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QUASIPARTICLES IN HOT GAUGE THEORIES*

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Because of long range interactions, the elementary excitations in a quark-gluon plasma have an unusual, non exponential damping.

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1. Introduction

In the weak coupling regime, the basic degrees of freedom of an ultrarelativistic plasma, such as the quark-gluon plasma, can be classified according to a hierarchy of scales controlled by the temperature T ($T \gg m$, where m is the mass of the plasma constituents), and the coupling strength g (for a recent review, see *e.g.* [1]). Two types of degrees of freedom are important in the present discussion: *i*) The plasma particles, which have typical momenta of order T and a thermal wavelength of order $1/T$, comparable to their average relative distance ($r_0 \sim n^{-1/3} \sim 1/T$). *ii*) The collective excitations which develop at a particular wavelength $\sim 1/gT$. For example the inverse screening length is $\lambda_D^{-1} \sim \sqrt{g^2 n/T} \sim gT$. Since $g \ll 1$, $1/gT \gg 1/T$, and excitations at wavelength $1/gT$ necessarily involve many particles ($n\lambda_D^3 \sim 1/g^3 \gg 1$), *i.e.* they are collective. Similar considerations apply to cold and dense plasmas, with a chemical potential $\mu \gg m$. The study of these elementary excitations, and in particular of their lifetimes, has received much attention in the recent past [2–10].

Information about the lifetimes can be obtained from the retarded propagator $S_R(t, \boldsymbol{p})$. A usual expectation is that $S_R(t, \boldsymbol{p})$ decays exponentially in time, $S_R(t, \boldsymbol{p}) \sim \mathrm{e}^{-iE(p)t-\gamma(p)t}$. The exponential decay may be associated

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to a pole of the Fourier transform $S_R(\omega, \boldsymbol{p})$, located at $\omega = E(p) - i\gamma(p)$. A quasiparticle excitation is well defined if its lifetime $\sim 1/\gamma$ is much larger than the period $\sim 1/E$ of excitation, that is, if the damping rate γ is small compared to the energy E. In this case γ can generally be computed from the imaginary part of the on-shell self-energy $\Sigma(\omega = E(p), \boldsymbol{p})$. Such calculations suggest that $\gamma \sim g^2T$ for both the single-particle and the collective excitations. In the weak coupling regime $g \ll 1$, this is indeed small compared to the corresponding energies (of order T and gT, respectively), suggesting that the excitations are well defined, *i.e.*, are weakly damped. However, the computation of γ in perturbation theory is plagued with infrared divergences, which casts doubt on the validity of these statements.

The first attempts to calculate the damping rates were made in the early 80's. It was then found that, to one-loop order, the damping rate of the soft collective excitations in the hot QCD plasma was gauge-dependent, and could turn out negative in some gauges (see Ref. [2] for a survey of this problem). Pisarski [3], and Braaten and Pisarski [4], identified the resummation needed to obtain the screening corrections in a gauge-invariant way: the resummation of the so called "hard thermal loops" (HTL) [3–7]. Such screening corrections are sufficient to make finite the transport cross-sections [8], and also the damping rates of excitations with zero momentum [4]. At the same time, however, it has been remarked that the HTL resummation is not sufficient to render finite the damping rates of excitations with non-vanishing momenta. The remaining infrared divergences are due to collisions involving the exchange of long wavelength, quasistatic, magnetic photons (or gluons), which are not screened in the HTL approximation. Such divergences affect the computation of the damping rates of charged excitations, in both Abelian and non-Abelian gauge theories [9]. The problem appears for both soft ($p \sim gT$) and hard ($p \sim T$) quasiparticles. In QCD, one may be tempted to bypass this problem by the ad hoc introduction of an IR cutoff, the "magnetic mass" which is expected to appear dynamically from gluon self-interactions. However, since in QED it is known that no magnetic screening can occur [11], the solution of the problem must lie somewhere else.

As we have shown [10], the determination of the large time behavior of the retarded propagator of an electron in hot QED requires resummations for both the fermion propagator and the photon-electron vertex function. Such resummations, different in nature from the hard thermal loop resummations, lead to an unusual, non exponential, damping of the excitations. They modify the analytic structure of the retarded propagator, making it analytic in the vicinity of the mass shell.

To be more specific, consider the imaginary part of the fermion self-energy, near the mass shell, in the one-loop approximation:

$$\mathrm{Im}\,\Sigma_R^{(2)}(\omega, \simeq p) \simeq -\alpha T \ln \frac{\omega_p}{|\omega - p|}, \tag{1}$$

where $\alpha \equiv g^2/4\pi$, $\omega_p = gT/3$ (the plasma frequency), and the approximate equality means that only the singular term has been kept. For two or more photon loops, the mass-shell divergences are powerlike. However, no infrared divergences are encountered when the perturbation theory is carried out directly in the time representation: the inverse of the time acts then effectively as an infrared cutoff. For instance, the one-loop correction to the retarded propagator $S_R(t, \boldsymbol{p})$ at large times is given by

$$\delta S_R^{(2)}(t, \boldsymbol{p}) \simeq -it \int_0^t \mathrm{d}t'\, \mathrm{e}^{ipt'}\, \Sigma_R^{(2)}(t', \simeq p). \tag{2}$$

This expression is well defined, but the limit $t \to \infty$ of the integral over t' [which is precisely the on-shell self-energy $\Sigma_R^{(2)}(\omega = p)$] does not exist. We actually have:

$$\Sigma_R^{(2)}(t, \boldsymbol{p}) \simeq -i\alpha T \frac{\mathrm{e}^{-ipt}}{t}, \tag{3}$$

for $t \gg 1/\omega_p$ and, therefore,

$$\delta S_R^{(2)}(t, \boldsymbol{p}) \simeq -\alpha T t \int_{1/\omega_p}^{t} \frac{\mathrm{d}t'}{t'} = \alpha T t \ln(\omega_p t). \tag{4}$$

This correction exponentiates in an all-order calculation:

$$S_R(t, \boldsymbol{p}) \propto \exp\left(-\alpha T t \ln \omega_p t\right), \tag{5}$$

for $t \gg 1/\omega_p$. This result applies to both hard particles and soft collective excitations (with a slight modification). For a massless fermion with momentum $p \sim T$ or larger, or a massive ($m \gg T$) test particle, a more accurate result has been obtained:

$$S_R(t \gg 1/\omega_p) \propto \exp\{-\alpha T t [\ln(\omega_p t) + 0.12652 \cdots + \mathcal{O}(g)\}. \tag{6}$$

2. Lifetime of quasiparticles

Before I explain how this result was obtained, let me recall how, and why, infrared divergences occur in the calculation of the lifetime of a quasiparticle in a hot QED plasma. Physically, what limits the lifetime of a quasiparticle excitation is the collisions of the quasiparticle with the other particles in the plasma [8]. The collision rate can be estimated directly in the form $\gamma = n\sigma$, where $n \sim T^3$ is the density of plasma particles, and σ the collision cross section. Restricting ourselves first to the Coulomb interaction, we can write $\sigma = \int \mathrm{d}q^2(\mathrm{d}\sigma/\mathrm{d}q^2)$, with $\mathrm{d}\sigma/\mathrm{d}q^2 \sim g^4/q^4$. Thus,

$$\gamma \sim g^4 T^3 \int \mathrm{d}q^2 \frac{1}{q^4}, \tag{7}$$

which is badly infrared divergent. One knows, however, that in the plasma the Coulomb interaction is screened, so that the effective electric photon propagator is not $1/q^2$ but $1/(q^2 + m_D^2)$, where $m_D \sim gT$ is the Debye screening mass. With this correction taken into account, the collision rate becomes

$$\gamma \sim g^4 T^3 \frac{1}{m_D^2} \sim g^2 T, \tag{8}$$

which is now finite, and of order g^2T, as announced.

However, screening corrections are not sufficient to eliminate all the divergences due to the magnetic interactions. To see that, consider the transverse part of the photon polarization tensor $\Pi(q_0, q)$. At small frequency and momentum, it is imaginary:

$$\Pi(q_0, q) \approx i\frac{3\pi}{4}\omega_{pl}^2 \frac{q_0}{q}. \tag{9}$$

When its contribution is included in the magnetic photon propagator, one obtains the following contribution to γ:

$$\gamma \sim g^4 T^3 \int \mathrm{d}q \int_{-q}^{q} \mathrm{d}q_0 \frac{1}{q^4 + (3\pi\omega_{pl}^2 q_0/4q)^2}. \tag{10}$$

The integral over q_0 can be calculated, with the result

$$\gamma \sim g^2 T \int^{\omega_{pl}} \frac{\mathrm{d}q}{q} \tag{11}$$

which remains logarithmically divergent.

3. The contribution of static photons

The physical origin of the remaining infrared divergences can be traced back to the collisions involving the exchange of very soft, unscreened, magnetic photons. This is reflected in the fact that the dominant contribution to the integral (10) is concentrated at very small q_0. In fact,

$$\frac{1}{q^4+(3\pi\omega_{pl}^2 q_0/4q)^2} \sim_{q\to 0} \frac{4}{3\omega_{pl}^2}\frac{\delta(q_0)}{q}. \tag{12}$$

A similar observation can be made when calculating γ from the imaginary part of the self energy, on the unperturbed mass-shell $\omega = p$:

$$\gamma \;=\; -\frac{1}{4p}\,\mathrm{tr}\left(\not{p}\,\mathrm{Im}\,\Sigma(\omega+i\eta,\boldsymbol{p})\right)\Big|_{\omega=p}. \tag{13}$$

In the Matsubara formalism, we have:

$$\Sigma(p) = -\,g^2 T\sum_{q_0=i\omega_m}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\;\gamma_\mu\,S_0(p+q)\,\gamma_\nu\,D^{\mu\nu}(q)\,, \tag{14}$$

where $\boldsymbol{k}=\boldsymbol{p}+\boldsymbol{q}$, $p_0=i\omega_n=i(2n+1)\pi T$, and $\omega_m=2\pi mT$, with integers n and m. It may be verified that the infrared logarithmic divergence in Eq. (11) arises entirely from the magnetic contribution of the term $q_0=i\omega_m=0$ in the Matsubara sum of Eq. (14). One finds similar divergences in all the diagrams contributing to Σ, which have an arbitrary number of magnetic photon lines carrying zero Matsubara frequency.

The fact that the dominant divergences are, in the Matsubara formalism, concentrated in the sector with zero Matsubara frequency, is an important simplification which allows us to resum them in closed form. Thus, one can ignore fermion loop insertions on static photon propagators (the transverse polarisation tensor at zero frequency is proportional to $\boldsymbol{q}^2$, and represents a minor modification of the photon propagator). Therefore, in order to isolate the dominant divergences, we may use the "quenched approximation", in which the retarded fermion propagator can be written as the following functional integral

$$S_R(x,y) = \int[\mathrm{d}\boldsymbol{A}]\,G_R(x,y|\boldsymbol{A})\exp\left\{-\tfrac{1}{2}\left(\boldsymbol{A},D_0^{-1}\boldsymbol{A}\right)_0\right\}, \tag{15}$$

where $G(x,y|\boldsymbol{A})$ is the fermion propagator in the presence of a static background gauge field, and

$$\left(\boldsymbol{A},D_0^{-1}\boldsymbol{A}\right)_0 \;=\; \frac{1}{T}\int\mathrm{d}^3x\,\mathrm{d}^3y\,A^i(\boldsymbol{x})\,D_{0\,ij}^{-1}(\boldsymbol{x}-\boldsymbol{y})A^j(\boldsymbol{y}). \tag{16}$$

In this equation, the factor $1/T$ has its origin in the restriction to the zero Matsubara frequency. The propagator $D_{0\,ij}$ is that of a free static photon,

$$D_0^{ij}(\boldsymbol{x}) \equiv \int \frac{\mathrm{d}^3q}{(2\pi)^3}\, \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{x}}\, D_0^{ij}(\boldsymbol{q}) \tag{17}$$

with, in the Coulomb gauge, $D_0^{ij}(\boldsymbol{q}) = \delta^{ij}/q^2$. (The final result can be shown to be gauge independent.)

4. The Bloch–Nordsieck approximation

In the kinematical regime of interest, an approximate expression for $G_R(x,y|\boldsymbol{A})$ is obtained by neglecting the recoil of the fermion in the successive emissions or absorptions of very soft photons. More precisely, we can approximate a typical fermion propagator entering the perturbative expansion of $G_R(x,y|A)$ by

$$S_0(\omega,\boldsymbol{p}+\boldsymbol{q}) \;=\; \frac{-\omega\gamma_0+(\boldsymbol{p}+\boldsymbol{q})\cdot\boldsymbol{\gamma}}{(\omega+i\eta)^2-\varepsilon_{p+q}^2} \;\simeq\; \frac{-1}{\omega-\varepsilon_p-\boldsymbol{v}\cdot\boldsymbol{q}+i\eta}\,\frac{\gamma_0-\hat{\boldsymbol{p}}\cdot\boldsymbol{\gamma}}{2}, \tag{18}$$

where $\boldsymbol{q}$ is a linear combination of the internal photons momenta and $\boldsymbol{v}\equiv\partial\varepsilon_p/\partial\boldsymbol{p}$ ($\boldsymbol{v}=\hat{\boldsymbol{p}}$ for the ultrarelativistic fermion). This is the familiar structure encountered in most treatments of infrared divergences in QED and which is economically exploited within the Bloch–Nordsieck model (see, *e.g.* [12]). In this model, $G(x,y|A)$, satisfies the following equation ($D_\mu=\partial_\mu+igA_\mu$)

$$-i\,(v\cdot D_x)\,G(x,y|A) \;=\; \delta(x-y), \tag{19}$$

which can be solved *exactly*. For retarded boundary conditions, and for static fields:

$$G_R(x,y|\boldsymbol{A}) \;=\; i\,\theta(x_0-y_0)\,\delta^{(3)}\left(\boldsymbol{x}-\boldsymbol{y}-\boldsymbol{v}(x_0-y_0)\right)U(x,y)\,, \tag{20}$$

where $U(x,y)$ is the parallel transporter

$$U(x,x-vt)=\exp\left\{ig\int_0^t \mathrm{d}s\,\boldsymbol{v}\cdot\boldsymbol{A}(\boldsymbol{x}-\boldsymbol{v}s)\right\} \tag{21}$$

which involves the integral of the gauge potential along the straight line trajectory of the particle.

The retarded propagator $S_R(x-y)$ is calculated by inserting the expressions (20), (21) of $G_R(x,y|\boldsymbol{A})$ in the functional integral (15). Its Fourier transform with respect to $\boldsymbol{x}-\boldsymbol{y}$ can be written as

$$S_R(t,\boldsymbol{p}) \;=\; i\theta(t)\mathrm{e}^{-it(\boldsymbol{v}\cdot\boldsymbol{p})}\,\Delta(t), \tag{22}$$

where

$$\Delta(t) \equiv \int [\mathrm{d}\boldsymbol{A}]\, U(x, x-vt) \exp\left\{-\tfrac{1}{2}\left(\boldsymbol{A}, D_0^{-1}\boldsymbol{A}\right)_0\right\} \tag{23}$$

contains all the non-trivial time dependence. The functional integral is easily done:

$$\Delta(t) = \exp\left\{-\frac{g^2}{2}T\int_0^t \mathrm{d}s_1 \int_0^t \mathrm{d}s_2\, v^i\, D_0^{ij}(\boldsymbol{v}(s_2-s_2))\, v^j\right\}. \tag{24}$$

In this equation, $D_0^{ij}(\boldsymbol{x})$ is the coordinate space representation of the magnetostatic photon propagator (see Eq. (17)).

The s_1 and s_2 integrations in Eq. (24) can be done by going to the Fourier representation. One obtains thus:

$$\Delta(t) = \exp\left\{-g^2 T\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\frac{\tilde{D}(\boldsymbol{q})}{(\boldsymbol{v}\cdot\boldsymbol{q})^2}\Big(1-\cos t(\boldsymbol{v}\cdot\boldsymbol{q})\Big)\right\}, \tag{25}$$

where $v^i\, D_0^{ij}(\boldsymbol{q})v^j \;\equiv\; \tilde{D}(\boldsymbol{q})$. The integral in Eq. (25) has no infrared divergence, but one can verify that the expansion of $\Delta(t)$ in powers of g^2 generates the most singular pieces of the usual perturbative expansion for the self-energy. (The integral in Eq. (25) presents an ultraviolet logarithmic divergence. However, one should recall that the restriction to the static photon mode implies that such an integral is to be cut off at momenta $q \sim \omega_{pl}$.)

Calculating the integral in Eq. (25), one finds that at times $t \gg 1/\omega_{pl}$ the function $\Delta(t)$ is of the form ($\alpha = g^2/4\pi$)

$$\Delta(\omega_{pl}t \gg 1) \simeq \exp\Big(-\alpha T t \ln \omega_{pl} t\Big). \tag{26}$$

A measure of the decay time τ is given by

$$\frac{1}{\tau} = \alpha T \ln \omega_{pl}\tau = \alpha T\left(\ln\frac{\omega_{pl}}{\alpha T} - \ln\ln\frac{\omega_{pl}}{\alpha T} + \ldots\right). \tag{27}$$

Since $\alpha T \sim g\omega_{pl}$, $\tau \sim 1/(g^2 T \ln(1/g))$. This corresponds to a damping rate $\gamma \sim 1/\tau \sim g^2 T \ln(1/g)$, similar to that obtained in a one loop calculation with an infrared cut-off $\sim g^2 T$.

However, contrary to what perturbation theory predicts, $\Delta(t)$ is decreasing faster than any exponential. It follows that the Fourier transform

$$S_R(\omega,\boldsymbol{p}) \;=\; \int\limits_{-\infty}^{\infty} \mathrm{d}t\,\mathrm{e}^{-i\omega t} S_R(t,\boldsymbol{p}) \;=\; i\int\limits_{0}^{\infty} \mathrm{d}t\,\mathrm{e}^{it(\omega-\boldsymbol{v}\cdot\boldsymbol{p}+i\eta)}\,\Delta(t), \qquad (28)$$

exists for *any* complex (and finite) ω. Thus, the retarded propagator $S_R(\omega)$ is an entire function, with sole singularity at $\mathrm{Im}\,\omega \to -\infty$.

In order to obtain the more accurate result (6), one needs to include the contributions of the non vanishing Matsubara frequencies. I refer to the last of Refs. [10] for details.

5. Conclusions

An important conclusion of this work is that quasiparticles exist and have a damping rate small compared to their energy. The weak coupling calculations are consistent, but non trivial resummations are necessary. This provides a sound basis for the calculation of the transport properties of the quark-gluon plasma.

The results can be extended to massive particles and collective modes, and also to QCD. For QCD, however, the damping rates cannot be obtained in closed form, but only through an Euclidean functional integral which could, in principle, be calculated on a lattice.

Finally, it is interesting to note that the peculiar decay of quasiparticle excitations could be also observed in other physical systems with long range interactions or correlations [13, 14].

Most of the work presented in this lecture has been done in collaboration with Edmond Iancu. I wish to thank M. Polyakov for pointing out to me the reference [14] in which a strikingly similar study is carried out for a totally unrelated problem: that of the propagation of light in a random medium, with a long-range noise correlation function.

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