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DECAY OF TOP QUARKS IN $e^+e^- \to t\bar{t}$ NEAR THRESHOLD* **

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We discuss the physics involved in the production and decay chain of top quarks in $e^+e^- \to t\bar{t} \to bl^+\nu\bar{b}W^-$ near the top quark threshold. We elucidate the effects of the final-state interactions. Namely, we clarify how the evolution and decay of t in this process are affected by the chromostatic field generated by the other color charge ($\bar{t}$ or $\bar{b}$) nearby. Furthermore, we propose and calculate inclusive and exclusive observables in the threshold region. The latter depends only on the decay of *free* polarized top quarks, and thus it can be calculated without knowledge of bound-state effects or the final-state interactions.

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1. Introduction

In this article we consider the top quark pair-production in the threshold region, $E_{\rm cm} \simeq 2m_t$, and their subsequent decays:

$$e^+e^- \to t\,\bar{t} \to bl^+\nu\,\bar{b}W^- \tag{1}$$

(Fig. 1). We concentrate on the semi-leptonic decays of t, while $\bar{t}$ may decay either semi-leptonically or hadronically ($W^- \to l^-\bar{\nu}$ or $W^- \to q\bar{q}'$). From the practical point of view, the top threshold region is of interest since it is considered to be a serious candidate for the first stage operation of next generation linear e^+e^- colliders. There, we will be able to study immensely on the various properties of the top quark, using the largest cross section and the highly polarized top quark sample [1,2]. From the theoretical viewpoint, recently a clear description of the decay of top quarks in the threshold region has been obtained including the full $\mathcal{O}(\alpha_s)$ corrections, in particular the corrections from the final-state interactions have been calculated [3]. Prior to the calculation of the l^+ differential distribution an inclusive quantity, the mean value $\langle n\ell\rangle$ of the charged lepton four-momentum projection on an arbitrarily chosen four-vector n, was proposed as an observable sensitive to the top quark polarization, and this quantity was calculated including the final-state interactions [4].

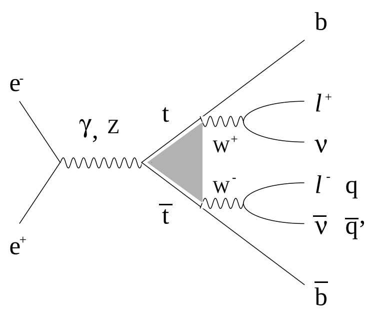

Fig. 1. The production and decay chain of top quark, $e^+e^- \to t\bar{t} \to bl^+\nu\bar{b}W^-$, in the threshold region $E_{\rm cm} \simeq 2m_t$.

For introduction, let us remind the readers of the production and decay chain of the process (1) far above the threshold, $E_{\rm cm} \gg 2m_t$. It is well known that in this kinematical region the production and decay processes of the top quark factorize in the narrow-width approximation:

$$\frac{d\sigma(e^+e^- \to t\bar{t} \to bl^+\nu\bar{b}W^-)}{d^3\boldsymbol{p}_t d\Phi_{bl^+\nu}}$$

$$= \frac{d\sigma(e^+e^- \to t\bar{t})}{d^3\boldsymbol{p}_t} \times \frac{1}{\Gamma_t} \frac{d\Gamma_{t\to bl^+\nu}(\boldsymbol{S})}{d\Phi_{bl^+\nu}} + \mathcal{O}\left(\frac{\Gamma_t}{m_t}\right) . \tag{2}$$

The left-hand-side denotes the differential cross section for which the three momentum of top quark ($\boldsymbol{p}_t$) and the kinematical variables of its decay products ($d\Phi_{bl^+\nu}$) are fixed, while the variables associated with the anti-top side are integrated over. The right-hand-side shows that the cross section can be written as a product of the production cross section for unpolarized top quarks and the decay differential cross section from free polarized top quarks. $\boldsymbol{S}$ represents the polarization vector of the top quark, which is given as a function of the e^+e^- beam polarization, the electroweak charges of e^- and t, and the kinematical variables $E_{\rm cm}$ and $\boldsymbol{p}_t$.

The above factorization formula constitutes the basis for various analyses of top quark decays at $E_{\rm cm} \gg 2m_t$. In particular, it justifies the detailed analyses of the decay of *free* polarized top quarks. Decay processes of polarized top quarks are of special interest. Due to the short lifetime of the top quark, there will be no top-hadron formations. Hence, all the spin information of the top quark will be transferred to its decay daughters [5], and the energy-angular distributions of the decay products are calculable as purely partonic processes. Therefore, we may take full advantage of the spin information in studying the top quark properties through its various decay modes [2, 6, 7]. It is known [7] that using the helicity analyses each of eight different form factors associated with the production and decay vertices of top quark can be tested to 5-10% accuracy relative to the Standard-Model values.

As an elegant example, we quote the energy-angular distribution of charged leptons l^+ in the semi-leptonic decay of the top quark. In leading order, the l^+ distribution has a form where the energy and angular dependences are factorized [8, 9]:

$$\frac{d\Gamma_{t\to bl^+\nu}(\boldsymbol{S})}{dE_l d\Omega_l} = h(E_l)\,(1 + |\boldsymbol{S}|\cos\theta_l) + (\mathcal{O}(\alpha_s) \text{ correction}). \tag{3}$$

Here E_l, Ω_l, and θ_l denote, respectively, the l^+ energy, the solid angle of l^+, and the angle between the l^+ direction and the top polarization vector $\boldsymbol{S}$, all of which are defined in the top-quark rest frame[1]. Hence, we may measure the top-quark polarization with maximal sensitivity using the l^+ angular distribution.

Now we return to the case of our interest: the same production and decay chain near the top-quark production threshold. There are two differences from the above case:

[1] The full $\mathcal{O}(\alpha_s)$ corrections to this formula have been calculated [10]. The factorization property still holds to a very good approximation including these corrections.

- $t\bar{t}$ boundstate effects
- final-state interactions

Both are unique to the threshold region. It is the latter effects that we focus on and elucidate in this article, which stem from the fact that the top quark decays in the chromostatic field generated by the other charge ($\bar{t}$ or $\bar{b}$). The effects are sizable near the threshold region because both t and $\bar{t}$ are slow and stay close to each other. A specific consequence of the final-state interactions is that the factorization of the production and decay processes, Eq. (2), is violated[2].

In Section 2, we present intuitive pictures of the final-state interaction effects and also the explicit formula for the cross section. Then in Section 3 we define a new observable in the threshold region which depends only on the decay of free top quarks [3]. In Section 4 the moments $\langle nl \rangle$ are described which are sensitive to the polarization of the top quarks [4]. A summary is given in Section 5.

2. Final-state interactions

As already mentioned, we are interested in how the production and decay chain of the top quark is affected by an existence of the other color charge close by. More specifically, the final-state interactions are those effects which are induced by the gluon exchange between t and $\bar{b}$ ($\bar{t}$ and b) or between b and $\bar{b}$ (Fig. 2). The size of the corrections is at the 10% level, hence it is necessary to incorporate their effects in precision studies of top-quark production and decay near the threshold.

2.1. Intuitive pictures

Let us discuss what we expect from physics ground [3].

- *Top Momentum Distribution*

Perhaps it is easiest to understand the effect of final-state interactions on the top momentum distribution. Fig. 3(a) shows the momentum distribution with (solid) and without (dashed) the final-state interactions. We see that the average momentum is reduced due to the interaction. To understand this, consider for example the case where t decays first. Fig. 3(b) shows the attractive force between $\bar{t}$ and b, which deflects the trajectory of b. Since

[2] Although we give no account for the boundstate effects we note here an important result: the boundstate effects affect only the production cross section $d\sigma(e^+e^- \to t\bar{t})/d^3\boldsymbol{p}_t$ and the top polarization vector $\boldsymbol{S}$. Hence, the factorization form of the production and decay processes, Eq. (2), is preserved by this effect.

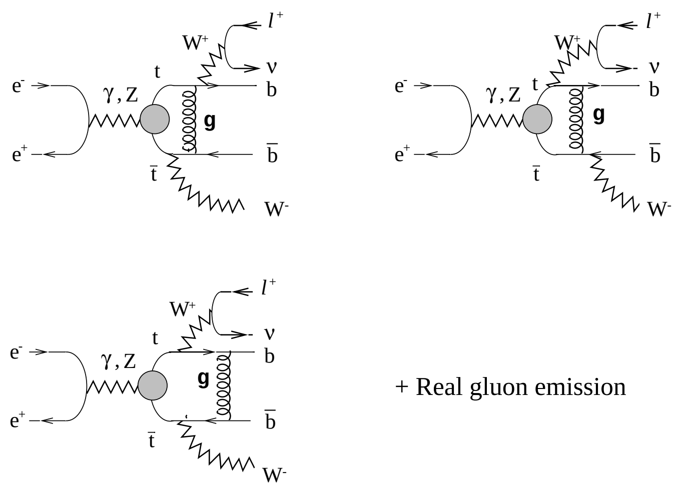

Fig. 2. Diagrams for the final-state interactions for $e^+e^- \to t\bar{t} \to bl^+\nu\bar{b}W^-$.

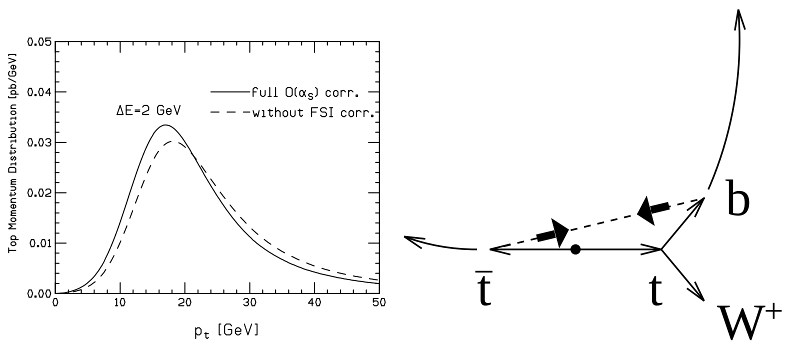

Fig. 3. (a) — The top momentum distribution with (solid) and without (dashed) the final-state interaction corrections for $\alpha_s(M_Z) = 0.118$ and $m_t = 175$ GeV. (b) — Attractive force between $\bar{t}$ and b (from t decay). The momentum transfer $\delta \boldsymbol{p}_b = -\delta \boldsymbol{p}_{\bar{t}}$ due to the attraction is indicated by thick arrows.

$\boldsymbol{p}_t$ is reconstructed from the bW^+ momenta at time $\tau \to \infty$, it is obvious that the reconstructed momentum $|\boldsymbol{p}_t| = |\boldsymbol{p}_b + \boldsymbol{p}_{W^+}|$ is decreased by the attraction.

- *Forward-Backward Asymmetric Distribution*

Next we consider the $\cos\theta_{te}$ distribution of the top quark. (θ_{te} denotes the angle between t and e^- in the $t\bar{t}$ c.m. frame.) We consider the case where $\bar{t}$ decays first and examine the interaction between t and $\bar{b}$. The t and $\bar{t}$ pair-produced near threshold in e^+e^- collisions have their spins approximately parallel or anti-parallel to the e^- beam direction ($\hat{\boldsymbol{n}}_\parallel$) and the spins are always oriented parallel to each other [11]. On the other hand, the decay of $\bar{t}$ occurs via a $V{-}A$ coupling, and $\bar{b}$ is emitted preferably in the spin direction of the parent $\bar{t}$, see Fig. 4. More precisely, the excess of the $\bar{b}$'s emitted in the $\bar{t}$ spin direction over those emitted in the opposite direction is given by $\kappa = (m_t^2 - 2M_W^2)/(m_t^2 + 2M_W^2)$.

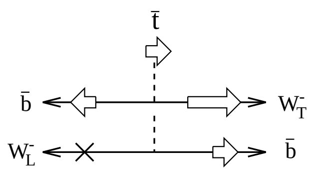

Fig. 4. Typical configurations in the decay of $\bar{t}$ with definite spin orientation. Transverse W^- (W_T^-) tends to be emitted in the direction of the $\bar{t}$ spin orientation, while longitudinal W^- (W_L^-) is emitted in the opposite direction due to helicity conservation. For $m_t \simeq 175$ GeV, $\bar{t}$ decays mainly to W_L^-, hence $\bar{b}$ is emitted more in the $\bar{t}$ spin direction.

Now suppose t and $\bar{t}$ have their spins in the $\hat{\boldsymbol{n}}_\parallel$ direction. Then $\bar{b}$ will be emitted dominantly in the $\hat{\boldsymbol{n}}_\parallel$ direction. One can see from Fig. 5(a) that in this case t is always attracted to the forward direction due to the attractive force between t and $\bar{b}$. The direction of the attractive force will be opposite

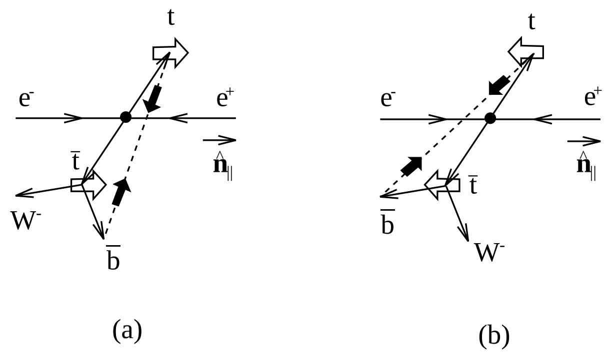

Fig. 5. Attractive force between t and $\bar{b}$ when the t and $\bar{t}$ spins are oriented in the (a) $\hat{\boldsymbol{n}}_\parallel$ direction, and in the (b) $-\hat{\boldsymbol{n}}_\parallel$ direction. The momentum transfer $\delta\boldsymbol{p}_b = -\delta\boldsymbol{p}_{\bar{t}}$ due to the attraction is indicated by thick black arrows.

if t and $\bar{t}$ have their spins in the $-\hat{\boldsymbol{n}}_\parallel$ direction (Fig. 5(b)). Thus, polarized top quarks will be pulled in a definite (forward or backward) direction, and we may expect that a forward-backward asymmetric distribution of the top quark $\sim \kappa\, S_\parallel \cos\theta_{te}$ is generated by the final-state interaction. ($S_\parallel$ denotes the $\hat{\boldsymbol{n}}_\parallel$-component of the top polarization vector $\boldsymbol{S}$.)

• *Top-Quark Polarization Vector*

From Fig. 5 we can also learn the effect of the final-state interaction on the top polarization vector. We have seen that if the t and $\bar{t}$ spins are oriented in the $\hat{\boldsymbol{n}}_\parallel$ direction, t will be attracted to the forward direction due to the attraction by $\bar{b}$, and oppositely attracted to the backward direction if the t and $\bar{t}$ spins are in the $-\hat{\boldsymbol{n}}_\parallel$ direction. This means that in the forward region ($\cos\theta_{te} \simeq 1$) the number of t's with spin in $\hat{\boldsymbol{n}}_\parallel$ direction increases, whereas in the backward region the number of those with spin in the opposite direction increases. Or equivalently, the $\hat{\boldsymbol{n}}_\parallel$-component of the top-quark polarization vector increases in the forward region and decreases in the backward region. We may thus conjecture that the top-quark polarization vector is modified as $\delta S_\parallel \sim \kappa \cos\theta_{te}$ due to the interaction between t and $\bar{b}$.

• l^+ *Energy-Angular Distribution*

Finally let us examine the effect of the attraction between b and $\bar{t}$ on the l^+ energy-angular distribution in the semi-leptonic decay of t. The b-quark from t decay will be attracted in the direction of $\bar{t}$ due to the gluon exchange between these two particles. We show schematically typical configurations of the particles in the top-quark semileptonic decay in Fig. 6. It can be seen

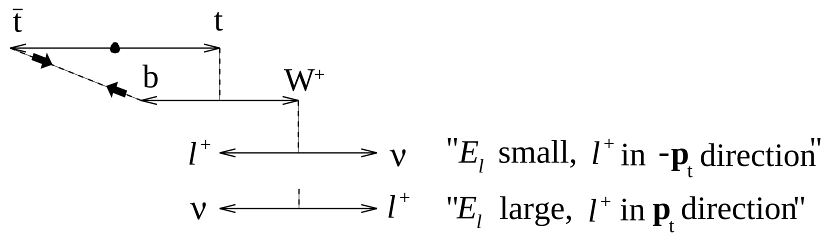

Fig. 6. Typical configurations of the particles in semileptonic decay of t when the b-quark is emitted in the $\bar{t}$ direction. Due to the boost by W^+, there will be an energy-angle correlation of l^+.

that if the probability for b being emitted in the $\bar{t}$ direction increases, correspondingly the probability for particular l^+ energy-angular configurations increases. These configurations are either "E_l is small and l^+ emitted in $-\boldsymbol{p}_t$ direction" or "E_l is large and l^+ emitted in $\boldsymbol{p}_t$ direction".

2.2. Lepton energy-angular distribution near $t\bar{t}$ threshold

Now we are in a position to present the formula for the charged lepton energy-angular distribution in the decay of top quarks that are produced via $e^+e^- \to t\bar{t}$ near threshold.

Without including the final-state interactions, the differential distribution of t and l^+ has a form where the production and decay processes of the top quark are factorized, Eq. (2). Including the final-state interactions, the factorization of production and decay processes is destroyed. The formula including the full $\mathcal{O}(\alpha_s)$ corrections[3] is given by [3]

$$\frac{d\sigma(e^+e^- \to t\bar{t} \to bl^+\nu\bar{b}W^-)}{d^3\boldsymbol{p}_t dE_l d\Omega_l} = \frac{d\sigma(e^+e^- \to t\bar{t})}{d^3\boldsymbol{p}_t} \times (1 + \delta_0 + \delta_1 \cos\theta_{te})$$
$$\times \frac{1}{\Gamma_t} \frac{d\Gamma_{t\to bl^+\nu}(\boldsymbol{S} + \delta\boldsymbol{S})}{dE_l d\Omega_l} \times [1 + \xi(|\boldsymbol{p}_t|, E, E_l, \cos\theta_{lt})] \; . \tag{4}$$

Here, the first line on the right-hand-side shows that there are corrections to the top-quark production cross section, while the second line shows that the correction to the decay distribution of l^+ is accounted for by a modification of the parent top-quark polarization vector, and finally there is a non-factorizable correction ξ which cannot be assigned either to the production or the decay process alone.

We have already seen in Fig. 3(a) that the top momentum distribution is modified by δ_0 to take a lower average momentum. The forward-backward asymmetric distribution and the top polarization vector get corrections as

$$\delta_1 \cos\theta_{te} = \kappa S_\parallel \cos\theta_{te} \times \frac{1}{2}\psi_{\rm R}, \tag{5}$$

$$\delta\boldsymbol{S} = \left[1 - (S_\parallel)^2\right] \times \kappa \cos\theta_{te} \times \frac{1}{2}\psi_{\rm R} \cdot \hat{\boldsymbol{n}}_\parallel \, , \tag{6}$$

where

$$\psi_2({\rm p}, E) \;=\; 2\int \frac{d^3k}{(2\pi)^3} V(|\boldsymbol{k} - \boldsymbol{p}_t|) \frac{\boldsymbol{p}_t \cdot (\boldsymbol{k} - \boldsymbol{p}_t)}{|\boldsymbol{p}_t||\boldsymbol{k} - \boldsymbol{p}_t|^2} \frac{G({\rm k}, E)}{G({\rm p}, E)}$$
$$\times \left(1 - \frac{\Gamma_t}{|\boldsymbol{k} - \boldsymbol{p}_t|} \arctan \frac{|\boldsymbol{k} - \boldsymbol{p}_t|}{\Gamma_t}\right) \tag{7}$$

$$\psi_{\rm R}({\rm p}, E) \;=\; \Re\, \psi_2({\rm p}, E). \tag{8}$$

and ${\rm p} = |\boldsymbol{p}_t|$. The above formulas (5) and (6) have exactly the forms that we anticipated in the previous subsection if $\psi_{\rm R}$ is positive. Indeed, the numerical evaluation in Ref. [4] shows that $\psi_{\rm R}(|\boldsymbol{p}_t|, E) \gtrsim 0$ holds in the entire threshold region[4].

[3] In order to take into account the QCD binding effects properly in the threshold region, we have to systematically rearrange the perturbative expansion. Namely, we first resum all the leading Coulomb singularities $\sim (\alpha_s/\beta)^n$, take the result as the leading order contribution, and then calculate higher order corrections, which are essentially resummations of the terms $\sim \alpha_s^{n+1}/\beta^n$, α_s^{n+2}/β^n, ...

[4] It shows that the force between b and $\bar{t}$ ($\bar{b}$ and t) is attractive in the entire threshold region. Note that the sign of $\psi_{\rm R}$ will be reversed if the force is repulsive.

We show the $\cos\theta_{lt}$ and E_l dependences of the non-factorizable correction ξ as a 3-dimensional plot in Fig. 7. One can see that ξ takes comparatively

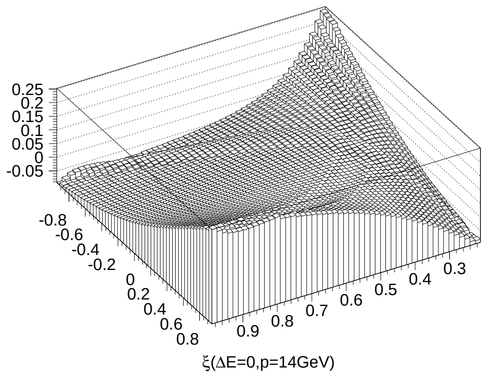

Fig. 7. A three-dimensional plot of ξ as a function of $2E_l/m_t$ (x-axis) and $\cos\theta_{te}$ (y-axis).

large positive values for either "small E_l and $\cos\theta_{lt} \simeq -1$" or "large E_l and $\cos\theta_{lt} \simeq +1$". Oppositely, in the other two corners of the E_l–$\cos\theta_{lt}$ plane ξ becomes negative. These features are consistent with our previous qualitative argument. The typical magnitude of ξ is 10–20%.

Thus, the theoretical prediction for the distribution of l^+ from the top decay is under good control in the $t\bar{t}$ threshold region, together with a good qualitative understanding.

3. New observable proper to the decay process

We have seen that the final-state interactions destroy the factorization of the production and decay cross sections of the top quark. In order to study the decay of top quarks in a clean environment in the threshold region, it would be useful if we could find an observable which depends only on the decay process of free polarized top quarks, $d\Gamma_{t\to bl^+\nu}(\boldsymbol{S})/dE_l d\Omega_l$. In fact, such an observable can be constructed, which at the same time preserves most of the differential information of the l^+ energy-angular distribution.

We can show (with sufficient reasoning) that the non-factorizable correction factor $\xi(|\boldsymbol{p}_t|, E, E_l, \cos\theta_{lt})$ is invariant under a transformation of the l^+ kinematical variables

$$E_l \;\to\; E_l' = 2m_t \left(\frac{m_t^2 + M_W^2}{M_W^2} - \frac{2m_t}{E_l} \right)^{-1} , \tag{9}$$

$$\boldsymbol{p}_l \;\to\; \boldsymbol{p}_l' \qquad \text{such that} \qquad \cos\theta_{lt} \to -\cos\theta_{lt}. \tag{10}$$

Using this symmetry, it is possible to cancel out not only the non-factorizable correction but also the top production cross section by taking an appropriate ratio of cross sections.

Let us define an observable as

$$\overline{A}(E_l, a = \boldsymbol{S}\cdot\boldsymbol{p}_l) \equiv \frac{\displaystyle\int d^3\boldsymbol{p}_t d\Omega_l \; \delta(\boldsymbol{S}\cdot\boldsymbol{p}_l - a) \left[\frac{d\sigma(e^+e^- \to t\bar{t} \to bl^+\nu\bar{b}W^-)}{d^3\boldsymbol{p}_t dE_l d\Omega_l}\right]_{E_l,\boldsymbol{p}_l}}{\displaystyle\int d^3\boldsymbol{p}_t d\Omega_l \; \delta(\boldsymbol{S}\cdot\boldsymbol{p}_l + a) \left[\frac{d\sigma(e^+e^- \to t\bar{t} \to bl^+\nu\bar{b}W^-)}{d^3\boldsymbol{p}_t dE_l d\Omega_l}\right]_{E_l',\boldsymbol{p}_l}}. \tag{11}$$

Here, the top-quark polarization vector $\boldsymbol{S}$ in the delta functions depends on $\boldsymbol{p}_t$ [3]. The numerator and denominator, respectively, depend on two external kinematical variables (the lepton energy and the lepton angle from the parent top-quark polarization vector), and all other variables are integrated out before taking the ratio.

Then substituting the differential distribution Eq. (4), one can show that theoretically $\overline{A}$ is determined solely from the decay distribution of free polarized top quarks:

$$\overline{A}(E_l, a) = \frac{\left[\frac{d\Gamma_{t\to bl^+\nu}(\boldsymbol{S})}{dE_l d\Omega_l}\right]_{E_l,\boldsymbol{S}\cdot\boldsymbol{p}_l = a}}{\left[\frac{d\Gamma_{t\to bl^+\nu}(\boldsymbol{S})}{dE_l d\Omega_l}\right]_{E_l',\boldsymbol{S}\cdot\boldsymbol{p}_l = -a}}. \tag{12}$$

This is a general formula that is valid even if the decay vertices of the top quark deviate from the standard-model forms.

This quantity will be useful from the theoretical point of view. If one claims that he calculates $\overline{A}(E_l, a)$ defined in Eq. (11) in the threshold region, it can be calculated without including any bound-state effects or final-state interaction corrections but only from a decay distribution of free polarized top quarks via Eq. (12).

4. Moments of the charged lepton energy–angular distribution

A useful information on the process (1) can be obtained by studying the average values of the scalar products $n \cdot l = n_\mu l^\mu$, where l^μ denotes the four momentum of the charged lepton and n^μ is a fixed four vector. One can consider the moments $\langle nl \rangle$ for a fixed three-momentum $\boldsymbol{p}_t$ of the top quark,

$$\langle nl \rangle \equiv \left(\frac{d\sigma}{\mathrm{d}\mathrm{p} d\Omega_p}\right)^{-1} \int dE_l d\Omega_l \, \frac{d\sigma(e^+e^- \to bl\nu\bar{b}W^-)}{\mathrm{d}\mathrm{p} d\Omega_p d\Omega_l dE_l} \, n\cdot l \tag{13}$$

or, integrating over the direction of the top quark, the moments

$$\langle\langle nl \rangle\rangle \equiv \left(\frac{d\sigma}{d\mathrm{p}}\right)^{-1} \int dE_l d\Omega_l d\Omega_p \, \frac{d\sigma(e^+e^- \to bl\nu\bar{b}W^-)}{d\mathrm{p} d\Omega_p d\Omega_l dE_l} \, n\cdot l \tag{14}$$

or, finally the moments $\langle\langle\langle nl \rangle\rangle\rangle$ integrated also over $\mathrm{p} = |\boldsymbol{p}_t|$.

Let $\hat{\boldsymbol{n}}_{e^-}$ denote the unit three vector along the direction of the electron beam and $\hat{\boldsymbol{n}}_t = \boldsymbol{p}_t/\mathrm{p}$ be the unit three vector in a direction parallel with $\boldsymbol{p}_t$ in the rest frame of $t\bar{t}$. In the threshold region the process of top quark pair production is dominated by the S wave contribution. Thus it is natural to align the reference system in the production plane with the beam direction. Then the basis in space consists of

$$\hat{\boldsymbol{n}}_\parallel = \hat{\boldsymbol{n}}_{e^-}, \quad \hat{\boldsymbol{n}}_\mathrm{N} = \frac{\hat{\boldsymbol{n}}_{e^-} \times \hat{\boldsymbol{n}}_t}{|\hat{\boldsymbol{n}}_{e^-} \times \hat{\boldsymbol{n}}_t|}, \quad \hat{\boldsymbol{n}}_\perp = \hat{\boldsymbol{n}}_\mathrm{N} \times \hat{\boldsymbol{n}}_\parallel, \tag{15}$$

and the basis in space–time can be chosen as follows:

$$n_{(0)} = (1,0,0,0)\,, \quad n_\parallel = \big(0, \hat{\boldsymbol{n}}_\parallel\big)\,, \quad n_\perp = (0, \hat{\boldsymbol{n}}_\perp)\,, \quad n_\mathrm{N} = (0, \hat{\boldsymbol{n}}_\mathrm{N})\,. \tag{16}$$

Neglecting terms of order β^2 the four momentum of the top quark reads

$$t^\mu \approx (m_t, \boldsymbol{p}_t) \;=\; (t_0, t_\parallel, t_\perp, t_\mathrm{N}) \;=\; (m_t, \mathrm{p}\cos\vartheta, \mathrm{p}\sin\vartheta, 0). \tag{17}$$

Assuming $V-A$ Lorentz structure of tbW vertex a closed formula for the moments $\langle nl \rangle$ has been derived in [4]. This formula can be written in the following way[5]

$$\langle nl \rangle = \mathrm{BR}(t \to bl\nu)\frac{1+2y+3y^2}{4(1+2y)}\left[n\cdot t + \frac{m_t}{3} n\cdot(\mathcal{P}+\delta\mathcal{P})\right], \tag{18}$$

with $y = \frac{m_W^2}{m_t^2}$. The four vector $\mathcal{P}$ characterizes the polarization of the top quark if rescattering corrections are neglected. It includes effects of interference between S and P partial waves proportional to β [4,11]

$$\begin{aligned}
\mathcal{P}_0(\boldsymbol{p}_t, E, \chi) &= \frac{\mathrm{p}}{m_t} C_\parallel^0(\chi)\cos\vartheta, && (19)\\
\mathcal{P}_\parallel(\boldsymbol{p}_t, E, \chi) &= C_\parallel^0(\chi) + C_\parallel^1(\chi)\,\varphi_\mathrm{R}(\mathrm{p}, E)\cos\vartheta, && (20)\\
\mathcal{P}_\perp(\boldsymbol{p}_t, E, \chi) &= C_\perp(\chi)\,\varphi_\mathrm{R}(\mathrm{p}, E)\sin\vartheta, && (21)\\
\mathcal{P}_\mathrm{N}(\boldsymbol{p}_t, E, \chi) &= C_\mathrm{N}(\chi)\varphi_\mathrm{I}(\mathrm{p}, E)\sin\vartheta\;, && (22)
\end{aligned}$$

[5] Eq.(18) is equivalent to Eq. (95) in [4] because our $\delta\mathcal{P}_0$ is not equal to zero.

where

$$\psi_{\mathrm{I}}(\mathrm{p}, E) \;=\; \Im\, \psi_2(\mathrm{p}, E), \tag{23}$$

$$\chi = (P_+ - P_-)/(1 - P_+ P_-), \tag{24}$$

and $P_\pm$ denote the longitudinal polarizations of positron and electron respectively. The coefficients $C_i(\chi)$ are known functions of the electroweak charges of electron and top quark, see Fig.8.

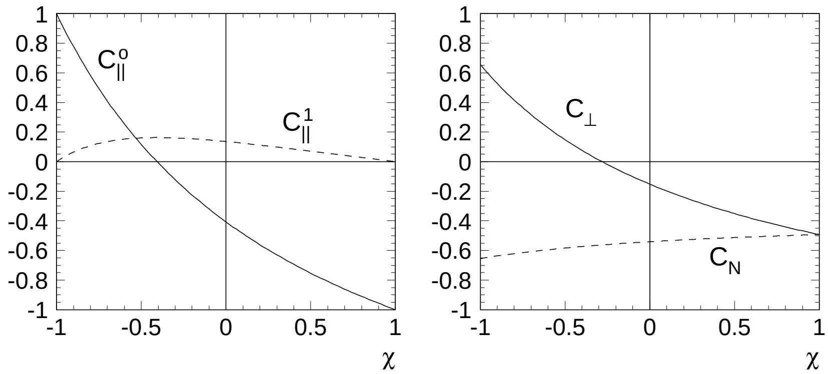

Fig. 8. The coefficients $C_i(\chi)$ for $\sqrt{s}/2 = m_t = 180$ GeV.

The functions $\varphi_{\mathrm{R}}(\mathrm{p}, E)$ and $\varphi_{\mathrm{I}}(\mathrm{p}, E)$ are proportional to the real and imaginary parts of the ratio of P and S wave Green functions $F(\mathrm{p}, E)$ and $G(\mathrm{p}, E)$ for the $t\bar{t}$ system.

The four vector $\delta\mathcal{P}$ includes effects of order α_s rescattering corrections. Neglecting all terms non-leading in β one obtains

$$\begin{aligned}
\delta\mathcal{P}_0 &= \frac{3y(1-y)}{(1+2y)(1+2y+3y^2)} C_\parallel^0 \psi_{\mathrm{R}}(\mathrm{p}, E)\cos\vartheta, && (25)\\
\delta\mathcal{P}_\parallel &= \left[\frac{2+3y-5y^2-12y^3}{(1+2y)(1+2y+3y^2)} - \kappa (C_\parallel^0)^2\right]\psi_{\mathrm{R}}(\mathrm{p}, E)\cos\vartheta \\
&\quad + \frac{1-4y+3y^2}{4(1+2y+3y^2)}(1-3\cos^2\vartheta) C_\parallel^0 \psi_3(\mathrm{p}, E), && (26)\\
\delta\mathcal{P}_\perp &= \frac{3(1-3y^2)}{2(1+2y+3y^2)}\psi_{\mathrm{R}}(\mathrm{p}, E)\sin\vartheta \\
&\quad - \frac{3(1-4y+3y^2)}{8(1+2y+3y^2)} C_\parallel^0 \psi_3(\mathrm{p}, E)\sin 2\vartheta, && (27)\\
\delta\mathcal{P}_{\mathrm{N}} &= 0, && (28)
\end{aligned}$$

where

$$\psi_3(\mathrm{p}, E) \; = \; \Im \int \frac{d^3k}{(2\pi)^3} V(|\boldsymbol{k} - \boldsymbol{p}_t|) \frac{G(\mathrm{k}, E)}{G(\mathrm{p}, E)} \frac{\arctan \frac{|\boldsymbol{k}-\boldsymbol{p}_t|}{\Gamma_t}}{|\boldsymbol{k} - \boldsymbol{p}_t|} \times \left[3 \left(\frac{\boldsymbol{p}_t \cdot (\boldsymbol{k} - \boldsymbol{p}_t)}{|\boldsymbol{p}_t||\boldsymbol{k} - \boldsymbol{p}_t|} \right)^2 - 1 \right] . \tag{29}$$

5. Summary

We have seen how the production and decay of top quarks in the $t\bar{t}$ threshold region are affected by the final-state interactions.

Studies of the decay of top quarks in the $t\bar{t}$ threshold region have just been started. First, an inclusive observable $\langle n\ell \rangle$ was calculated, which is sensitive to the top polarization. Recently, the differential decay distribution of l^+ in the top-quark semileptonic decay has been calculated. The final-state interactions modify the top-quark production cross section, the top-quark polarization vector, and also gives rise to a non-factorizable correction at the level of 10–20%.

We defined a new observable $\overline{A}(E_l, a)$ in the threshold region, which depends only on the decay process of free polarized top quarks. This quantity can be calculated (including e.g. anomalous top-quark decay vertices) without any knowledge of the bound-state effects or the final-state interactions, but assuming the highly polarized top quark samples expected in the $t\bar{t}$ threshold region. Further studies in this direction are demanded.

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