Vol. 29 (1998) ACTA PHYSICA POLONICA B No 7

QUANTUM FLUCTUATIONS IN CURVED SPACE *

R. Schaeffer

Service de Physique Théorique, C.E. Saclay
91191 Gif-sur-Yvette, France.

and U. Moschella

Istituto di Scienze Matematiche Fisiche e Chimiche
Via Lucini 3, 22100 Como
and INFN sez. di Milano, Italy

(Received January 21, 1997)

Even for free fields, canonical quantization is problematic when the space-time is not flat. There is a problem in identifying the proper degrees of freedom of the quantum field. In particular, for open Friedmann–Robertson–Walker universes modes which are not L^2-normalizable may exist, and there is a controversy whether or not they contribute to the quantum fluctuations. We have shown unambiguously that these modes are allowed by quantum mechanics. Their appearance turns out to be an essential ingredient if one wants to insure invariance properties of the correlation functions in the maximal symmetric de Sitter case.

PACS numbers: 04.62.+v

1. Introduction

It is believed that the present matter irregularities in the Universe at Mpc scales, leading to the formation of galaxies and large clusters, have been generated from tiny seeds in the very primordial Universe. The latter result from quantum fluctuations of some specific field [4, 13] at an epoch where astrophysical length scales and microscopic scales were of the same order of magnitude.

The corresponding amplitude at a given scale is thus related to the amplitude of the energy–density fluctuations induced by these quantum effects.

* Presented at the XXXVII Cracow School of Theoretical Physics, Zakopane, Poland, May 30–June 10, 1997.

The latter may in principle be calculated from the fundamental laws of physics.

The time evolution of this tiny amplitude is well controlled and is obtained by linearizing the Einstein equations around a background (spatially) homogeneous Universe. In the standard case the scale of these fluctuations also evolves at the same rate as the microscopic scale.

On the other side, during inflation [7], the size of the microscopic scale, related to the typical energy-density of the vacuum which is the same at the beginning and at the end of inflation, remains constant in time whereas the scale of the fluctuations follows the expansion factor which grows exponentially.

After the phase transition which causes inflation is completed, both scales grow again at the same rate (up to the present epoch if inflation occurs only once) but with an offset related to the duration of the inflation. The microscopic scale at the end of inflation reflects itself in the heat bath created at this epoch (the microwave background). In the course of the evolution, the background solution of the Einstein equations becomes unstable against small fluctuations, the amplitude of the latter grows with time. Both the background (at 3K, that is at mm scales) and its fluctuations (at 1000 Mpc scale) have been well measured by the COBE satellite [12,16]. The difference (27 orders of magnitude) in scales that were originally similar, is caused by inflation. The calculation of quantum fluctuations is especially difficult for open universes. Curvature of space is at the origin of qualitatively new phenomena. We describe here the problems studied in the last few years in the light of recent progress made in this subject [10, 11].

2. The inflating Universe and curved spaces

The problem of explaining the large scale structure of the Universe is thus linked to the problem of calculating the correlation functions of a certain quantum field at the epoch of inflation. At this epoch the equation of state is dominated by the (constant) vacuum energy density ε_v and the expansion parameter a obeys the equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\varepsilon_v}{3c^2} + \frac{c^2}{a^2R_s^2} = \frac{c^2}{R_v^2} + \frac{c^2}{a^2R_s^2}\,, \tag{1}$$

where $-6/R_s^2$ is the comovingly constant spatial curvature and $-12/R_v^2$ is the constant space-time curvature. The usual density parameter Ω is defined by $\Omega = 8\pi G\varepsilon_v a^2/(3c^2\dot{a}^2)$. The corresponding space-time metric reads

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1+\frac{r^2}{R_S^2}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{2}$$

with

$$\frac{\ddot{a}}{a} = \frac{c^2}{R_v^2}\,. \tag{3}$$

Eq. (1) can be solved as follows

$$a(t) = \frac{R_v}{R_s}\sinh\frac{ct}{R_v}\,. \tag{4}$$

When the constant space-time curvature is not zero, we get the de Sitter metric, which is the relevant case for inflation.

A convenient way to represent the four-dimensional de Sitter space-time is to consider an embedding of it in a five-dimensional ambient Minkowski space.

Let us denote by $X^{(\mu)}\,,\quad \mu = 0,\ldots,4$, the coordinates of a five-vector X of the ambient space; the de Sitter Universe can then be identified with the one-sheeted hyperboloid with equation

$${X^{(0)}}^2 - {X^{(1)}}^2 - {X^{(2)}}^2 - {X^{(3)}}^2 - {X^{(4)}}^2 = -R_v^2\,. \tag{5}$$

The de Sitter metric is obtained as the restriction of the metric of the ambient Minkowski space-time

$$ds^2 = d{X^{(0)}}^2 - d{X^{(1)}}^2 - d{X^{(2)}}^2 - d{X^{(3)}}^2 - d{X^{(4)}}^2 \tag{6}$$

to the de Sitter manifold.

One interesting point is that the de Sitter metric can be used to describe inflating universes with closed, flat or open spatial sections [14]. This can be obtained by different choices of "cosmic time" (see Figs 1, 2 and 3).

The coordinate system adapted to the open model represented in Fig. 1 is the following:

$$\begin{aligned}
X^{(0)} &= R_v \sinh\frac{ct}{R_v}\cosh\frac{\rho}{R_s}\,,\\
X^{(1)} &= R_v \sinh\frac{ct}{R_v}\sinh\frac{\rho}{R_s}\sin\theta\sin\phi\,,\\
X^{(2)} &= R_v \sinh\frac{ct}{R_v}\sinh\frac{\rho}{R_s}\sin\theta\cos\phi\,,\\
X^{(3)} &= R_v \sinh\frac{ct}{R_v}\sinh\frac{\rho}{R_s}\cos\theta\,,\\
X^{(4)} &= R_v \cosh\frac{ct}{R_v}\,.
\end{aligned}$$

In these coordinates the metric reads

$$ds^2 = c^2 dt^2 - R_v^2 \sinh^2 \frac{ct}{R_v} \left[\frac{d\rho^2}{R_s^2} + \sinh^2 \frac{\rho}{R_s} \, (d\theta^2 + \sin^2\theta \, d\phi^2) \right] .$$

The spatial manifold of the open inflating Universe at cosmic time t is then visualized as the intersection of the de Sitter hyperboloid by the plane $X^4 = R_v \cosh ct/R_v$ with the condition $X^0 > 0$ (see Fig. 1).

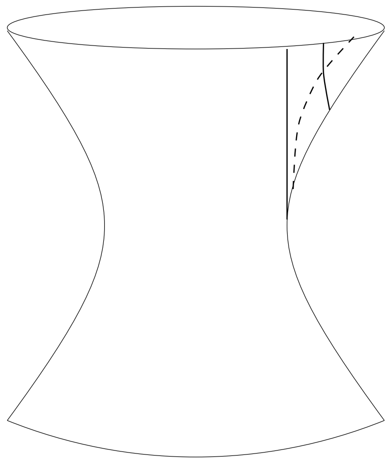

Fig. 1. Open model

A look at Figs 2 and 3 shows that the corresponding events may also be regarded as belonging to spatially flat or open universes. Of course, they will not occur at identical values of the corresponding "cosmic times" (surfaces of constant cosmic times are the solid lines in the figures while dashed lines represent the comoving space points).

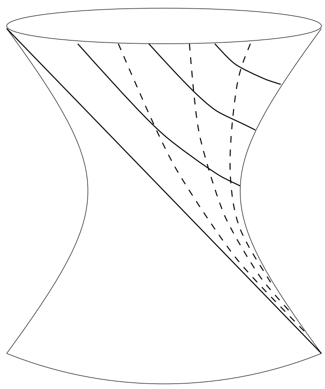

Fig. 2. Flat model

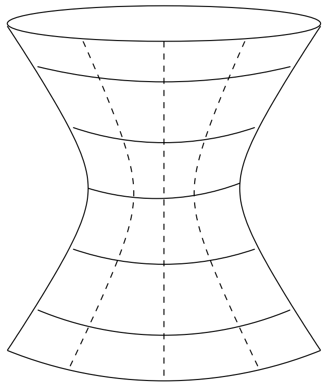

Fig. 3. Closed model

3. Quantum fluctuations and canonical quantization

The early Universe is usually described as a system whose state is defined by one parameter $\bar{\phi}$. We restrict our attention to quantum states which are homogeneous in space, *i.e.* $\bar{\phi}$ is an overall constant. All the observables, such as for instance the energy density are specified in terms of $\bar{\phi}$.

The quantum fluctuations of the system are then determined by the quantum fluctuations of the field ϕ, which deviates from the overall constant mean value by a space-time dependent quantity $\phi(x)$. The action is given by

$$S = \int \tfrac{1}{2}\sqrt{-g}\,\{g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - V(\phi)\}\, d^d x\,. \tag{7}$$

The gravitational effects of the background are inscribed into the metric $g^{\mu\nu}(x)$. The field satisfies a Klein–Gordon type equation $\Box\phi = -\frac{\partial V}{\partial\phi}$, where $\Box = (-g)^{-1/2}\partial_\mu(-g)^{1/2}g^{\mu\nu}\partial_\nu$ is the Laplace–Beltrami operator.

The potential has a minimum at $\phi = 0$, and behaves there as

$$V(\phi) \approx m^2\phi^2 + V_0. \tag{8}$$

We have, however, to keep in mind that we work close to conditions where a phase transition occurs, with a possible opening of the potential barrier when the mass m vanishes. The problem is to calculate the fluctuations of ϕ near the minimum of this potential. Since the potential has to be nearly flat around its minimum the quantum fluctuations are well described by a free Klein–Gordon field:

$$\Box\phi + m^2\phi = 0. \tag{9}$$

These fluctuations are translated into density fluctuations, and then evolved according with the linearized Einstein equations up to the present epoch.

The problem we focus on here [10, 11] is whether canonical quantization could provide for the correct answer when space and/or space-time are curved.

4. Canonical quantization

The usual canonical quantization of a free Klein–Gordon field (interacting only with the background) goes as follows. One introduces the Klein–Gordon scalar product

$$(\phi_1, \phi_2) = -i \int_{\Sigma} \bar{\phi}_1(x) \overleftrightarrow{\partial}_\mu \phi_2(x) d\Sigma^\mu, \tag{10}$$

where Σ is a suitable space like hypersurface and $d\Sigma$ is the associated volume element, and looks for a complete set "positive frequency" solutions ϕ_K of (9), labeled by some parameter K orthonormal with respect to that scalar-product (and therefore normalizable). The quantum field can then be expanded as

$$\hat{\phi}\,(x) = \sum_K \left(\phi_K(x) a_K^+ + \phi_K^*(x) a_K\right) , \tag{11}$$

where ϕ_K^* is the complex conjugate of ϕ_K; canonical quantization is achieved by assuming the commutation rules (CCR)

$$[a_K, a_{K'}^\dagger] = \delta_{K,K'}, \qquad [a_K, a_{K'}] = 0, \qquad [a_K^\dagger, a_{K'}^\dagger] = 0,$$

and by choosing the corresponding vacuum.

In many cases, time and space variables in (12) may be separated, then the constant introduced by the separation of variables provides for K.

Quantum fluctuations are described by the two-point vacuum expectation value of the field ϕ

$$W(x, x') = \langle \hat{\phi}\,(x)\, \hat{\phi}\,(x') \rangle = \sum_K \phi_K^*(x) \phi_K(x')\,. \tag{12}$$

Canonical quantization gives the two-point function as a sum over a complete set of normalizable functions times their complex conjugates. This expansion automatically induces the positivity properties of $W(x, y)$.

However, the difficulties and the ambiguities inherent in the canonical quantization of fields on a gravitational background appear here clearly: in fact the previous mode expansion is generally based on an arbitrary choice of local coordinates which may or may not extend to the whole space. In particular the spacelike hypersurface that one might want to use for physical reasons may fail to be a complete Cauchy surface and therefore the complete set found may be not sufficient to describe all the degrees of freedom of the field. Moreover, it is in general impossible to characterize the physically relevant vacuum states as the fundamental states for the energy in the usual sense and one has to give some physical prescription to single out such states. Let us now explore the different possible values for R_s and R_v (we do not consider closed universes). In the following we will write a solution $\phi(x)$ of the Klein–Gordon equation as the product $\chi(t)\Psi(\boldsymbol{r})$.

5. Flat space-time and flat space

Separation of variables in Minkowski space leads to the equations

$$\Delta\Psi_k + k^2\Psi_k = 0\,, \tag{13}$$

$$\partial^2\chi_k/\partial t^2 + \omega^2\chi_k = 0\,. \tag{14}$$

The constant of separation of variables is a three-dimensional vector $\boldsymbol{k}$ with modulus k. The Wightman vacuum of the Klein–Gordon field is obtained by retaining only the modes corresponding to $\omega > 0$ with $\omega^2 = k^2 + m^2$ (positive energy solutions) and all k from 0 to infinity (spectrum of the operator Δ in $\mathrm{L}^2(\boldsymbol{R}^3)$). Other specifications give thermal representations, many particle states, *etc.*

It is worth to note that the k spectrum extends down to the $k = 0$ mode (which corresponds to a constant shift of ϕ). These modes may be crucial to describe a phase transition which implies an overall change of the order parameter $\bar{\phi}$ (that describes the new vacuum of the system).

6. Curved space-time, flat space

In this case, studied first by Bunch and Davies [4], the scale factor enters non-trivially in Eq. (13) which is modified as follows:

$$\frac{1}{a^3}\frac{\partial}{\partial t}a^3\frac{\partial}{\partial t}\chi_k + \left(\frac{k^2}{a^2} + m^2\right)\chi_k = 0 \tag{15}$$

with $a(t) = R_v \exp(ct/R_v)$. Several prescriptions have been indicated to recognize the analogues of the positive energy solutions. The adiabatic prescription suggests that one should retain the solutions of Eq. (15) which goes over the solution of (13) in the large k limit. The vacuum chosen by this rule is referred to as the Bunch–Davies vacuum. There is still some ambiguity in applying this recipe. A more precise characterization of the Bunch–Davies vacuum can be given by the local Hadamard condition [8]. More recently, this particular representation has been characterized [3] by the global analyticity properties of a class of space-time waves in the complexified de Sitter manifold, in which separation of variables is avoided. This leads to the same choice of the "positive energy" modes, which are in this way fully justified.

The equation in the space variables is the same as in the Minkowski case, and so is the labelling of the spatial modes associated to the Laplace operator $\varDelta$.

7. Curved space-time, curved space

For curved spatial sections, Eq. (15) holds with $a(t) = (R_v/R_s)\sinh(ct/R_v)$. The comoving spatial section can be represented as the manifold $\varSigma$

$$\varSigma = \{x \in \boldsymbol{R}^d : \boldsymbol{x} \cdot \boldsymbol{x} = R_s^2\} \tag{16}$$

embedded in a Minkowski ambient space $\boldsymbol{R}^4$ whose metric is issued from the product $\boldsymbol{x} \cdot \boldsymbol{y} = \boldsymbol{x}^{(0)}\boldsymbol{y}^{(0)} - \boldsymbol{x}^{(1)}\boldsymbol{y}^{(1)} - \boldsymbol{x}^{(2)}\boldsymbol{y}^{(2)} - \boldsymbol{x}^{(3)}\boldsymbol{y}^{(3)}$.

The equation for the spatial wave function is replaced by

$$\Delta \varPsi_{iq} + k^2 \varPsi_{iq} = 0\,, \tag{17}$$

where now Δ is the Laplace–Beltrami operator for the curved space metric. The corresponding eigenfunctions may be conveniently parametrized as follows [6]

$$\varPsi_{iq}^{(d-1)}(\boldsymbol{x}, \boldsymbol{\xi}) = (\boldsymbol{x} \cdot \boldsymbol{\xi})^{-\frac{1}{R_s} - iq}\,; \tag{18}$$

where

$$q \in \boldsymbol{C}: \quad k^2 = \frac{1}{R_s^2} + q^2\,, \quad \boldsymbol{\xi} = (\xi^{(0)}, \ldots \xi^{(3)}), \quad \boldsymbol{\xi} \cdot \boldsymbol{\xi} = 0 \quad \xi^0 > 0\,,$$

k^2 is positive when $q \in \boldsymbol{R}$ or q is imaginary with $|q| \leq (d-2)/2$; k can be interpreted as the modulus of the wave number and $\boldsymbol{\xi}$ as the angular direction.

The spherically symmetric solutions of Eq. (17) are proportional to

$$f_{iq}(\rho) = \frac{\sin(q\rho)}{R_s q \sinh \frac{\rho}{R_s}}, \tag{19}$$

and can be seen to go over to the usual flat-space spherical waves for large R_s.

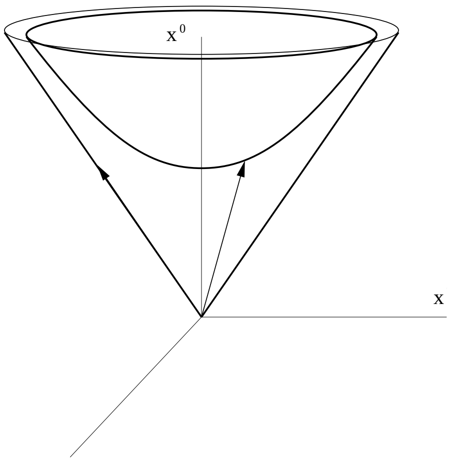

Fig. 4. Momentum space for a negatively curved space

The problem [9] is now to understand which modes have to be included in the sum (12). More precisely, should k run from 0 to infinity, or should it be q? The L^2 scalar product

$$R_s^2 \int f^*_{iq_1}(\rho) f_{iq_2}(\rho) \sinh^2 \frac{\rho}{R_s} d\rho = \frac{1}{q_1 q_2} \int \sin(q_1\rho)\sin(q_2\rho) d\rho \tag{20}$$

is finite for real values of q but for imaginary values of $q = i\tilde{q}$

$$R_s^2 \int f^*_{i\tilde{q}_1}(\rho) f_{\tilde{q}_2}(\rho) \sinh^2 \frac{\rho}{R_s} d\rho = \frac{1}{\tilde{q}_1 \tilde{q}_2} \int \sinh(\tilde{q}_1\rho)\sinh(\tilde{q}_2\rho) d\rho \tag{21}$$

diverges exponentially. To include only the modes with real values of q, however, has drastic consequences: *whatever the (real) value of* q, *the wave function (18) decreases faster than* $\exp(-\rho/R_s)$ *at large* ρ. This would be a quite specific prediction, *i.e.* the absence of any large scale fluctuations in curved space. Furthermore, it is seen that the mode which is spatially constant (relevant to describe a transition between ground states if homogeneity is to be preserved) corresponds to $k = 0$. Therefore, one may expect the appearance of all modes down to $k = 0$. The modes that are labeled by imaginary q with $0 < |q| < 1/R_s$ *which seem to be physically needed are not* L^2*-normalizable.*

8. Issue of de Sitter invariance

In Minkowski space, a general argument based on Lorentz invariance shows that the two-point function (12) depends only on the invariant variable $(x-y)^2$. Note that, however, none of the terms in the expansion exhibits this invariance. Had we omitted one of these terms, the invariance would have been broken.

A similar property holds in the de Sitter case: de Sitter invariant two-point functions can again be shown to depend only on $(X-Y)^2$ (X and Y are here five-vectors restricted to de Sitter manifold). This holds without any reference to coordinate systems. The two-point function depends only on the two events X and Y and not on the choice of coordinate systems (remember that the same event can be seen as belonging to a flat, open or closed Universe). Only the way X and Y are related to space and time coordinates differs.

If we then restrict our attention to de Sitter invariant two-point functions we can deduce the expansion for two-point functions in the open Universe coordinates from that calculated *e.g.* in the spatially flat case [4].

In [15] the Bunch–Davies two-point function has been reconstructed in open coordinates by using a method inspired by canonical quantization.

However, the standard textbook canonical formalism (see for instance [5], chapter 5) requires several modifications. Separation of variables leads to a second order equation for the time dependent factor of the modes, which has two independent solutions. Following the standard prescriptions of canonical quantization in generic open FRW universes literally one should choose one particular solution to be "positive frequency". In the open de Sitter case the differential equation for $\chi(t)$ can be identified with the Legendre equation (by a suitable change of variables). An argument based on the analyticity of the modes in the time variables indicates that one can choose the function

$$\chi_{iq}(t) = P^{iq}_{-1/2+i\nu}(\cosh t) \tag{22}$$

with

$$\nu^2 = m^2 R_v^2 - 9/4 \tag{23}$$

as positive frequency solution. However, this is not sufficient to get a de Sitter invariant two-point function and, as has been noted in [15]. One has to add also a second series of modes constructed with the help of the following solution.

$$\varphi_{iq}(t) = P^{iq}_{-1/2+i\nu}(-\cosh t + i\varepsilon)\,. \tag{24}$$

Thus one is led to retain the two independent solutions of the time dependent equation!

The point is that the relevant spatial manifold in the open de Sitter model is not a complete Cauchy surface for the Klein–Gordon equation, but, roughly speaking, there is another half of the Cauchy surface on the other side of the de Sitter manifold (see Fig. 1). Therefore, spacetime modes which do not appear as independent, if the Klein–Gordon product is calculated by integrating only on the physical spatial manifold, can be interpreted as independent if one integrates suitable extensions of them on a true Cauchy surface.

In a general open FRW Universe one does not have access to information of this kind, which regards the global structure of the space-time manifold, and is lead to work only with the spatial manifold *as if it were a Cauchy surface*. Therefore, in the general open FRW case, the standard prescriptions of canonical quantization would eventually lead to a "wrong" vacuum.

The situation is even worse for the modes arising from the super-curvature spatial waves (*i.e.* the modes of the form $\chi_{iq}(t)\Psi_{iq}(x)$ for imaginary values of q). Since these modes are not normalizable on the relevant spatial manifold (and even on the manifold obtained by including the mirror image of the spatial manifold used), the use of canonical quantization for a generic open FRW Universe necessarily leads to the conclusion that these modes do not contribute to quantum field expansions.

However, a clever insight lead the authors of [15] to introduce an additional mode normalized on a compact section of the de Sitter hyperboloid, where there are no divergences and to continue the resulting mode to the physical Universe. This mode enters the field expansion for masses lower than a critical mass.

Unfortunately there are some points in the treatment given in [15] that are not completely clarified. Furthermore, the whole procedure is hopeless in generic open FRW Universe, where standard canonical quantization necessarily leads to the conclusion that these modes do not contribute to quantum field expansions. As a consequence, many subsequent papers on the subject could not follow these suggestions, fearing above all that the unsolved problem of the exponential divergences could be a sign of some inconsistency in the treatment. A look to the literature shows extreme confusion on the matter.

In a recent work [10] we have studied the problem of finding the mode decomposition of a *given* two-point function on an open de Sitter Universe. The method we have used is based on a Laplace-type transform [2, 3] suitably adapted to curved spaces. It is possible to obtain in this way the sought representation (12) of the two-point function; the latter appears as a Källen–Lehmann-type decomposition in which the modes arising from the separation of variables enter explicitly and directly with the right normalization. This calculation takes advantage of the proven analyticity properties [3] that the Bunch–Davies two-point function possesses.

The result is the following:

$$W(X,X') = \int\limits_{-\infty+\frac{i}{R_s}}^{+\infty+\frac{i}{R_s}} dm(q) \int d\boldsymbol{\xi}\, \chi_{iq}(t)\varphi_{iq}(t')\Psi_{iq}(\boldsymbol{x},\boldsymbol{\xi})\Psi_{-iq}(\boldsymbol{x}',\boldsymbol{\xi})\,, \tag{25}$$

where the five-vectors are restricted to the (relevant region of the) de Sitter space-time; $dm(q)$ and $d\boldsymbol{\xi}$ are suitable measures [10].

The interesting result here is that the integral is not over the real values of q. This result emerges naturally in our approach and we do not have to distinguish special cases and we have not to postulate it to guarantee de Sitter invariance. The use [10] of Laplace-type transform provides calculations free of the divergences encountered by all previous authors when working in the open de Sitter Universe, and can be readily generalized to a generic Friedmann–Robertson–Walker Universe that does not possess the de Sitter symmetry.

In the de Sitter case, for masses lower than the critical mass $m^2 = 2R_v^2$, Eq. (25) can be recasted in the following form [10]:

$$\begin{aligned} W(X,X') \;=\; & \int\limits_0^\infty d\mu(q) \int d\boldsymbol{\xi}\chi_{iq}(t)\Psi_{iq}(\boldsymbol{x},\boldsymbol{\xi})\chi^*_{iq}(t')\Psi^*_{iq}(\boldsymbol{x}',\boldsymbol{\xi}) \\ & + \int\limits_0^\infty d\sigma(q) \int d\boldsymbol{\xi}\varphi_{iq}(t)\Psi_{iq}(\boldsymbol{x},\boldsymbol{\xi})\varphi^*_{iq}(t')\Psi^*_{iq}(\boldsymbol{x}',\boldsymbol{\xi}) \\ & + A(\tilde{q}_m) \int d\boldsymbol{\xi}\chi_{\tilde{q}_m}(t)\Psi_{\tilde{q}_m}(\boldsymbol{x},\boldsymbol{\xi})\chi_{\tilde{q}_m}(t')\Psi_{-q_m}(\boldsymbol{x}',\boldsymbol{\xi})\,, \end{aligned} \tag{26}$$

where $\tilde{q}_m R_s = (9/4 - m^2R_v^2)^{1/2} - 1/2$, $d\mu$ and $d\sigma$ are suitable measures and A is a constant [10]. It can be viewed as the sum over the modes labelled by the modulus of the momentum k and the direction $\boldsymbol{\xi}$. The appearance of two terms for given $k > 1/R_s$ is linked to the fact that the open de Sitter Universe does not contain a complete Cauchy surface (for its geodesical completion). For the discrete value $k_m < 1/R_s$ there is a single contribution (for a given direction $\boldsymbol{\xi}$). This mode corresponds to a product of spatial functions which *are not related by complex conjugation.* This is out of reach of the usual canonical quantization [15] procedure. Our procedure has a generalization to open universes which do not possess the de Sitter symmetry [10]. Our method also gives as a by-product a general structure for the two-point function on a general Friedmann–Robertson–Walker Universe.

9. Conclusions

For open FRW models new modes appear in some specific cases (in the de Sitter case for Klein–Gordon fields whose mass is lower than critical) which one would not naturally include using canonical quantization because they are not L^2-normalizable. Although their possible existence had been suggested a few years ago [9, 15], the situation stayed confused. We have recently indicated one correct way to handle these modes [10, 11], showing that they indeed are representing quantum fluctuations larger than the curvature scale and are allowed by quantum mechanics. In curved spaces, these modes represent a new and specific contribution to the quantum field, potentially important at large scales (and small masses).

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