

DISCRETE MODEL FOR THE INTERFACE
EVOLUTION IN A $2d$ SPACE WITH SOME
WELL-DEFINED SET OF
ANNIHILATION–CREATION RULES*

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A simple discrete model for the evolution of the interface (or front) in a $2d$ (square) lattice, based on a complete as well as natural set of a few stochastic rules, has been examined. The interface is initially assumed to be a vertical straight-line which is made of elementary unit pieces called further particles. Once one of the particles is chosen at random it is pushed either left or right, drawing two new horizontal units (particles). Then the process continues to proceed into both main directions, following the rules (reversible and irreversible) that generally rely on creating and annihilating at random the vertical as well as horizontal particles. The scaling properties of the system have been analyzed, recognizing the front as a subdiffusive (anomalous) macromolecular chain or “lattice animal”, with a few residual parts, on the one hand, and as a rough surface with overhangs in a $1 + 1$ -space, on the other. In the former, it turns out that the current length of the front scales more or less like a polymeric chain under an attractive (qualitatively: “suppressing”) potential field, while in the latter it seems at a first glance that the problem may fall into an universality class characteristic of the nonlinear nonconservative dynamics with possibly nonconservative noise, exemplified by some dynamics of rough surfaces or interfaces.

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1. Introduction

An expanding interest in kinetics as well as scaling properties of randomly evolving interfaces (surfaces) or fronts, being practically known in physics and chemistry as well as biology as boundaries, "lines of phase separation", (bio)membranes, active zones, (slowly) percolating flows, etc., is of major interest to material scientists, physicists, and chemists but recently also attracts interest of technologists or chemical processing engineers [1]. The objects in question are mostly known as grain (or domain) boundaries [2], fluctuating interfaces [3], spreading or invading fronts and/or crystal surfaces [4], etc. They may represent a behaviour of such nonequilibrium phenomena like *e.g.*, grain coalescence in alloys, fracture propagation in solid materials, damage spreading in mechanical, electrical as well as biological systems, wetting, roughening, evolution of the crystallization front, growth and volume increase of bubbles or even invasion of bacterial colonies [4]. The kinetic behaviour of many of them still attracts some effort of researchers in many disciplines, and is a subject of careful considerations. In particular, scaling properties which always assume some invariance of the system properties under a scaling rule (or, a set of scaling rules) are of interest, mostly to physicists which, under the self-similarity assumption(s), try to conclude on quite general static as well as dynamic properties of the system under study [5].

There is neither commonly accepted nor general theory which is able to deal successfully with all of the phenomena listed above. This is due to the fact that each of the complex systems mentioned has its own characteristics that may be quite different from the characteristics of another system "chosen from the list". In general, two main directions are observed when dealing with them:

1. Extensive Monte Carlo (MC) simulations; cellular automata (CA) as well as molecular dynamics (MD) are also used (it is more fashionable to use CA in last years) [6].
2. Analytical studies on dynamics or kinetics of some evolving fronts (*e.g.*, Kardar–Parisi–Zhang (KPZ) equation; directed polymers; Burgers equation; master equation for the diffusive and/or convective front propagation; Langevin equation for the evolution of a fluctuating chemical wave front [7], etc.; ratchet concept [8]; percolation systems; note that the concept of fractality of any type is frequently used for description of the above mentioned phenomena; *cf.* [9] for a general overview; also, a scaling concept borrowed from polymer physics is very much pronounced here [5].

In this study, we wish to explore a problem of the evolution of a random front on a $2d$ lattice, based on some well-known physical tool, *i.e.*, the scaling approach, but invoking two kinetic-geometrical concepts which, at a first look, seem differ apparently from one another (one will probably see that it would be a certain novelty in studying such phenomena).

The first concept comes from polymer physics and is called: a random chain concept. To be more concrete, we have in mind a macromolecule which grows under a potential (typically, the Lennard–Jones-like potential) field, where, however, its attractive (or: “supressing”; say also: “contracting”) part dominates which results in a scaling law, like $length \propto time^{1/3}$, in a long times (or large beads number) regime [10]. In our case the role of the attractive part of the potential field is played by some well-defined set of annihilation-creation rules, where the annihilation part takes undoubtedly favour in the course of time; equivalently, one can state that the sub-set of the creation rules applied in this study would mimic the repulsive (hard core) part of the potential [11].

The second concept, in turn, comes originally from the materials or surface sciences, rather, and relies on exploiting the dynamics of possibly rough surfaces or nonsolid-solid interfaces, or even to some extent, interfaces between two phases of the same type, like liquid-liquid or solid-solid interfaces. In such a situation, a particular attention has to be paid to how does the surface width (designated by w) scale with time t , in a short times regime, or, which is a scaling law width versus the system (lattice) size, say L , but around the long times limit or above a saturation time? Choosing the line $x = 0$ that means the left border of the square lattice as a referenced line, we are able to calculate the height as well as the width of the surface which, in our opinion, possesses quite realistic geometrical characteristics, with some overhangs, islands, spikes (or cusps), “necks”, etc. (*cf.* Fig. 1).

By performing some extensive computer simulations (on a square lattice of 256×256 , with the number of time or evolution steps of 2.5×10^6 ; the number of simulations was 82 though a certain number of some “small-scale” trails has also been done) we have been able to pick up the scaling exponent (denoted further by $\bar{\nu}$) assigned to the first concept to be quite close to $1/3$ (although, it is also very close to $1/4$, but the value has a less popular or physically clear interpretation as the former one; for instance, in the anomalous random walk (RW) concept it corresponds to a RW with the anomalous RW exponent $d_w = 4$; for the normal RW $d_w = 2$, and this case is usually related to the Gaussian chain or to a RW or Brownian motion in an isotropic space, whereas the anomaly due to $d_w = 4$ is very often assigned to the so-called RW with obstacles or realized in an anisotropic space; this anisotropy or some kind of “inhomogeneity” in the system, may arise from either geometrical, *e.g.* steric, hindrances, or other factors, like macromolecule-solvent

interactions; *cf.* [9, 11]). As for the scaling exponents characteristic of our system as a rough surface, we have observed, by applying the essential sampling method [12], some crude tendencies of the system to fall into the category (universality class) of dynamical systems with both nonconservative dynamics as well as noise, like the KPZ system characteristic of rough surfaces, directed polymers or complex (turbulent) hydrodynamical systems [1]; this, however, cannot be understood as a firm result, but as a preliminary report on first observations and tendencies, rather. Because, there exists a formal correspondence between the Langevin approach (at least, for the Edwards–Wilkinson (EW) model for rough surfaces) to interface statistics and the diffusion-limited reactions [13], our problem can also be seen in such terms, like $A + A^- > 0$ or more generally $A + B^- > 0$ reactions (for species of type A or B), which are probably the most primitive as well as nontrivial reactions in our system [14].

2. Description of the problem

2.1. In terms of anomalous Random Walk (RW)

There is no doubts that from the beginning the front under study proceeds a random walk. It means that because the object is at $t = 0$ a vertical straight line consisting of a number (256 in our computer experiment) of particles (or, unit bars; see above), we simply choose at random one of the particles and push it also at random either left or right. After this step two horizontal particles are created (a creation act is being proceeded) so that, in the next time step, there is also a possibility to choose (always, at random) one of the two horizontal particles though the probability of choosing one of them is obviously small at this stage (it is equal to $1/129$, in our case). Then, the RW relies on pushing either vertical or horizontal particles, that means, on creating some new necessary particles ("bridges") after the next evolution step. The creation step ensures to keep the front more or less continuous or compact during all successive motion steps, though some overhangs, neighboring islands, say, residual parts, can emerge even after the several evolution steps. There are, however, some necessary (again!) annihilation steps associated with each creation act. They are thought of to allow a physically realistic character of the front (*e.g.*, to avoid to create some artificial internal structure of the front, and towards having a certain advancing possibly nonartificial as well as natural crystallization front). During the successive simulation steps we are always looking for how does the current overall front length l behave (no matter, whether there is a single trajectory or a certain number of them *i.e.*, when some residual sub-trajectories appear; *cf.* Fig. 1 for details) in the course of time t . Since

we observe a power-law dependence, like

$$l \propto t^\nu, \quad (1)$$

(for $t > 0$, and typically $t \gg 1$) where ν is a critical (scaling) exponent of the RW, we are interested in measuring the value of ν .

2.2. In terms of rough interfaces

There is also possible to investigate some kinetic or dynamic properties of the object in question in terms of rough surfaces because a quite realistic surface or interface is created on the square lattice (Fig. 1). Now, one can have a 1+1-dimensional problem because we see that in such a case dynamic surface properties have to be investigated considering a straight line as a reference line (a basal plane or so). Let us then choose the left border of the square lattice as the basal plane, *i.e.* a plane of $x = 0$ in our simulations. This having in mind, we wish to examine some scaling rules, like

$$w \propto t^\beta \quad (2)$$

(for $t \ll t_s$; note that w corresponds to the surface width which will be specified below), where β is the growth exponent of the process and t_s (usually, much greater than zero) stands for a measure of the stationarity (saturation) of the process studied that has to be chosen experimentally [1], or

$$w_s \propto L^\alpha \quad (3)$$

(for $t \gg t_s$; note that w_s corresponds to the saturation surface width), where α is the roughness exponent of the surface and L stands for the size of the lattice ($L = 256$, in our case), or finally

$$t_s \propto L^\gamma, \quad (4)$$

where γ represents the so-called dynamic exponent. Notice that the equality $\gamma = \alpha/\beta$ should hold (see Ref. [1], again), in this case.

3. Specification of the front

3.1. Algorithm

The following algorithm is explored in our simulations:

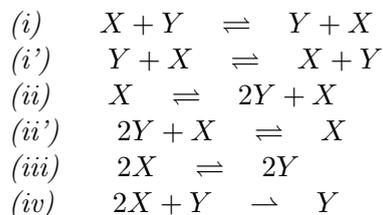
- 0) initialize a configuration of Y -units by inserting a vertical straight-line
- 1) draw a X or Y -unit (particle) by a uniform random number

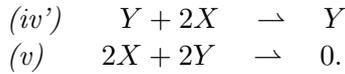
- 2) choose the direction of shifting of X or Y
- 3) look for the place into which the particle has to be shifted
- 4) if it is an occupied place, follow the corresponding irreversible rule after examining the vicinity, if not, choose one of the corresponding reversible rules in this same way (after examining the vicinity, too)
- 5) re-calculate the length of the front and other quantities estimated and determine the coordinates of the new units created, and delete the coordinates of the units removed
- 6) actualize any possible counters
- 7) jump back to point 1) unless a stop command appears
- 8) finish the process by performing some calculations and/or drawing pictures.

3.2. Rules of random motion

The front consists of elementary X - and Y -units or particles (initially only Y -units constitute the front). By X -particle (Y -particle) we understand a horizontal (a vertical) elementary piece of the front which is of length of the lattice unit. Two basic properties of the particles are (obviously, inspite constituting the whole front) that (a) they can be created and/or annihilated during the front evolution (b) because the front evolves (*i.e.*, changes its position in space and time, but in this study we are exclusively interested in its temporal behaviour), they are shifted at random under the only restriction that for X -particles the either shifting possibility is in Y -direction (upwards or downwards), and *vice versa*, Y -particles can exclusively be shifted in X -direction (left or right); in other words, the principle of perpendicular shifts is applied during the whole process (it is important to mention that the only boundary conditions imposed are the periodic boundary conditions).

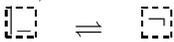
After the first move two X -units are created. Then the motion of the front relies exclusively on creation and/or annihilation of small groups of X - and Y -s step by step. We have roughly assumed 8 basic “moving rules”, *i.e.* shifts of X or Y -units into their perpendicular directions. They may be either onto another unit or into an empty spot on a square lattice, and they may generally be listed as:



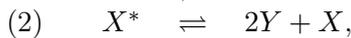


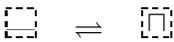
We have 8 basic *a priori* rules, 4 onto an empty and 4 onto an occupied (by another particle) spot on the lattice, respectively, but among them there are most probably (see discussion below) 5 of reversible type (from (i) to (iii), see above) and 3 of irreversible type (the remaining ones), and this way it was plugged in our simulation process. We see, however, that the 8 rules implemented are not independent of each other, and there are only 5 distinctly different rules which are at most very weakly related with one another. It is worth emphasizing here that the rules appear to be natural for the square lattice because they rely on some “conceptual destruction” of a square following the above written schemes, or, more generally, on taking off the sides X and Y of a rectangle). Notice that basically the system includes 50 percent of the total number of rules that are of annihilative nature, 37.5 percent of rules which have no account for the annihilation-creation process of the interface (they can be called neutral, like $Y + X \rightleftharpoons X + Y$), and only 12.5 percent of rules that represent a pure creation process (see above). Because of the 4 elementary sub-directions in $2d$ lattice, however, we have 32 elementary shifts in the system. For explaining the rules in a more precise way, let us designate a particle to be shifted in X -(Y) direction by Y^* (X^*). Now, one can draw the 5 most independent and unrepeatable rules schematically or even in a picturesque way as follows:



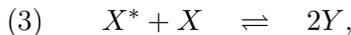
or in a diagram-like form: 

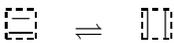
(comment on a strong or direct reversibility: one can take the same particle and move it back; the rule corresponds to a neutral shift);



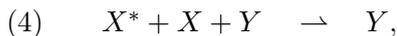
or in a diagram-like form: 

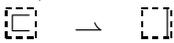
(comment on strong or direct reversibility: again, one can take the same particle and move it back; the rule represents a creative shift);



or in a diagram-like form: 

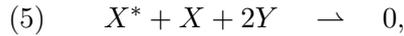
(comment on a weak or rather indirect reversibility: it is for the first time irreversible since one cannot take the same particle; nevertheless, there exists the reverse reaction $2Y \rightarrow 2X$, and hence it is reversible; note that the rule is neither annihilative nor creative);



or in a diagram-like form: 

(comment on an undoubtful (so to say!) irreversibility: it is not reversible

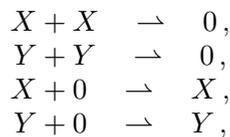
but there is a reverse reaction, $Y^* \rightarrow X + Y + X$, which has already its counterpart as the rule 2, and therefore is really irreversible; the rule commented is not balanced by a back reaction: in reality, it is a kind of surface tension which means this physical quantity that plays always a profound role, *e.g.* in the crystal growth [15] or formation of bubbles, foams or many colloidal assemblies; the rule looks fairly annihilative);



or in a diagram-like form:  \rightarrow 

(comment on the irreversibility: the rule is irreversible as well as very annihilative; note that the open dot or small circle from the right-hand side of the rule means “nothing” or a state which is physically irrelevant).

It is probably sufficient to say that by the reversibility we have meant here a physical state which is easy to restore from one of the preceding states (possibly, from the preceding one). If it is not the case, we call the rule irreversible. One has to realize that above in the diagrams, for simplicity, shifts upwards of the horizontal X -particle (signed with $*$) are presented, and dashed sides of the elementary cell (or square) are the empty sides while the sides drawn with both dashed and continuous lines are the occupied lattice places or simply X - or Y -particles. Note that, basically, the most elementary “chemical reactions” which cause some changes in the front structure are the following:



and in this sense the system can probably be treated using some superficial findings of the diffusion-limited reaction approach [13,14] (yet, we have applied the 8 first rules stated in this sub-chapter, exclusively!). It is worth stating that the two first of them possess a nontrivial anomalous chemical reaction kinetics, with the critical dimension $d_w = 2$ [14], whereas the two remaining are of standard (say, Debye) kinetics, either. One should be also aware that not too much is known *a priori* about the chemical reaction rates of all the reactions mentioned in this sub-chapter, and at a first look one can predict rather more qualitatively than quantitatively which directions for the reversible reactions are the most probable or whether a certain direction of an irreversible reaction is privileged or rather hard to realize. The general informations about whether the reaction is of annihilative or of creative, or even of neutral nature, can surely help in this case (see some discussion above).

3.3. The measured quantities

According to what has been written in Section 2, the measured quantities are as follows:

$$l \equiv l(t) = \sum_i^{N_m} (X_i + Y_i), \quad (5)$$

where N_m stands for a number of runs while $X_i \equiv X_i(t)$ as well as $Y_i \equiv Y_i(t)$ are the numbers of X - and Y -s got for i -th run, respectively; in fact, we are interested in the l versus t behaviour averaged over 82 simulations performed so that we have picked up an averaged scaling exponent $\bar{\nu}$. The next quantity is

$$w \equiv w(L, t) = \sqrt{\frac{1}{L} \sum_j^L [h(j, t) - \bar{h}(t)]^2}, \quad (6)$$

where

$$\bar{h}(t) = \frac{1}{L} \sum_j^L h(j, t), \quad (7)$$

and where $h(j, t)$ stands for the current height measured from the reference level (since, *cf.* Fig. 1, we may have this definition ambiguous, because of the fact that more than one particle can correspond to a single j so that we have applied here the concept of ‘‘inertia’’ and we have weighted, for each j , like for the traditionally measured radius of giration [5, 10], the values of $h(j, t)$, just to ensure our estimations unambiguous; see Ref. [1]). Last but not least, let us state clearly that the saturation time t_s was estimated directly from the computer experiment(s) by observing at which time step the first saturation takes place (from this time step, it was rigorously checked whether the current front length l changes by 20 percent or less).

4. Numerical results

First, let us point out that for visual demonstration of the process examined, we offer three snapshots shown in Fig. 1. Here, Fig. 1a presents an initial stage of the front after only 108 time steps while Figs. 1b-c display some intermediate and late stages of the front evolution, *i.e.*, after 2045 and 3994 time steps, respectively (for a proper visualization, we have simulated the process on the lattice 20×20 up to 4000 time steps).

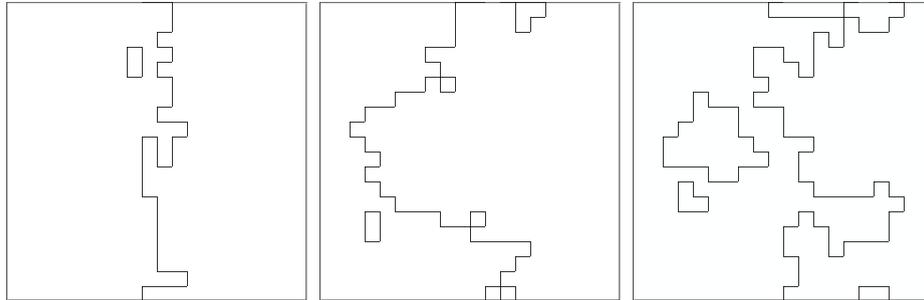


Fig. 1. A snapshot of the interface on the 20×20 square lattice realized up to 4000 time steps: a — an initial stage for 108 time steps; b — an intermediate stage for 2045 time steps; c — a late stage for 3994 time steps; note that during later stages of the simulation some residual subdiffusive trajectories emerge.

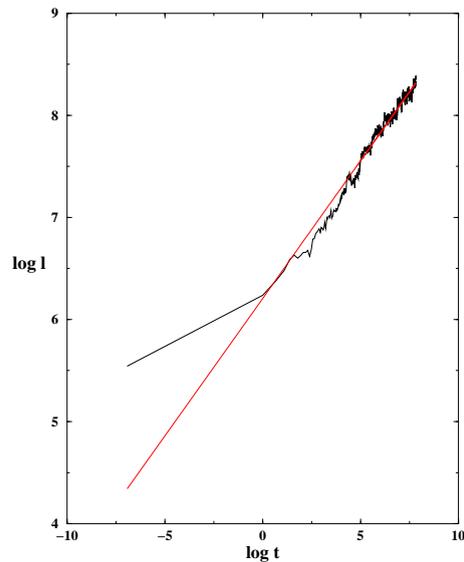


Fig. 2. A log-log length l versus time t dependence with the linear regression fitting for the slope of ca. 0.27 within a confidence level of about 98 percent for the simulation performed on 256×256 square lattice up to 2.500.000 time steps; here, one of the bests fits is presented.

In Fig. 2, one has the log-log length l versus time t dependence, with some best linear regression fitting within the confidence level of ca. 98 percent, and with the slope about 0.27. In Fig. 3, one observes the log-log length l versus time t dependence, with some typical linear regression fitting within the confidence level of ca. 96 percent, and with the slope about 0.32. In Fig. 4, in turn, one sees the log-log length l versus time t

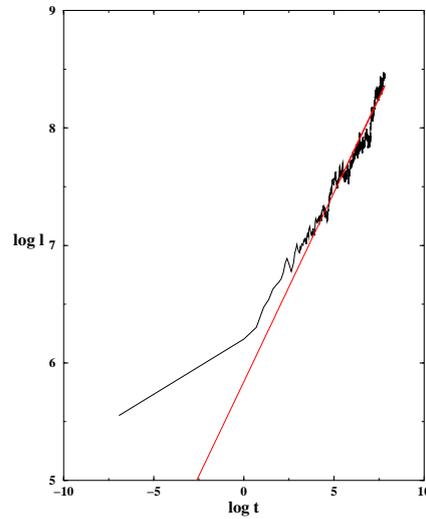


Fig. 3. A log-log length l versus time t dependence with the linear regression fitting for the slope of ca. 0.32 within a confidence level of about 96 percent for the simulation performed on 256×256 square lattice up to 2.500.000 time steps; here, one of the typical or intermediate fits is presented.

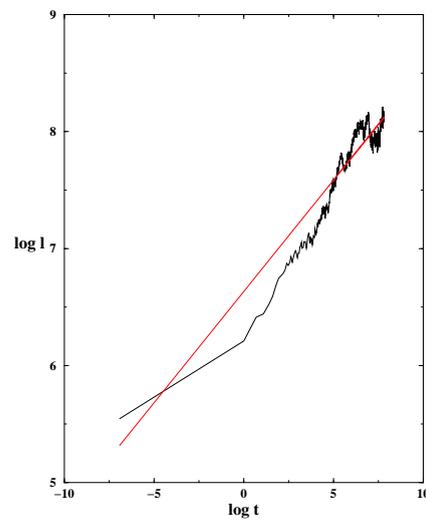


Fig. 4. A log-log length l versus time t dependence with the linear regression fitting for the slope of ca. 0.19 within a confidence level of about 85.5 percent for the simulation performed on 256×256 square lattice up to 2.500.000 time steps; here, one of the worst or “misleading” fits is presented (but, fortunately, they are in a very distinct minority).

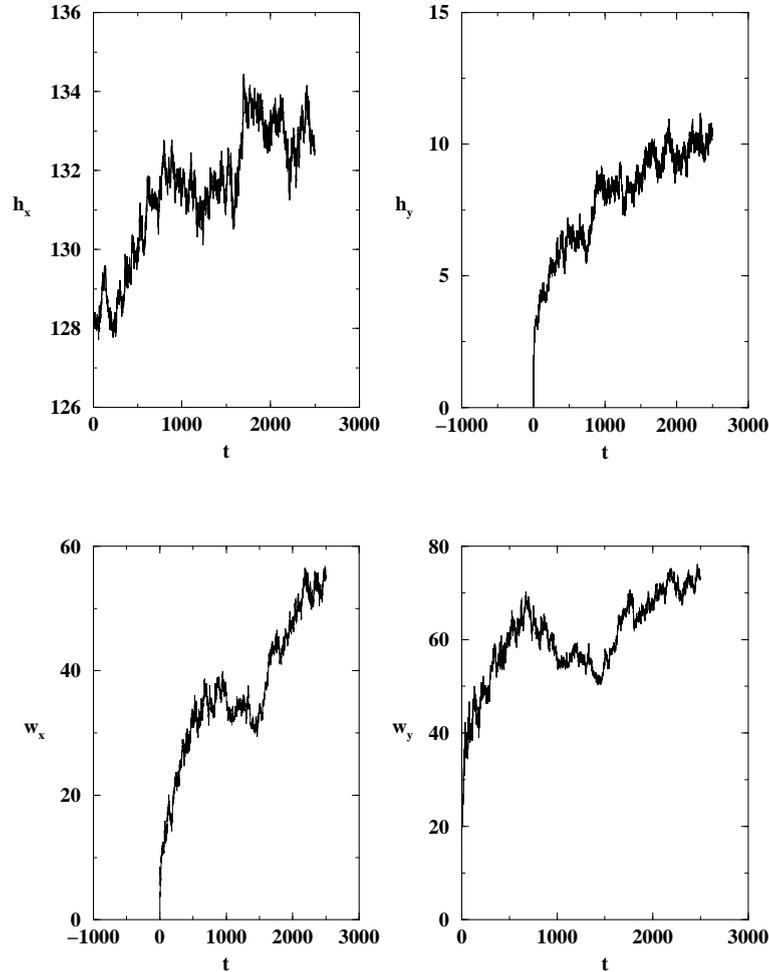


Fig. 5. The rough surface characteristics of the system is presented as a typical realization in which the heights as well as the widths of the surface, measured in both main perpendicular directions x and y , are emphasized; note fairly irregular character of the curves, and realize that a first stationary (or, saturation) time is “suspected” to be here below 1000.000 time steps (all the values of the x -axis must be multiplied by 1000, in reality); obviously, the saturation time t_s (see chaps. 3-4) that we have used must be a value somehow averaged over 82 computer simulations, and we have picked up it to be about 928.000 elementary time steps.

dependence, with one of the worst linear regression fittings got, *i.e.*, within the confidence level of ca. 85.5 percent, and with the slope about 0.19. As to the estimation of $\bar{\nu}$ (see above) we have picked up that $\bar{\nu} = 0.27 \pm 0.037$, within the averaged (over 82 simulations) confidence level about 0.96 ± 0.02 ,

and the averaged intercept value [12] ca. 6.16 ± 0.2 ; the fluctuations of all the averages estimated were found to be less than 0.0016. This is the main numerical result of our work that shows up that we simply have to do with an anomalous RW process, which means, that its purely Brownian character is destroyed (or suppressed) by the annihilation-creation rules applied. In Fig. 5, we show the rough surface characteristics of the system, just by presenting a typical realization in which the heights as well as the widths (see above) of the surface, measured in both perpendicular main directions x and y , are presented (see Fig. 1, to realize what is going on). The exponents named in Section 3 have been measured for both the directions mentioned, but we wish to present here exclusively the values got for x direction, that means, a typical case (*cf.* [1]). A crude estimation of them, done by an essential sampling method (*i.e.*, by averaging over a sub-space of ca. 15 percent of all available data chosen from the adequate time intervals; *cf.* Eqs. (2) and (3)), gives $\bar{\alpha} = 0.434 \pm 0.039$, $\bar{\beta} = 0.311 \pm 0.02$ and $\bar{\gamma} \cong 1.232$ at an averaged saturation time t_s about 928.000 of elementary evolution steps (note that we have 2.500.000 steps, totally, where the bares denote, as previously, some averaging over 82 simulation data). Notice, that because of a crudeness of the measurements $\bar{\alpha}/\bar{\beta}$ differs from γ by more than 11 percent. But, it is worth stating here that our estimation could enable to take, as a serious candidate for some future investigations, the description of rough surface dynamics based on KPZ-equation, for which $\bar{\alpha} = 0.5$, $\bar{\beta} = 0.333$ so that $\bar{\gamma} = 1.5$. In consequence, we may have here a process which is recognized (see Ref. [1], again) as a nonlinear process with nonconservative dynamics as well as noise, that means, extremally difficult to deal with.

5. Conclusions

For having our study more versatile or proceeded in various directions, we have treated the process in question twofold. First, we have dealt with the front evolution as an anomalous RW, and we have arrived at the conclusion that it must be an subdiffusive process, where the pure Brownian motion must be damped (or, maybe, interrupted) by the annihilation-creation acts, where, however, the annihilation prevails, in the course of time.

Second, to benefit from the studies on rough surfaces [1], we have studied our process in terms of the rough surfaces, and we have estimated, roughly, the basic exponents usually got from the computer simulations or certain simplest analytical cases [16]. We have obtained some numerical indication that the processes could be understood in terms of KPZ-description for the interface heigth $h \equiv h(y, t)$, measured in each y point and obeying a partial differential equation $h_t = Dh_{yy} + \delta h_y^2 + \eta$ (where: D and δ are positive constants, and η stands for a nonconservative noise) [16], but we obviously

need more careful analysis to state it for sure. The question may arise here: why the process cannot be considered, for example, by using Das Sarma–Tamborenea description due to a one-dimensional molecular beam epitaxy phenomenon [17] or more widely, to kinetic growth with surface relaxation [18] (at the beginning of our studies, we have tried to associate 5 most essential annihilation-creation rules of our process with 5 terms contained in Eq. (5) of [18], but our numerical findings indicate rather the KPZ-system as a *precursor* of ours; *cf.* Section 3, Subsect. 3.2), which seems to be more “flexible” for understanding a complexity of such phenomena, or, by invoking a recent approach offered by Constanza [19] (in fact, our first small-scale computer simulations indicated that it might be the case).

From the above mentioned, it appears to be worthy stating that two exponent values measured, namely $\bar{\nu}$ and $\bar{\beta}$, *i.e.* the anomalous RW and the growth exponents, respectively, are close to each other (is it got accidentally here? would it be an indirect proof that the two approaches utilized may co-exist or can be effectively applied? note that they are mostly measured in different time regimes). One further remark seems to be necessary. Due to the irreversible reactions the process finally (at very long times) should break down if we do not add proper boundary conditions. The question is whether the initially two open ends at the boundary could collapse? For instance, if one would allow that they can go around the whole lattice. Presuming that the open ends are able to do this a set of connected interfaces (fronts) inside the lattice may result. The interfaces are not necessarily further connected to the border of the lattice. In that case, the dynamics would eventually break down with probability one (it is ergodic and hence the state with $X = 0$ and $Y = 0$ must be reached). It means, that either one has to incorporate boundary conditions, say, the open ends are fixed at $x = 0$ and $x = L$ (think again that L is the size of the lattice; it does not imply, however, that there y -values are fixed) or we are looking only at an intermediate state. On the contrary, that means, while acting against a break down of the front, we have to preserve (*cf.* Subsection 3.2) a domination of the creative shifts (rule (2)) which leads to keep the process extremally reversible (also, some neutral shifts, like those given by rule (1) or in some sense by rule (3), do not destroy this strategy). Let us also mention explicitly that there is no direct comparison between the system studied in this work and some precise numerical results obtained by Jiang and Ebner [6]. One can also realize that some more careful consideration of our modelling in terms of a competition between two physical mechanisms (annihilation and creation), and a certain interpretations of the numerical results (exponents) obtained from the computer simulations “in a spirit” of some competition exponents [20, 16] would help in full understanding the process that we have studied.

Finally, let us notice that for going a step further just to make our study more realistic from the physical viewpoint, one is simply allowed to play on rules (1)–(5); *cf.* Subsection 3.2. Such a play would, for instance, be directed towards having them more repulsive or attractive (a charged interface can be investigated when one will use rules (3)–(5) *i.e.*, mostly those of irreversible nature, with a probability greater than for the two remaining), or, favoring a surface tension effect by applying rule (4) in a more pronounced way. Certain useful ideas on how to get the behaviour of the front more interesting under various physical (or technological) circumstances have also been gathered in [1] (see, for example, the Wolf–Villain or Lai–Das Sarma models, pp. 154–162, with the surface relaxation effect and the activation energy concept of Arrhenius-type, included). For comparative (future) studies, it can be useful to look at an interface (Ising-type) model in which the overhangs are disallowed (*cf.*, [21], and Ref. [17] therein).

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