# SPIN EFFECTS IN VECTOR MESON PRODUCTION AT LEP* 

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Spin observables may reveal much deeper properties of non perturbative hadronic physics than unpolarized quantities. We discuss the polarization of hadrons produced in $e^{+} e^{-}$annihilation at LEP. We show how final state $q \bar{q}$ interactions may give origin to non zero values of the off-diagonal element $\rho_{1,-1}$ of the helicity density matrix of vector mesons: some predictions are given for $K^{*}, \phi, D^{*}$ and $B^{*}$ in agreement with recent OPAL data. We also discuss the relative amount of vector and pseudovector meson states and the probability of helicity zero vector states. Similar measurements in other processes are suggested.

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## 1. $\rho_{1,-1}(V)$ in the process $e^{-} e^{+} \rightarrow q \bar{q} \rightarrow V+X$

The spin properties of hadrons inclusively produced in high energy interactions are related to the fundamental properties of quarks and gluons and to their elementary interactions in a much more subtle way than unpolarized quantities. They test unusual basic dynamical properties and reveal how the usual hadronization models - successful in predicting unpolarized cross-sections - may not be adequate to describe spin effects, say the fragmentation of a polarized quark.

We consider here the spin properties of hadrons created at LEP. It was pointed out in Refs [1] and [2] that final state interactions between the $q$ and $\bar{q}$ produced in $e^{+} e^{-}$annihilations - usually neglected, but indeed necessary - might give origin to non zero spin observables which would otherwise

[^0]be forced to vanish: the off-diagonal element $\rho_{1,-1}$ of the helicity density matrix of vector mesons may be sizeably different from zero [1] due to a coherent fragmentation process which takes into account $q \bar{q}$ interactions. The incoherent fragmentation of a single independent quark leads instead to zero values for such off-diagonal elements.

We present predictions [3] for $\rho_{1,-1}$ of several vector mesons $V$ provided they are produced in two jet events, carry a large momentum or energy fraction $z=2 E_{V} / \sqrt{s}$, and have a small transverse momentum $p_{T}$ inside the jet. Our estimates are in agreement with the existing data and are crucially related both to the presence of final state interactions and to the Standard Model couplings of the elementary $e^{-} e^{+} \rightarrow q \bar{q}$ interaction.

The helicity density matrix of a hadron $h$ inclusively produced in the two jet event $e^{-} e^{+} \rightarrow q \bar{q} \rightarrow h+X$ can be written as $[1,2]$

$$
\begin{equation*}
\rho_{\lambda_{h} \lambda_{h}^{\prime}}(h)=\frac{1}{N_{h}} \sum_{q, X, \lambda_{X}, \lambda_{q}, \lambda_{\bar{q}}, \lambda_{q}^{\prime}, \lambda_{\bar{q}}^{\prime}} D_{\lambda_{h} \lambda_{X} ; \lambda_{q}, \lambda_{\bar{q}}} \rho_{\lambda_{q}, \lambda_{\bar{q}}, \lambda_{q}^{\prime}, \lambda_{\bar{q}}^{\prime}}(q \bar{q}) D_{\lambda_{h}^{\prime} \lambda_{X} ; ;_{q}^{\prime}, \lambda_{\bar{q}}^{\prime}}^{*}, \tag{1}
\end{equation*}
$$

where $\rho(q \bar{q})$ is the helicity density matrix of the $q \bar{q}$ state created in the annihilation of the unpolarized $e^{+}$and $e^{-}$,

$$
\begin{equation*}
\rho_{\lambda_{q}, \lambda_{\bar{q}} ; \lambda_{q}^{\prime}, \lambda_{\bar{q}}^{\prime}}(q \bar{q})=\frac{1}{4 N_{q \bar{q}}} \sum_{\lambda_{-}, \lambda_{+}} M_{\lambda_{q} \lambda_{\bar{q}} ; \lambda_{-} \lambda_{+}} M_{\lambda_{q^{\prime}}^{\prime} \lambda_{\bar{q}}, \lambda_{-} \lambda_{+}}^{*} . \tag{2}
\end{equation*}
$$

The M's are the helicity amplitudes for the $e^{-} e^{+} \rightarrow q \bar{q}$ process and the $D$ 's are the fragmentation amplitudes, i.e. the helicity amplitudes for the process $q \bar{q} \rightarrow h+X$; the $\sum_{X, \lambda_{X}}$ stands for the phase space integration and the sum over spins of all the unobserved particles, grouped into a state $X$. The normalization factors $N_{h}$ and $N_{q \bar{q}}$ are given by:
$N_{h}=\sum_{q, X ; \lambda_{h}, \lambda_{X}, \lambda_{q}, \lambda_{\bar{q}}, \lambda_{q}^{\prime}, \lambda_{\bar{q}}^{\prime}} D_{\lambda_{h} \lambda_{X} ; \lambda_{q}, \lambda_{\bar{q}}} \rho_{\lambda_{q}, \lambda_{\bar{q}} ; \lambda_{q}^{\prime}, \lambda_{\bar{q}}}(q \bar{q}) D_{\lambda_{h} \lambda_{X} ; \lambda_{q}^{\prime}, \lambda_{\bar{q}}^{\prime}}^{*}=\sum_{q} D_{q}^{h}$,
where $D_{q}^{h}$ is the usual fragmentation function of quark $q$ into hadron $h$, and

$$
\begin{equation*}
N_{q \bar{q}}=\frac{1}{4} \sum_{\lambda_{q}, \lambda_{\bar{q}} ; \lambda_{-}, \lambda_{+}}\left|M_{\lambda_{q} \lambda_{\bar{q}} ; \lambda_{-} \lambda_{+}}\right|^{2} . \tag{3}
\end{equation*}
$$

The helicity density matrix for the $q \bar{q}$ state can be computed in the Standard Model and its non zero elements are given by

$$
\begin{align*}
& \rho_{+-;+-}(q \bar{q})=1-\rho_{-+;-+}(q \bar{q}) \simeq \frac{1}{2} \frac{\left(g_{V}-g_{A}\right)_{q}^{2}}{\left(g_{V}^{2}+g_{A}^{2}\right)_{q}},  \tag{5}\\
& \rho_{+-;-+}(q \bar{q})=\rho_{-+;+-}^{*}(q \bar{q}) \simeq \frac{1}{2} \frac{\left(g_{V}^{2}-g_{A}^{2}\right)_{q}}{\left(g_{V}^{2}+g_{A}^{2}\right)_{q}} \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta} . \tag{6}
\end{align*}
$$

These expressions are simple but approximate and hold at the $Z_{0}$ pole, neglecting electromagnetic contributions, masses and terms proportional to $g_{V}^{l}$; the full correct expressions can be found in Ref. [3].

Notice that, inserting the values of the coupling constants

$$
\begin{array}{ll}
g_{V}^{u, c, t}=\frac{1}{2}-\frac{4}{3} \sin ^{2} \theta_{W}, & g_{A}^{u, c, t}=\frac{1}{2} \\
g_{V}^{d, s, b}=-\frac{1}{2}+\frac{2}{3} \sin ^{2} \theta_{W}, & g_{A}^{d, s, b}=-\frac{1}{2} \tag{7}
\end{array}
$$

one has

$$
\begin{align*}
& \rho_{+-;-+}(u \bar{u}, c \bar{c}, t \bar{t}) \simeq-0.36 \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta} \\
& \rho_{+-;-+}(d \bar{d}, s \bar{s}, b \bar{b}) \simeq-0.17 \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta} \tag{8}
\end{align*}
$$

Eq. (8) clearly shows the $\theta$ dependence of $\rho_{+-;-+}(q \bar{q})$. In case of pure electromagnetic interactions $\left(\sqrt{s} \ll M_{Z}\right)$ one has exactly:

$$
\begin{equation*}
\rho_{+-;-+}^{\gamma}(q \bar{q})=\frac{1}{2} \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta} . \tag{9}
\end{equation*}
$$

Notice that Eqs (8) and (9) have the same angular dependence, but a different sign for the coefficient in front, which is negative for the $Z$ contribution.

By using the above equations for $\rho(q \bar{q})$ into Eq. (1) one obtains the most general expression of $\rho(h)$ in terms of the $q \bar{q}$ spin state and the unknown fragmentation amplitudes.

Despite the ignorance of the fragmentation process some predictions can be made [3] by considering the production of hadrons almost collinear with the parent jet: the $q \bar{q} \rightarrow h+X$ fragmentation is then essentially a c.m. forward process and the unknown $D$ amplitudes must satisfy the angular momentum conservation relation [4]

$$
\begin{equation*}
D_{\lambda_{h} \lambda_{X} ; \lambda_{q}, \lambda_{\bar{q}}} \sim\left(\sin \frac{\theta_{h}}{2}\right)^{\left|\lambda_{h}-\lambda_{X}-\lambda_{q}+\lambda_{\bar{q}}\right|} \tag{10}
\end{equation*}
$$

where $\theta_{h}$ is the angle between the hadron momentum, $\boldsymbol{h}=z \boldsymbol{q}+\boldsymbol{p}_{T}$, and the quark momentum $\boldsymbol{q}$, that is

$$
\begin{equation*}
\sin \theta_{h} \simeq \frac{2 p_{T}}{z \sqrt{s}} \tag{11}
\end{equation*}
$$

The bilinear combinations of fragmentation amplitudes contributing to $\rho(h)$ are then not suppressed by powers of $\left(p_{T} / z \sqrt{s}\right)$ only if the exponent in

Eq. (10) is zero, which greatly reduces the number of relevant helicity configurations.

The fragmentation process is a parity conserving one and the fragmentation amplitudes must then also satisfy the forward parity relationship

$$
\begin{equation*}
D_{-\lambda_{h}-\lambda_{X} ;-+}=(-1)^{S_{h}+S_{X}+\lambda_{h}-\lambda_{X}} D_{\lambda_{h} \lambda_{X} ;+-} \tag{12}
\end{equation*}
$$

Before presenting analytical and numerical results for the coherent quark fragmentation let us remember that in case of incoherent single quark fragmentation Eq. (1) becomes

$$
\begin{equation*}
\rho_{\lambda_{h} \lambda_{h}^{\prime}}(h)=\frac{1}{N_{h}} \sum_{q, X, \lambda_{X}, \lambda_{q}, \lambda_{q}^{\prime}} D_{\lambda_{h} \lambda_{X} ; \lambda_{q}} \rho_{\lambda_{q} \lambda_{q}^{\prime}} D_{\lambda_{h} \lambda_{X} ; \lambda_{q}^{\prime}}^{*} \tag{13}
\end{equation*}
$$

where $\rho(q)$ is the quark $q$ helicity density matrix related to $\rho(q \bar{q})$ by

$$
\begin{equation*}
\rho_{\lambda_{q} \lambda_{q}^{\prime}}=\sum_{\lambda_{\bar{q}}} \rho_{\lambda_{q}, \lambda_{\bar{q}} ; \lambda_{q}^{\prime}, \lambda_{\bar{q}}}(q \bar{q}) . \tag{14}
\end{equation*}
$$

In such a case angular momentum conservation for the collinear quark fragmentation requires $\lambda_{q}=\lambda_{h}+\lambda_{X}$; the Standard Model computation of $\rho(q)$ gives only non zero diagonal terms $\left[\rho_{++}(q)=\rho_{+-;+-}(q \bar{q})\right.$ and $\rho_{--}(q)=$ $\left.\rho_{-+;-+}(q \bar{q})\right]$, and one ends up with the usual probabilistic expression

$$
\begin{equation*}
\rho_{\lambda_{h} \lambda_{h}}(h)=\frac{1}{N_{h}} \sum_{q, \lambda_{q}} \rho_{\lambda_{q} \lambda_{q}} D_{q, \lambda_{q}}^{h, \lambda_{h}}, \tag{15}
\end{equation*}
$$

where $D_{q, \lambda_{q}}^{h, \lambda_{h}}$ is the polarized fragmentation function of a $q$ with helicity $\lambda_{q}$ into a hadron $h$ with helicity $\lambda_{h}$. Off-diagonal elements of $\rho(h)$ are all zero.

$$
\text { 2. } e^{-} e^{+} \rightarrow B X\left(S_{B}=1 / 2, p_{T} / \sqrt{s} \rightarrow 0\right)
$$

Let us consider first the case in which $h$ is a spin $1 / 2$ baryon. It was shown in Ref. [2] that in such a case the coherent quark fragmentation only induces small corrections to the usual incoherent description

$$
\begin{align*}
\rho_{++}(B) & =\frac{1}{N_{B}} \sum_{q}\left[\rho_{+-;+-}(q \bar{q}) D_{q,+}^{B,+}+\rho_{-+;-+}(q \bar{q}) D_{q,-}^{B,+}\right]  \tag{16}\\
\rho_{+-}(B) & =\mathcal{O}\left[\left(\frac{p_{T}}{z \sqrt{s}}\right)\right] . \tag{17}
\end{align*}
$$

That is, the diagonal elements of $\rho(B)$ are the same as those given by the usual probabilistic formula (15), with small corrections of the order of $\left(p_{T} / z \sqrt{s}\right)^{2}$, while off-diagonal elements are of the order $\left(p_{T} / z \sqrt{s}\right)$ and vanish in the $p_{T} / \sqrt{s} \rightarrow 0$ limit.

The matrix elements of $\rho(B)$ are related to the longitudinal $\left(P_{z}\right)$ and transverse $\left(P_{y}\right)$ polarization of the baryon:

$$
\begin{equation*}
P_{z}=2 \rho_{++}-1, \quad P_{y}=-2 \operatorname{Im} \rho_{+-} . \tag{18}
\end{equation*}
$$

Some data are available on $\Lambda$ polarization, both longitudinal and transverse, from ALEPH Collaboration [5] and they do agree with the above equations. In particular the transverse polarization, at $\sqrt{s}=M_{Z}, p_{T} \simeq 0.5 \mathrm{GeV} / c$ and $z \simeq 0.5$ is indeed of the order $1 \%$, as expected from Eq. (17).

$$
\text { 3. } e^{-} e^{+} \rightarrow V X\left(S_{V}=1, p_{T} / \sqrt{s} \rightarrow 0\right)
$$

In case of final spin 1 vector mesons one has, always in the limit of small $p_{T}[1,3]$

$$
\begin{align*}
\rho_{00}(V) & =\frac{1}{N_{V}} \sum_{q} D_{q,+}^{V, 0},  \tag{19}\\
\rho_{11}(V) & =\frac{1}{N_{V}} \sum_{q}\left[\rho_{+-;+-}(q \bar{q}) D_{q,+}^{V, 1}+\rho_{-+;-+}(q \bar{q}) D_{q,-}^{V, 1}\right]  \tag{20}\\
\rho_{1,-1}(V) & =\frac{1}{N_{V}} \sum_{q, X} D_{10 ;+-} D_{-10 ;-+}^{*} \rho_{+--+-}(q \bar{q}) . \tag{21}
\end{align*}
$$

Again, the diagonal elements have the usual probabilistic expression; however, there is now an off-diagonal element, $\rho_{1,-1}$, which may survive even in the $p_{T} / \sqrt{s} \rightarrow 0$ limit. In the sequel we shall concentrate on it. Let us first notice that, in the collinear limit, one has

$$
\begin{align*}
& D_{q,+}^{V, 0}=\sum_{X}\left|D_{0-1 ;+-}\right|^{2}=D_{q,-}^{V, 0},  \tag{22}\\
& D_{q,+}^{V, 1}=\sum_{X}^{X,}\left|D_{10 ;+-}\right|^{2}=D_{q,-}^{V,-1},  \tag{23}\\
& D_{q,-}^{V, 1}=\sum_{X}\left|D_{12 ;-+}\right|^{2}=D_{q,+}^{V,-1}, \tag{24}
\end{align*}
$$

with $D_{q}^{V}=D_{q,+}^{V, 0}+D_{q,+}^{V, 1}+D_{q,+}^{V,-1}$ and $N_{V}=\sum_{q} D_{q}^{V}$. We also notice that the two fragmentation amplitudes appearing in Eq. (21) are related by parity and their product is always real. $\rho_{00}$ and $\rho_{1,-1}$ can be measured through the angular distribution of two body decays of $V$.

In order to give numerical estimates of $\rho_{1,-1}$ we make some plausible assumptions

$$
\begin{align*}
D_{q,-}^{h, 1} & =D_{q,+}^{h,-1}=0,  \tag{25}\\
D_{q,+}^{h, 0} & =\alpha_{q}^{V} D_{q,+}^{h, 1} \tag{26}
\end{align*}
$$

The first of these assumptions simply means that quarks with helicity $1 / 2$ $(-1 / 2)$ cannot fragment into vector mesons with helicity $-1(+1)$. This is true for valence quarks assuming vector meson wave functions with no orbital angular momentum, like in $\operatorname{SU}(6)$. The second assumption is also true in $\mathrm{SU}(6)$ with $\alpha_{q}^{V}=1 / 2$ for any valence $q$ and $V$. Rather than taking $\alpha_{q}^{V}=1 / 2$ we prefer to relate the value of $\alpha_{q}^{V}$ to the value of $\rho_{00}(V)$ which can be or has been measured. In fact, always in the $p_{T} \rightarrow 0$ limit, one has [3]

$$
\begin{equation*}
\rho_{00}(V)=\frac{\sum_{q} \alpha_{q}^{V} D_{q,+}^{h, 1}}{\sum_{q}\left(1+\alpha_{q}^{V}\right) D_{q,+}^{h, 1}} . \tag{27}
\end{equation*}
$$

If $\alpha_{q}^{V}$ is the same for all valence quarks in $V\left(\alpha_{q}^{V}=\alpha^{V}\right)$ one has, for the valence quark contribution:

$$
\begin{equation*}
\alpha^{V}=\frac{\rho_{00}(V)}{1-\rho_{00}(V)} . \tag{28}
\end{equation*}
$$

Finally, one obtains [3]

$$
\begin{equation*}
\rho_{1,-1}(V) \simeq\left[1-\rho_{0,0}(V)\right] \frac{\sum_{q} D_{q,+}^{V, 1} \rho_{+-;-+}(q \bar{q})}{\sum_{q} D_{q,+}^{V, 1}} . \tag{29}
\end{equation*}
$$

We shall now consider some specific cases in which we expect Eq. (29) to hold; let us remind once more that our conclusions apply to spin 1 vector mesons produced in $e^{-} e^{+} \rightarrow q \bar{q} \rightarrow V+X$ processes in the limit of small $p_{T}$ and large $z$, i.e., to vector mesons produced in two jet events ( $\left.e^{-} e^{+} \rightarrow q \bar{q}\right)$ and collinear with one of them $\left(p_{T}=0\right)$, which is the jet generated by a quark which is a valence quark for the observed vector meson (large $z$ ). These conditions should be met more easily in the production of heavy vector mesons.

One obtains [3]:

$$
\begin{align*}
\rho_{1,-1}\left(B^{*}\right) & \simeq\left[1-\rho_{0,0}\left(B^{*}\right)\right] \rho_{+-;-+}(b \bar{b}),  \tag{30}\\
\rho_{1,-1}\left(D^{*}\right) & \simeq\left[1-\rho_{0,0}\left(D^{*}\right)\right] \rho_{+-;-+}(c \bar{c}),  \tag{31}\\
\rho_{1,-1}(\phi) & \simeq\left[1-\rho_{0,0}(\phi)\right] \rho_{+-;-+}(s \bar{s}), \tag{32}
\end{align*}
$$

$$
\begin{align*}
\rho_{1,-1}(\rho) & \simeq \frac{1}{2}\left[1-\rho_{0,0}(\rho)\right]\left[\rho_{+-;-+}(u \bar{u})+\rho_{+-;-+}(d \bar{d})\right]  \tag{33}\\
\rho_{1,-1}\left(K^{* \pm}\right) & \simeq \frac{1}{2}\left[1-\rho_{0,0}\left(K^{* \pm}\right)\right]\left[\rho_{+-;-+}(u \bar{u})+\rho_{+-;-+}(s \bar{s})\right]  \tag{34}\\
\rho_{1,-1}\left(K^{* 0}\right) & \simeq \frac{1}{2}\left[1-\rho_{0,0}\left(K^{* 0}\right)\right]\left[\rho_{+-;-+}(d \bar{d})+\rho_{+-;-+}(s \bar{s})\right] . \tag{35}
\end{align*}
$$

Eqs (30)-(35) show how the value of $\rho_{1,-1}(V)$ are simply related to the off-diagonal helicity density matrix element $\rho_{+-;-+}(q \bar{q})$ of the $q \bar{q}$ pair created in the elementary $e^{-} e^{+} \rightarrow q \bar{q}$ process; such off-diagonal elements would not appear in the incoherent independent fragmentation of a single quark, yielding $\rho_{1,-1}(V)=0$.

By inserting into the above equations the value of $\rho_{00}$ when available [6] and the expressions of $\rho_{+-;-+}$, Eq. (8), one has:

$$
\begin{align*}
\rho_{1,-1}\left(B^{*}\right) & \simeq-(0.109 \pm 0.015) \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta}  \tag{36}\\
\rho_{1,-1}\left(D^{*}\right) & \simeq-(0.216 \pm 0.007) \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta}  \tag{37}\\
\rho_{1,-1}(\phi) & \simeq-(0.078 \pm 0.014) \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta}  \tag{38}\\
\rho_{1,-1}\left(K^{* 0}\right) & \simeq-0.170\left[1-\rho_{0,0}\left(K^{* 0}\right)\right] \frac{\sin ^{2} \theta}{1+\cos ^{2} \theta} . \tag{39}
\end{align*}
$$

Finally, in case one collects all meson produced at different angles in the full available $\theta$ range (say $\alpha<\theta<\pi-\alpha,|\cos \theta|<\cos \alpha$ ) an average should be taken in $\theta$, weighting the different values of $\rho_{1,-1}(\theta)$ with the cross-section for the $e^{-} e^{+} \rightarrow V+X$ process; this gives [3]:

$$
\begin{align*}
\left\langle\rho_{1,-1}\left(B^{*}\right)\right\rangle_{[\alpha, \pi-\alpha]} & \simeq-(0.109 \pm 0.015) \frac{3-\cos ^{2} \alpha}{3+\cos ^{2} \alpha}  \tag{40}\\
\left\langle\rho_{1,-1}\left(D^{*}\right)\right\rangle_{[\alpha, \pi-\alpha]} & \simeq-(0.216 \pm 0.007) \frac{3-\cos ^{2} \alpha}{3+\cos ^{2} \alpha}  \tag{41}\\
\left\langle\rho_{1,-1}(\phi)\right\rangle_{[\alpha, \pi-\alpha]} & \simeq-(0.078 \pm 0.014) \frac{3-\cos ^{2} \alpha}{3+\cos ^{2} \alpha}  \tag{42}\\
\left\langle\rho_{1,-1}\left(K^{* 0}\right)\right\rangle_{[\alpha, \pi-\alpha]} & \simeq-0.170\left[1-\rho_{0,0}\left(K^{*}\right)\right] \frac{3-\cos ^{2} \alpha}{3+\cos ^{2} \alpha} \tag{43}
\end{align*}
$$

These results have to be compared with data [6]

$$
\begin{array}{rllll}
\rho_{1,-1}\left(D^{*}\right) & =-0.039 \pm 0.016 & \text { for } & z>0.5 & \\
\cos \alpha=0.9,(44) \\
\rho_{1,-1}(\phi) & =-0.110 \pm 0.070 & \text { for } & z>0.7 & \\
\cos \alpha=0.9,(45) \\
\rho_{1,-1}\left(K^{* 0}\right) & =-0.090 \pm 0.030 & \text { for } & z>0.3 & \\
\cos \alpha=0.9
\end{array}
$$

which shows a good qualitative agreement with the theoretical predictions. We notice that while the mere fact that $\rho_{1,-1}$ differs from zero is due to a coherent fragmentation of the $q \bar{q}$ pair, the actual numerical values depend on the Standard Model coupling constants; for example, $\rho_{1,-1}$ would be positive at smaller energies, at which the one gamma exchange dominates, while it is negative at LEP energy where the one $Z$ exchange dominates. $\rho_{1,-1}$ has also a peculiar dependence on the meson production angle, being small at small and large angles and maximum at $\theta=\pi / 2$. Such angular dependence has been tested in case of $K^{* 0}$ production and indeed one has [6], in agreement with Eqs (35) and (8),

$$
\begin{equation*}
\left[\frac{\rho_{1,-1}}{1-\rho_{00}}\right]_{|\cos \theta|<0.5} \cdot\left[\frac{\rho_{1,-1}}{1-\rho_{00}}\right]_{|\cos \theta|>0.5}^{-1}=1.5 \pm 0.7 . \tag{47}
\end{equation*}
$$

## 4. Diagonal elements of $\rho(V)$ and $P_{V} \equiv V /(V+P)$

Let us consider now the diagonal element $\rho_{00}(V)$ - for which the probabilistic interpretation, Eq. (19) holds - together with the production of pseudoscalar mesons; that is, we consider the production of the pseudoscalar mesons $P=K, D, B$ and the corresponding vector mesons $V=K^{*}, D^{*}, B^{*}$ : data are available on $\rho_{00}(V)$ and the ratio of vector to vector + pseudoscalar mesons, $P_{V} \equiv V /(V+P)[6,7]$.

We denote by $P_{S}^{\lambda}$ the probability that the fragmenting quark produces a meson with spin $S$ and helicity $\lambda$ and consider only the production of vector and pseudovector mesons. Then we have $[8,9]$ :

$$
\begin{equation*}
\rho_{00}(V)=\frac{P_{1}^{0}}{P_{1}^{ \pm 1}+P_{1}^{0}}, \quad P_{V}=P_{1}^{ \pm 1}+P_{1}^{0} \tag{48}
\end{equation*}
$$

with $P_{1}^{ \pm 1}=P_{1}^{1}+P_{1}^{-1}$ and $P_{1}^{ \pm 1}+P_{1}^{0}+P_{0}^{0}=1$.
In terms of the fragmentation functions this reads:

$$
\begin{equation*}
P_{0}^{0}=\frac{D_{q}^{P}}{D_{q}^{V}+D_{q}^{P}}, \quad P_{1}^{0}=\frac{D_{q}^{V, 0}}{D_{q}^{V}+D_{q}^{P}}, \quad P_{1}^{ \pm 1}=\frac{D_{q}^{V, 1}+D_{q}^{V,-1}}{D_{q}^{V}+D_{q}^{P}} . \tag{49}
\end{equation*}
$$

Notice that, by parity invariance, the above quantities are independent of the quark helicity.

Statistical spin counting would give

$$
\begin{equation*}
P_{1}^{ \pm 1}=0.5, \quad P_{1}^{0}=0.25, \quad P_{0}^{0}=0.25 \tag{50}
\end{equation*}
$$

that is

$$
\begin{equation*}
\rho_{00}=\frac{1}{3}, \quad P_{V}=\frac{3}{4}, \tag{51}
\end{equation*}
$$

so that the vector meson alignment is zero:

$$
\begin{equation*}
A=\frac{1}{2}\left(3 \rho_{00}-1\right)=0 \tag{52}
\end{equation*}
$$

From data [7] one obtains [9]

$$
\begin{array}{lll}
P_{1}^{ \pm 1}\left(K^{*}\right)=0.34 \pm 0.06, & P_{1}^{0}\left(K^{*}\right)=0.41 \pm 0.07, & P_{0}^{0}(K)=0.25 \pm 0.10 \\
P_{1}^{ \pm 1}\left(D^{*}\right)=0.34 \pm 0.04, & P_{1}^{0}\left(D^{*}\right)=0.23 \pm 0.03, & P_{0}^{0}(D)=0.43 \pm 0.06 \\
P_{1}^{ \pm 1}\left(B^{*}\right)=0.49 \pm 0.09, & P_{1}^{0}\left(B^{*}\right)=0.27 \pm 0.08, & P_{0}^{0}(B)=0.24 \pm 0.09 \tag{54}
\end{array}
$$

The simultaneous measurements of the ratio of vector to vector $+\mathrm{pseu}-$ dovector mesons and $\rho_{00}(V)$ supply basic information on the fragmentation of quarks which does not depend on the helicity of the quark, but on the spin and helicity of the final meson. The data available for $K, D$ and $B$ mesons show clear deviations from simple statistical spin counting; such information could be of crucial importance for the correct formulation of quark fragmentation Monte Carlo programs, which at the moment widely assume simple relative statistical probabilities.

Let us consider our results, Eqs (53)-(55). For strange mesons the data agree with spin counting in the amount of $K$ versus $K^{*}$, but, among vector mesons, helicity zero states seem to be favoured; these are in absolute the most abundantly produced, $P_{1}^{0}\left(K^{*}\right)=0.41$. For charmed mesons results differ from the spin counting values (50), suggesting a prevalence of pseudoscalar states, $P_{0}^{0}(D)=0.43$, and, among vector mesons, of helicity 0 states. The heavy $b$-mesons, instead, are produced in good agreement with statistical spin counting rules, as one expects.

## 5. $\rho_{1,-1}(V)$ in other processes and conclusions

The results discussed here are encouraging; indeed measurements of offdiagonal and diagonal elements of $\rho(V)$ give valuable information on the hadronization process and test the underlying elementary dynamics. It would be very helpful to have more and more detailed data, possibly with a selection of final hadrons with the required features for our results to hold.

It would be interesting to test the coherent fragmentation of quarks in other processes [10], like $\gamma \gamma \rightarrow V X, p p \rightarrow D^{*} X$ and $\gamma p \rightarrow V X$ or $\gamma^{*} p \rightarrow$ $V X$. The first two processes are similar to $e^{-} e^{+} \rightarrow V X$ in that a $q \bar{q}$ pair is created which then fragments coherently into the observed vector meson; one assumes that the dominating elementary process in $p p \rightarrow D^{*} X$ is $g g \rightarrow c \bar{c}$.

In both these cases one has for $\rho_{+-;-+}(q \bar{q})$ the same value as in Eq. (9), so that one expects a positive value of $\rho_{1,-1}(V)$.

In the case of the real photo-production of vector mesons the quark fragmentation is in general a more complicated interaction of the struck quark with the remnants of the proton and it might be more difficult to obtain numerical predictions. However, if one observes $D^{*}$ mesons one can assume or select kinematical regions for which the underlying elementary interaction is $\gamma g \rightarrow c \bar{c}$ : again, one would have the same $\rho_{+-;-+}(c \bar{c})$ as in Eq. (9), and one would expect a positive value of $\rho_{1,-1}\left(D^{*}\right)$. Similarly for the production of $\phi$ or $B^{*}$.

The production of vector mesons like $D^{*}$ or $B^{*}$ in DIS is even more interesting; the polarization of the virtual photon depends on $x$ and $Q^{2}$ and so does the value of $\rho_{+-;-+}(c \bar{c})$ [10]. An eventual dependence of $\rho_{1,-1}\left(D^{*}\right)$ on $x$ and $Q^{2}$ would then be an unambigous test of the hadronization mechanism and the elementary interaction. A similar situation can be obtained by considering $e^{+} e^{-}$or $\gamma \gamma$ processes with polarized initial particles: in such cases the value of $\rho_{+-;-+}(q \bar{q})$ strongly depends on the initial spin states and should change the measured value of $\rho_{1,-1}(V)$ [11].

Similar considerations hold for the diagonal elements of $\rho(V)$ and for $P_{V}$; the degree of universality of quark fragmentation could be tested by studying these same quantities in other processes, like the ones mentioned above. It would also be interesting to compare data on the production of spin $1 / 2$ and spin $3 / 2$ baryons.

To conclude, some non perturbative aspects of strong interactions can only be tackled by gathering experimental information and looking for patterns and regularities which might allow the formulation of correct phenomenological models. Spin dependent quantities are still in a first stage of consideration and development, so that even qualitative studies are meaningful; for example, it would indeed be interesting to perform the simple tests of coherent fragmentation effects suggested here.

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