}(r) = \left[\frac{m_Q^{-2} + m_{\overline{Q}}^{-2}}{4}\left(-\frac{1}{r}\frac{\partial V}{\partial r} + \frac{8\alpha_s}{3r^3}\right) + \frac{4\alpha_s}{3m_Q m_{\overline{Q}} r^3}\right]\vec{L}\cdot\left(\vec{S}_Q + \vec{S}_{\overline{Q}}\right), \tag{8}$$

$$V_T(\vec{r}) = \frac{4\alpha_s}{3m_Q m_{\overline{Q}} r^5}\left[3(\vec{S}_Q\cdot\vec{r})(\vec{S}_{\overline{Q}}\cdot\vec{r}) - r^2\vec{S}_Q\cdot\vec{S}_{\overline{Q}}\right]. \tag{9}$$

In these formulae the only undefined quantity is the coupling constant α_s, which should be running, *i.e.* depending on r. We propose the following procedure to define this function. The general formula is modelled on the well-known one-loop expression for $\alpha_s(p)$ with the replacement of the momentum scale p by $\frac{1}{r}$. We introduce tildes above α_s and Λ, when considered as functions of r, in order to avoid confusion with the corresponding quantities considered as functions of momentum. Thus

$$\tilde{\alpha}_s(r) = \frac{12\pi}{33 - 2n_{\mathrm{f}}}\frac{\left(\tilde{\Lambda}^{(n_{\mathrm{f}})}\right)^2 r^2 - 1}{\log\left(\left(\tilde{\Lambda}^{(n_{\mathrm{f}})}\right)^2 r^2\right)}. \tag{10}$$

Here the number of flavours n_{f} equals three for $r > m_c^{-1}$, equals four for $m_b^{-1} < r < m_c^{-1}$ and equals five for $r < m_b^{-1}$. The region $r > m_s^{-1}$ is of little interest for the present problem, therefore, we keep $n_{\mathrm{f}} = 3$ also for large values of r. The numerator is introduced in order to compensate the zero of the denominator at $r = \frac{1}{\tilde{\Lambda}^{(n_{\mathrm{f}})}}$. Its exact shape is of little importance. The parameter $\tilde{\Lambda}^{(4)}$ is obtained from the two conditions

$$\tilde{\alpha}_s\left(\frac{1}{m_c}\right) = \alpha_s(m_c), \tag{11}$$

$$\tilde{\alpha}_s\left(\frac{1}{m_b}\right) = \alpha_s(m_b). \tag{12}$$

These two equations give slightly different values of the parameter $\tilde{\Lambda}^{(4)}$ and we take the geometrical mean of the two results. However, the two solutions are so close to each other, that taking the arithmetic mean instead of the geometric one would make no difference at our level of precision. Then, $\tilde{\Lambda}^{(3)}$ and $\tilde{\Lambda}^{(5)}$ are calculated from the conditions that the function $\tilde{\alpha}_s(r)$ should be continuous at $r = m_c^{-1}$ and at $r = m_b^{-1}$. Interpreting α_s in the formulae for the potentials V_A, V_{LS} and V_T as the function $\tilde{\alpha}_s(r)$, we obtain the explicit form of the operator responsible for the fine splittings of the mass levels of the heavy quarkonia.

When calculating the splittings of the $\overline{b}b$ quarkonia, we find very good agreement with experiment for the splittings of the $1P$ and $2P$ states. The accuracies of our results are all within 1MeV. Most other models, however, are also successful in these predictions, typical errors being within 5 MeV. Also for the $3P$ states, where there is no data to compare with, our predictions are close to those of other authors. A qualitative difference between the predictions appears, however, for the higher L states. For instance, Kwong and Rosner [11] predict a splitting of about 1 MeV for the F-states and almost no splitting for the G-states, while we find a splitting of about 13 MeV for the $1F$ state and of about 10 MeV for the $1G$ state. Thus, the high L states will be of great interest for the comparison of models. For the $\overline{c}c$ systems our predictions for the splitting of the $1P$ state agree with experiment within about 5 MeV. For other states we are in rough agreement with other models, except for the $^3D_1$ state, where we predict a down shift of about 20 MeV, while other models find much smaller shifts.

For the $\overline{b}c$ system one can calculate the singlet-triplet mixing due to the operator V_A. These angles are very sensitive to the details of the models and comparing the results of our paper and of the papers [9] and [10] one sees that no two sets of predictions agree. According to our calculation the sines of the mixing angles for the $1P$, $2P$ and $1D$ states are respectively: 0.374, 0.385 and 0.244. Eichten and Quigg [9] find almost no mixing for the $1P$ state, while Gershtein and collaborators [10] find the biggest mixing for the $1D$ state.

Another interesting mixing effect is due to the operator V_T. This involves pairs of states differing by two units in orbital angular momentum. The mixing angles are very small — less than 10^{-3} — and the effect on the mass values is negligible. This effect, however, enhances considerably the leptonic decays of the quarkonia with angular momenta $L \geq 2$.

REFERENCES

[1] B. Besson, T. Skwarnicki, *Annu. Rev. Nucl. Part. Sci.* **43**, 333 (1993).
[2] S. Narison, *Phys. Lett.* **B387**, 162 (1996).
[3] C.T.H. Davies *et al.*, *Phys. Lett.* **B382**, 131 (1996).
[4] L. Motyka, K. Zalewski, *Z. Phys.* **C69**, 343.
[5] L. Motyka, K. Zalewski, *Eur. J. Phys.* in print.
[6] Particle Data Group, *Phys. Rev.* **D540**, 1 (1996).
[7] S.N. Gupta, J.M. Johnson, *Phys. Rev.* **D38**, 2075 (1996).
[8] W. Buchmüller, *Phys. Lett.* **B112**, 479 (1982).
[9] E.J. Eichten, C. Quigg, *Phys. Rev.* **D49**, 5845 (1994).
[10] S.S. Gershtein *et al.*, *Phys. Rev.* **D51**, 3613 (1995).
[11] W. Kwong, J.L. Rosner, *Phys. Rev.* **D38**, 279 (1988).