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HYPOTHETIC TIME–TEMPERATURE DUALITY AS A HINT FOR MODIFICATIONS IN QUANTUM DYNAMICS?

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The well-known formal analogy between time and absolute temperature, existing on the quantum level, is considered as a profound duality relationship requiring some modifications in the conventional quantum dynamics. They consist of tiny deviations from uniform time run in the physical spacetime, as well as of tiny deviations from unitary time evolution characteristic for the conventional quantum theory. The first deviations are conjectured to be produced by local changes of total average particle number. Then, they imply the second deviations exerting in turn influence upon this particle number. Two examples of the second deviations are described: hypothetic tiny violation of optical theorem for particle scattering, and hypothetic slow variation of the average number of probe particles contained in a sample situated in proximity of a big accelerator (producing abundantly particles on a target).

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1. Introduction

As is well known, in the quantum theory there exists a formal analogy [1] between the unitary time evolution described by the operator $\exp(-iHt/\hbar)$ and the thermal equilibrium connected with the operator $\exp(-H/kT)$ (so, the unitary time evolution may be also referred to as the "quantum dynamical equilibrium"). This may raise a natural question, as to whether the implied correspondence between time and absolute temperature,

$$it/\hbar \leftrightarrow 1/kT\,, \tag{1}$$

is only a formal analogy, appearing more or less accidentally in the structure of quantum theory, or is rather a profound relationship expressing a fundamental duality of time and absolute temperature (when H is time-independent, what is the generic situation from the theoretical viewpoint)[1]. In the second case, a fascinating possibility is open [2]: for quantum dynamical systems in some circumstances there might appear tiny but measurable deviations from the unitary time evolution, in analogy with the familiar deviations from the thermal equilibrium[2]. Since the former deviations are not observed yet (for instance, as deviations from the optical theorem [3]), there should be a natural mechanism making them very small indeed. Let us emphasize that they would manifest themselves as a quantum effect, caused by a thermodynamic-like structure of a properly modified quantum dynamics. Such an effect is not present in the Einsteinian classical theory of gravitation.

In this note we describe a thermodynamic-like model of quantum dynamics implying tiny deviations from the unitary time evolution as it is defined by the conventional state equation [4]

$$i\hbar \frac{d\psi(t)}{dt} = H\psi(t)\ , \quad H^\dagger = H \tag{2}$$

and an initial condition for the state vector $\psi(t)$ (here, in some phenomenological situations, H may be time-dependent). The model is based on the correspondence (1) between time and absolute temperature, and so, in analogy with thermodynamics, it may be called "chronodynamics".

2. Equations of chronodynamics

In the case of small deviations from the thermal equilibrium when $T \to T + \delta T(\vec{r}, t)$, the heat conductivity equation holds for $\delta T(\vec{r}, t)$. In a homogeneous matter medium, it takes the form

$$\left(\triangle - \frac{1}{\lambda_Q\, c} \frac{\partial}{\partial t} \right) \delta T(\vec{r}, t) = 0\,. \tag{3}$$

Here, $\delta T(\vec{r}, t) \equiv 0$ for the thermal equilibrium.

[1] It should be emphasized that the correspondence (1) implies on the quantum level a thermodynamic-like character of time t in contrast to its more familiar status on the classical level. Such a statistical-like character of time is mathematically independent of (but physically consistent with) the fundamental probabilistic interpretation of the wave function in conventional quantum theory and, naturally, the probabilistic interpretation of the density matrix in conventional statistical quantum theory, built up on top of the former.

[2] Then, both notions of time and temperature lose their conventional meaning defined by the operators $\exp(-iHt/\hbar)$ and $\exp(-H/kT)$, since now these operators work no longer in the sense of exact physical operations.

By analogy, in the case of small deviations from the unitary time evolution (*i.e.*, from quantum dynamical equilibrium) when the uniform time run would be a little deformed: $t \to t + \delta t(\vec{r}, t)$, a new conductivity equation should be valid for the inverse-time-deformation field

$$\varphi(\vec{r}, t) \equiv \frac{1}{t + \delta t(\vec{r}, t)} - \frac{1}{t} \simeq -\frac{\delta t(\vec{r}, t)}{t^2} \tag{4}$$

(since the field $-i\hbar\varphi(\vec{r}, t)$ would be related to the field $k\delta T(\vec{r}, t)$ through the correspondence (1)). In the vacuum, such an equation would have the form

$$\left(\triangle - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t}\right) \varphi(\vec{r}, t) = 0\,, \tag{5}$$

where $\lambda_\Gamma > 0$ would denote an unknown length-dimensional conductivity constant (in the vacuum). Here, $\delta t(\vec{r}, t) \equiv 0$ and so $\varphi(\vec{r}, t) \equiv 0$ for the unitary time evolution.

Now, let us take into account the identity

$$\begin{aligned} \left(\Box + \frac{1}{4\lambda_\Gamma^2}\right)\left[\varphi(\vec{r}, t) \exp\frac{c\,t}{2\lambda_\Gamma}\right] &= \exp\frac{c\,t}{2\lambda_\Gamma}\left(\Box - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t}\right)\varphi(\vec{r}, t) \\ &\simeq \exp\frac{c\,t}{2\lambda_\Gamma}\left(\triangle - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t}\right)\varphi(\vec{r}, t)\,, \end{aligned} \tag{6}$$

where the last step is valid if $(1/c^2)\partial^2\varphi/\partial t^2$ can be neglected nonrelativistically in comparison with $(1/\lambda_\Gamma c)\,\partial\varphi/\partial t$. Due to this identity, the conductivity equation (5) may be considered as a nonrelativistic approximation of tachyonic-type Klein–Gordon equation

$$\left(\Box + \frac{1}{4\lambda_\Gamma^2}\right) \chi(x) = 0 \tag{7}$$

for the new field

$$\chi(x) \equiv \varphi(\vec{r}, t) \exp\frac{c\,t}{2\lambda_\Gamma}\,. \tag{8}$$

Such a field equation for $\chi(x)$ is relativistic in the sense of special relativity, if $\chi(x)$ is a Lorentz covariant. Let us accept this equation and assume that $\chi(x)$ is simply a Lorentz scalar. Then, in contrast to $\chi(x)$, the inverse-time-deformation field $\varphi(\vec{r}, t) = \chi(x)\exp(-c\,t/2\lambda_\Gamma)$, satisfying relativistically the equation

$$\left(\Box - \frac{1}{\lambda_\Gamma c}\frac{\partial}{\partial t}\right) \varphi(\vec{r}, t) = 0 \tag{9}$$

in place of Eq. (5), is no Lorentz covariant. Of course, both fields $\varphi(\vec{r},t)$ and $\chi(x)$ are real-parameter-valued, since time t is always a real parameter [4]. This implies that they cannot transport quantum energy. It is important to observe that Eq. (7) allows for ultraluminal plane–wave solutions $\exp(-ik\cdot x)$ with $k_0 = \sqrt{\vec{k}^2 - 1/4\lambda_\Gamma^2} > 0$, beside the damped-in-time plane–wave solutions with $ik_0 = \sqrt{1/4\lambda_\Gamma^2 - \vec{k}^2} > 0$ (where $k_0 = -i|k_0|$).

Having accepted the homogeneous field equation (7), we come to a crucial point in our argument: we must guess the form of matter sources which are to be inserted into this equation in order to describe the production of time-run deformations $\delta t(\vec{r},t)$ by time-evolving matter and, *vice versa*, their influence on the time evolution of matter. Here, the term "matter" means all physical particles, both fermions and bosons (including photons).

The best guess we can offer is that the inhomogeneous field equation for the Lorentz scalar $\chi(x) \equiv \varphi(\vec{r},t)\exp(ct/2\lambda_\Gamma)$ ought to have the form

$$\left(\Box + \frac{1}{4\lambda_\Gamma^2}\right)\chi(x) = -g_\Gamma\lambda_\Gamma\partial_\mu j^\mu(x)\,, \tag{10}$$

where $g_\Gamma > 0$ is an unknown coupling constant and $(j^\mu(x)) = (c\rho(\vec{r},t), \vec{\jmath}(\vec{r},t))$ describes a total average matter four-current. In the simplest case of pure quantum states, such a current ought to be given by the spin-averaged expectation value

$$j^\mu(x) \equiv \langle\psi(t)|J^\mu(\vec{r})|\psi(t)\rangle_{\rm av} \tag{11}$$

with $(J^\mu(\vec{r}))$ denoting the operator of total particle four-current and $\psi(t)$ standing for the state vector of the considered quantum system (here, $\psi(t)$ and $J^\mu(\vec{r})$ are presented in the Schrödinger picture). Evidently, the state vector $\psi(t)$ in Eq. (11) must satisfy a state equation, modified in comparison with the conventional state equation (2) valid in the case of unitary time evolution (*i.e.*, in the case of quantum dynamical equilibrium which holds when $\delta t(\vec{r},t) \equiv 0$)[3].

The conjecture (10) means physically that time-run deformations are produced by local changes of the total average particle number and, *vice versa*, such local changes are influenced by time-run deformations.

[3] In the general case of mixed quantum states, $j^\mu(x) \equiv {\rm Tr}[J^\mu(\vec{r})\rho(t)]$, where $\rho(t)$ is the system's density matrix satisfying an extended Liouville–von Neumann equation, properly modified in comparison with its conventional form valid for the unitary time-evolution. Note that an extended form of this equation proposed in Ref. [5] is intended to modify the coventional quantum mechanics by introducing some decoherence effects at the microscopic scale, without simultaneously violating the probability conservation: $d{\rm Tr}\rho(t)/dt = 0$ (which is slightly violated in our model, *cf.* Eq. (20) and footnote[4]).

In order to complete the formal stucture of our model we must guess the modified state equation for $\psi(t)$. To this end let us observe that the correspondence (1) suggests for the first law of thermodynamics the modified form

$$dU = \delta W + \delta Q - i\delta\Gamma \tag{12}$$

with an imaginary term $-i\delta\Gamma$ added. Here, $-i\Gamma$ denotes a new thermodynamic-like quantity being an analogue of heat Q, when $-i\hbar/t$ takes over the role of kT according to Eq. (1). Let us call Γ "energy width" transferred to the quantum system from the physical spacetime playing in our model the role of an unavoidable part of surroundings for any matter system. In analogy with the familiar heat law

$$Q \propto \int d^3\,\vec{r}\,\rho(\vec{r},t)\,k\,\left[T + \delta T(\vec{r},t) - T\right]\,, \tag{13}$$

we expect to have the formula

$$-i\Gamma \propto \int d^3\,\vec{r}\,\rho(\vec{r},t)(-i\hbar)\left[\frac{1}{t+\delta\,t(\vec{r},t)} - \frac{1}{t}\right]\,, \tag{14}$$

because $-i\hbar/t$ corresponds to kT. Then, for the unitary time evolution where $\delta\,t(\vec{r},t) \equiv 0$, we get $\Gamma \equiv 0$.

The modified first law of thermodynamics (12) leads in a natural way to the modified state equation of the form

$$i\hbar\frac{d\,\psi(t)}{dt} = (H - i\mathbf{1}\Gamma)\;\psi(t) \quad , \quad H^\dagger = H\,, \tag{15}$$

where $\mathbf{1}$ is the unit operator. Evidently, the new time-evolution generator $H - i\mathbf{1}\Gamma$ is (generally) a non-Hermitian operator implying nonunitary time evolution. We will accept Eq. (15) and assume that in the nonrelativistic approximation (with respect to matter) the energy width in Eq. (15) can be presented as

$$\Gamma \equiv g_\Gamma\,\hbar \int d^3\vec{r}\,\rho(\vec{r},t)\,\varphi(\vec{r},t) \tag{16}$$

in consistency with the formula (14), while its relativistically exact form (with respect to matter) is

$$\Gamma \equiv g_\Gamma\,\hbar \int d^3\vec{r}\,\sqrt{j_\mu(x)\,j^\mu(x)}\,\varphi(\vec{r},t)\,. \tag{17}$$

Here, $g_\Gamma > 0$ denotes the same coupling constant as that introduced already in Eq. (10). It is expected to be very small in order to get tiny deviations from the unitary time evolution. Note from Eqs. (17) and (10) that

$\Gamma = O(g_\Gamma^2)$. It is important to observe from Eqs. (15) and (17) that the inverse-time-deformation field $\varphi(\vec{r},t)$, though it cannot transport quantum energy, can transfer to the quantum system the energy width.

Since $\varphi(\vec{r},t) = \chi(x)\exp(-ct/2\lambda_\Gamma)$ is no Lorentz covariant, the energy width Γ as given in Eq. (17) is no pure time component of a four-vector, and so, while in Eq. (15), introduces necessarily some deviations from the special relativity. They vanish (only) at $t \to 0$, what is well defined if the Lorentz scalar $\chi(x)$ is regular at $t = 0$ or has there a pole (for such a $\chi(x)$, the time-deformation field $\delta t(\vec{r},t)$, equal by Eq. (4) to $\delta t(\vec{r},t) = -\varphi(\vec{r},t)\,t^2\,[1+\varphi(\vec{r},t)\,t]^{-1}$, vanishes at $t = 0$). These deviations are caused by the factor $\exp(-ct/2\lambda_\Gamma)$ in $\varphi(\vec{r},t)$, so they may be avoided if in place of $\varphi(\vec{r},t)$ the scalar $\chi(x)$ is used in Eqs. (16) and (17). However, such a choice for Γ seems unnatural as long as the strict correspondence between Eqs. (13) and (14) has to be maintained. We will return to this potentially attractive choice at the end of Section 3.

Thus, in our model, due to the factor $\exp(-ct/2\lambda_\Gamma)$ involved in the inverse-time-deformation field $\varphi(\vec{r},t)$, the instant $t = 0$ is distinguished from all other instants in the evolution of Universe. This seems to suggest that time t in our model ought to be identified with the "cosmological time" $t \geq 0$ counted from the Big Bang as from its natural beginning at $t = 0$. In such a case, the analogy between the absolute temperature $T \geq 0$ and the inverse of time $t \geq 0$ really appeals to our imagination. For such reckoning of time we have $t =$ (running) age of Universe. Note from Eq. (15) that

$$\psi(t) = \psi^{(0)}(t)\exp\left[-\frac{1}{\hbar}\int_{t_0}^{t} dt'\,\Gamma(t')\right] , \tag{18}$$

where $\psi^{(0)}(t)$ satisfies the conventional state equation (2) describing the unitary time evolution. If the Hamiltonian H is time-independent, then

$$\psi^{(0)}(t) = \exp\left[-\frac{i}{\hbar}H(t-t_0)\right]\psi^{(0)}(t_0)\,. \tag{19}$$

We can see from Eq. (18) that the norm of $\psi(t)$ is equal to

$$\langle\psi(t)|\psi(t)\rangle^{1/2} = \exp\left[-\frac{1}{\hbar}\int_{t_0}^{t} dt'\,\Gamma(t')\right] , \tag{20}$$

so, generally, in contrast to $\langle\psi^{(0)}(t)|\psi^{(0)}(t)\rangle^{1/2} = 1$, it changes (very slowly) in time[4]. The formal reason for this variation in time is that in our thermody-

[4] In the general case of mixed quantum states, $\mathrm{Tr}\rho(t) = \sum_n \rho_n \exp[-(2/\hbar)\int_{t_0}^t dt'\,\Gamma_n(t')]$ for $\rho(t) = \sum_n |\psi_n(t)\rangle\rho_n\langle\psi_n(t)|$ with $\sum_n \rho_n = 1$ and $H\psi_n(t_0) = E_n\psi_n(t_0)$. Thus, generally, it changes (very slowly) in time.

namic-like model of quantum dynamics the physical spacetime is not included into the quantum system as its part (in spite of mutual interactions) [6], playing (in virtue of these interactions) the role analogical to that of a thermostat in thermodynamics. In particular, during the unitary time evolution of a quantum system, the physical spacetime behaves formally as if it maintained in the quantum system the conventional uniform run of time t, much like in the thermal equilibrium a thermostat keeps temperature T of a system uniform (and constant in time, though on cosmological scale the absolute temperature of Universe is also "running"). So, the term "chronodynamics" for our theory seems to be justified on a profound level.

Concluding this Section, we can see that Eqs. (10) and (15) (together with (11) and (17)) form a mixed set of two coupled equations for the parameter-valued field $\chi(x) \equiv \varphi(\vec{r},t)\exp(ct/2\lambda_\Gamma)$ describing time-run deformations $\delta t(\vec{r},t)$ and, on the other hand, for the state vector $\psi(t)$ of time-evolving matter. Because of the bilinear form appearing in Eq. (11) this set of equations is (strictly speaking) nonlinear with respect to the state vector $\psi(t)$, what (slightly) violates the superposition principle for $\psi(t)$. This perturbs the fundamental probability interpretation of $\psi(t)$.

However, in the lowest-order perturbative approximation with respect to g_Γ, where $\psi(t)$ in Eq. (11) is approximated by $\psi^{(0)}(t)$ satisfying the conventional state equation (2), the set of Eqs. (10) and (15) becomes linear with respect to $\psi^{(1)}(t)$. In fact, in this approximation

$$\left(\Box + \frac{1}{4\lambda_\Gamma^2}\right)\chi^{(1)}(x) = -g_\Gamma \lambda_\Gamma \partial_\mu j^{(0)\,\mu}(x) \tag{21}$$

with

$$j^{(0)\,\mu}(x) \equiv \langle\psi^{(0)}(t)|J^\mu(\vec{r})|\psi^{(0)}(t)\rangle_{\rm av}\,, \tag{22}$$

and

$$i\hbar\frac{d\,\psi^{(1)}(t)}{dt} = \left(H - i\mathbf{1}\Gamma^{(1)}\right)\psi^{(1)}(t) \;\;,\;\; H^\dagger = H \tag{23}$$

with

$$\Gamma^{(1)} \equiv g_\Gamma \hbar \int d^3\vec{r}\sqrt{j^{(0)}_\mu(x) j^{(0)\,\mu}(x)}\,\varphi^{(1)}(\vec{r},t) = O(g_\Gamma^2)\,. \tag{24}$$

Thus, the state vector $\psi^{(1)}(t)$ still may be interpreted probabilistically in spite of a (very slow) variation in time of its norm

$$\langle\psi^{(1)}(t)|\psi^{(1)}(t)\rangle^{1/2} = \exp\left[-\frac{1}{\hbar}\int\limits_{t_0}^{t} dt'\,\Gamma^{(1)}(t')\right]\,, \tag{25}$$

caused by the physical spacetime that is not included into the quantum system (in spite of mutual interactions) [6].

3. Consequences

There is a number of consequences of the hypothetic chronodynamics, caused by its departures from the conventional quantum theory. Most of them may consist of tiny (but *in spe* measurable) deviations from the unitary time evolution. Let us consider two examples.

(i) Violation of optical theorem

The lowest-order perturbative S matrix in chronodynamics is related to the conventional S matrix through the formula

$$S^{(1)} = S^{(0)} \exp\left[-\frac{1}{\hbar}\int\limits_{-\infty}^{\infty} dt \Gamma^{(1)}(t)\right] , \tag{26}$$

where $\Gamma^{(1)}(t) = O(g_\Gamma^2)$ determines the lowest-order unitarity defect (here, $S^{(0)\dagger}S^{(0)} = \mathbf{1} = S^{(0)}S^{(0)\dagger}$). In Eq. (26) the interval $-\infty\,,\,\infty$ symbolizes a time interval very long in comparison with a short reaction time when $\Gamma^{(1)}(t) \neq 0$. Hence, in place of the conventional optical theorem

$$\sigma^{(0)}_{\mathrm{tot}\,i} = \frac{(2\pi)^4\hbar^2}{v_i}\frac{1}{\pi}\mathrm{Im}\,R^{(0)}_{ii} \tag{27}$$

we obtain

$$\sigma^{(1)}_{\mathrm{tot}\,i} = \frac{(2\pi)^4\hbar^2}{v_i}\frac{1}{\pi}\mathrm{Im}\,R^{(1)}_{ii} \exp\left[-\frac{1}{\hbar}\int\limits_{-\infty}^{\infty} dt \Gamma^{(1)}(t)\right] , \tag{28}$$

where $R^{(0)}_{f\,i}$ and $R^{(1)}_{f\,i}$ are elements of the reaction matrix $R^{(0)}$ and $R^{(1)}$, respectively, while the labels i and f refer to particular asymptotic states of two colliding particles (averaged over particle spins). Here, in addition,

$$\frac{\mathrm{Im}R^{(1)}_{ii}}{\mathrm{Im}R^{(0)}_{ii}} = \exp\left[-\frac{1}{\hbar}\int\limits_{-\infty}^{\infty} dt \Gamma^{(1)}(t)\right] . \tag{29}$$

To get an idea of the possible magnitude of unknown coupling constant g_Γ and time-deformation conductivity constant λ_Γ we pass to the next example.

(ii) Variation of average particle number

Consider a sample of gas consisting of some identical nonrelativistic particles, *e.g.* electrons or hydrogen atoms, situated in proximity of a pointlike target $\vec{r}_{\mathrm{S}}$ of a big accelerator producing $1/\tau$ particles of all sorts per unit of time. Then, according to Eq. (10), the accelerator during its stationary run excites time-run deformations $\delta t^{\rm ex}(\vec{r},t)$ described by the static field of the form

$$\chi^{\rm ex}(\vec{r}) = \frac{g_\Gamma \lambda_\Gamma}{\tau} \frac{\cos(|\vec{r}-\vec{r}_{\rm S}|/2\lambda_\Gamma)}{|\vec{r}-\vec{r}_{\rm S}|} \sum_{l\, m_l} c_{l\, m_l} Y_{l\, m_l}(\theta,\phi)\,, \tag{30}$$

where $\sum_{l\, m_l} c_{l\, m_l} Y_{l\, m_l}(0,0) = 1$ and $\theta\,,\ \phi$ are spherical angles for $\vec{r}-\vec{r}_{\rm S}$. This field satisfies the equation $(\triangle + 1/4\lambda_\Gamma^2)\,\chi^{\rm ex}(\vec{r}) = -4\pi g_\Gamma \lambda_\Gamma {\rm div}\vec{\jmath}(\vec{r})$, where

$${\rm div}\vec{\jmath}(\vec{r}) = \frac{1}{\tau}\left[\delta^3(\vec{r}-\vec{r}_{\rm S}) + \frac{\cos(|\vec{r}-\vec{r}_{\rm S}|/2\lambda_\Gamma)}{|\vec{r}-\vec{r}_{\rm S}|^3}\sum_{l\, m_l} c_{l\, m_l}\frac{l(l+1)}{4\pi}\, Y_{l\, m_l}(\theta,\phi)\right] \tag{31}$$

gives a mathematical model of the particle current $\vec{\jmath}(\vec{r})$ produced on the target $\vec{r}_{\rm S}$. The field $\chi^{\rm ex}(\vec{r})$, playing the role of an external field for our sample of probe particles, modifies the conventional wave function $\psi^{(0)}(\vec{r}_{\rm P},t) = \psi^{(0)}(\vec{r}_{\rm P})\exp\left[-(i/\hbar)E(t-t_0)\right]$ of any particle in the sample, leading in virtue of Eq. (18) to the following lowest-order perturbed wave function:

$$\psi^{(1)}(\vec{r}_{\rm P},t) = \psi^{(0)}(\vec{r}_{\rm P})\exp\left[-\frac{i}{\hbar}\,E\,(t-t_0)\right]\exp\left[-\frac{1}{\hbar}\int_{t_0}^{t} dt'\,\Gamma^{(1)}(t')\right]\,. \tag{32}$$

Here, by Eq. (16)

$$\Gamma^{(1)}(t) \equiv g_\Gamma \hbar \int_V d^3\vec{r}\rho^{(0)}(\vec{r})\,\chi^{\rm ex}(\vec{r})\,\exp\left(-\frac{c\,t}{2\lambda_\Gamma}\right) = O(g_\Gamma^2) \tag{33}$$

and so

$$\int_{t_0}^{t} dt'\Gamma^{(1)}(t') = \frac{2g_\Gamma\lambda_\Gamma\hbar}{c}\int_V d^3\vec{r}\rho^{(0)}(\vec{r})\chi^{\rm ex}(\vec{r})\exp\left(-\frac{c\,t_0}{2\lambda_\Gamma}\right)\left\{1-\exp\left[-\frac{c(t-t_0)}{2\lambda_\Gamma}\right]\right\}. \tag{34}$$

In our argument, the stationary run of accelerator is switched on at the moment t_0 and still lasts at the later moment t.

In this example, when using Eq. (11) with the operator of particle density of the form $(1/c)J^0(\vec{r}) = \delta^3(\vec{r}-\vec{r}_{\rm P})$, we get

$$\rho^{(0)}(\vec{r}) = \int_V d^3\vec{r}_{\rm P}\,|\psi^{(0)}(\vec{r}_{\rm P})|^2\,\delta^3(\vec{r}-\vec{r}_{\rm P}) = \frac{1}{V}\quad \text{for}\quad \vec{r}\in V \tag{35}$$

if we put $\psi^{(0)}(\vec{r}_{\rm P}) = (1/\sqrt{V})\exp(i\vec{k}\cdot\vec{r}_{\rm P})$. In a similar way, we have

$$\rho^{(1)}(\vec{r}) = \frac{1}{V}\exp\left[-\frac{2}{\hbar}\int_{t_0}^{t} dt'\,\Gamma^{(1)}(t')\right]\quad \text{for}\quad \vec{r}\in V\,. \tag{36}$$

Hence, the lowest-order perturbed average number of particles contained in the sample is given by

$$N^{(1)}(t) = N^{(0)} \exp\left[-\frac{2}{\hbar} N^{(0)} \int\limits_{t_0}^{t} dt' \Gamma^{(1)}(t')\right], \tag{37}$$

where, in particular for gas of hydrogen atoms, $N^{(0)} = N^{(1)}(t_0)$ is equal (in normal conditions) to the Loschmidt number $2.69 \times 10^{19} \mathrm{cm}^{-3}$ multiplied by the volume V. Thus, such an average number of particles changes in time during the stationary run of the accelerator. Of course, this variation is expected to be very slow. For a sample of charged particles, *e.g.* electrons, this conclusion implies unavoidably a simultaneous variation in time of the total average charge.

Using Eqs. (33), (35) and (30) we can write in Eq. (37)

$$\begin{aligned} \int\limits_{t_0}^{t} dt' \Gamma^{(1)}(t') &= \frac{2 g_\Gamma \lambda_\Gamma \hbar}{c} \frac{1}{V} \int\limits_{V} d^3\vec{r} \chi^{\mathrm{ex}}(\vec{r}) \exp\left(-\frac{c\, t_0}{2\lambda_\Gamma}\right) \left\{1 - \exp\left[-\frac{c(t-t_0)}{2\lambda_\Gamma}\right]\right\} \\ &\simeq \frac{2 g_\Gamma^2 \lambda_\Gamma^2 \hbar}{c\,\tau} \frac{\cos(d/2\lambda_\Gamma)}{d} \exp\left(-\frac{c\, t_0}{2\lambda_\Gamma}\right) \left\{1 - \exp\left[-\frac{c(t-t_0)}{2\lambda_\Gamma}\right]\right\}, \end{aligned} \tag{38}$$

where d denotes an average distance of the sample from the pointlike target of accelerator, while the average value of angular part of $\chi^{\mathrm{ex}}(\vec{r})$ over the sample is put equal to 1. If $c(t-t_0)/2\lambda_\Gamma \ll 1$ or $c(t-t_0)/2\lambda_\Gamma \gg 1$, this estimation gives

$$\begin{aligned} \int\limits_{t_0}^{t} dt'\, \Gamma^{(1)}(t') &\simeq \frac{g_\Gamma^2\, \lambda_\Gamma\, \hbar}{\tau} \frac{\cos(d/2\lambda_\Gamma)}{d} \exp\left(-\frac{c\, t_0}{2\lambda_\Gamma}\right)(t - t_0) \\ &\simeq \frac{g_\Gamma^2\, \lambda_\Gamma\, \hbar}{\tau\, d} \exp\left(-\frac{c\, t_0}{2\lambda_\Gamma}\right)(t-t_0) > 0 \end{aligned} \tag{39}$$

or

$$\begin{aligned} \int\limits_{t_0}^{t} dt' \Gamma^{(1)}(t') &\simeq \frac{2 g_\Gamma^2\, \lambda_\Gamma^2\, \hbar}{c\,\tau} \frac{\cos(d/2\lambda_\Gamma)}{d} \exp\left(-\frac{c\, t_0}{2\lambda_\Gamma}\right) \\ &\simeq \frac{2 g_\Gamma^2\, \lambda_\Gamma^2\, \hbar}{c\,\tau\, d} \exp\left(-\frac{c\, t_0}{2\lambda_\Gamma}\right) \simeq 0, \end{aligned} \tag{40}$$

respectively. Here, the last step is valid if, in addition, $d/2\lambda_\Gamma \ll 1$ or $d/2\lambda_\Gamma \gg 1$ (in the second case, because of $[\cos(d/2\lambda_\Gamma) - 1](d/2\lambda_\Gamma)^{-1} \to$

$-\pi\, d\delta(d) = 0$ for $\lambda_\Gamma \to 0$). We can see from Eqs. (37) and (39) or (40) that in the first case (where $c(t–t_0)/2\lambda_\Gamma \ll 1$ and $d/2\lambda_\Gamma \ll 1$) the number $N^{(1)}(t)$ decreases (very slowly) in time, or in the second case (where $c(t–t_0)/2\lambda_\Gamma \gg 1$ and $d/2\lambda_\Gamma \gg 1$) its variation in time is completely negligible, what excludes in this case the positive observation of possible nonunitarity effects.

In order to get an idea of the possible magnitude of unknown constants g_Γ and λ_Γ in the decrement factor in Eq. (37), let us consider our first case and put: $N^{(0)} = 2.69 \times 10^{19} V\text{cm}^{-3}$, the accelerator total production rate $1/\tau \sim 10^8\,\text{sec}^{-1}$, $V/d \sim 100\,\text{cm}^2$ and $t - t_0 \sim 1$ month $\sim 10^6$ sec. Then, making use of Eq. (39), we obtain

$$p^{(1)} \equiv \frac{2}{\hbar} N^{(0)} \int\limits_{t_0}^{t} dt'\, \Gamma^{(1)}(t') \sim 10^{36} g_\Gamma^2 \lambda_\Gamma \exp\left(-\frac{c\,t_0}{2\lambda_\Gamma}\right) \text{cm}^{-1}\,. \tag{41}$$

Hence, requiring for the decrement factor in Eq. (37) the lower bound lying (for instance) in the range

$$\exp(-p^{(1)}) \gtrsim 0.9900 \;\text{ to }\; 0.9999 \tag{42}$$

we get the estimation

$$g_\Gamma^2 \lambda_\Gamma \exp\left(-\frac{c\,t_0}{2\lambda_\Gamma}\right) \lesssim (10^{-38} \;\text{ to }\; 10^{-40})\,\text{cm} \tag{43}$$

(here, the exponent is practically fixed for time t_0 counted from the Big Bang).

Finally, in consistency with our first case, let us consider for the length-dimensional conductivity constant λ_Γ the extreme cosmological option: $\lambda_\Gamma \sim c \times$ age of Universe $\sim 10^{28}$ cm (if age of Universe $\sim 1.5 \times 10^{10}$ yr). Then, under our conjecture that time t ought to be counted from the Big Bang, we have $t_0 \sim t =$ age of Universe and $\exp(-ct_0/2\lambda_\Gamma) \sim \exp(-1/2) = 0.6065$, and so $g_\Gamma^2 \lesssim 10^{-66}$ to 10^{-68} from Eq. (43).

Concluding this Section, we can say that, at present, it seems difficult to estimate the magnitude of g_Γ and λ_Γ, unless we decide to accept for λ_Γ the above extreme option (or another cosmological option postulating also a big λ_Γ, favorable from the viewpoint of experimental discovery potential).

However, the situation changes essentially, if in Eqs. (16) and (17) (defining the energy width Γ) the scalar $\chi(x) \equiv \varphi(\vec{r}, t) \exp(c\,t/2\lambda_\Gamma)$ is used in place of $\varphi(\vec{r}, t)$ (what in Section 2 was considered an unnatural choice in view of the strict correspondence between Eqs. (13) and (14), though this choice is potentially attractive, giving no deviations from the special relativity). Now, for such a choice, the factor $\exp(-c\,t/2\lambda_\Gamma)$ disappears from

Eq. (33) and, in consequence, Eq. (34) is replaced by the formula

$$\int_{t_0}^{t} dt' \Gamma^{(1)}(t') = g_\Gamma \hbar \int_V d^3\vec{r} \rho^{(0)}(\vec{r})\, \chi^{\rm ex}(\vec{r})(t-t_0)\,. \tag{44}$$

Then, instead of Eq. (39) or (40), the relation

$$\int_{t_0}^{t} dt'\, \Gamma^{(1)}(t') \simeq \frac{g_\Gamma^2 \lambda_\Gamma \hbar}{\tau\, d}(t-t_0) > 0 \tag{45}$$

holds for any $c(t-t_0)/2\lambda_\Gamma$ in both the cases $d/2\lambda_\Gamma \ll 1$ and $d/2\lambda_\Gamma \gg 1$. Hence, with the use of lower bound (42), the estimation (43), but now without the factor $\exp(-c\,t_0/2\lambda_\Gamma)$ on its lhs, follows.

Thus, for the extreme cosmological option $\lambda_\Gamma \sim c\times$ age of Universe $\sim 10^{28}$ cm, consistent with our first case, the previous estimate $g_\Gamma^2 \lesssim 10^{-66}$ to 10^{-68} does not change in the new situation, since for this option the irrelevant factor $\exp(-c\,t_0/2\lambda_\Gamma) \sim \exp(-1/2) \sim 1$ (with $t_0 \sim t =$ age of Universe) was used previously in Eq. (43).

But, in this new situation, even the extreme microscopic option $\lambda_\Gamma \sim$ Planck length $= \hbar/M_{\rm PL}\,c \sim 10^{-33}$ cm, consistent with our second case, is experimentally not excluded *a priori*, because the factor $\exp(-c\,t/2\lambda_\Gamma)$ (being as small as $\exp(-61)$ for this option) is now absent from Eq. (33). For such an option the estimate $g_\Gamma^2 \lesssim 10^{-5}$ to 10^{-7} follows from Eq. (43), if the factor $-\exp(c\,t_0/2\lambda_\Gamma)$ on its lhs is omitted.

Note that Γ as given in Eq. (17) is replaced now by the choice

$$\Gamma \equiv g_\Gamma \hbar \int_V d^3\vec{r} \sqrt{j_\mu(x) j^\mu(x)}\, \chi(x) = g_\Gamma \hbar \exp \frac{c\,t}{2\lambda_\Gamma} \int_V d^3\vec{r} \sqrt{j_\mu(x) j^\mu(x)}\, \varphi(\vec{r},t) \tag{46}$$

which, in the nonrelativistic approximation (with respect to matter), is also consistent with Eq. (14), but only within a narrow time interval $0 \le t - t_0 \ll 2\lambda_\Gamma/c$ (for any $t_0 > 0$). In the case of a cosmological option, extremally of $\lambda_\Gamma \sim 10^{28}$ cm, such a time interval is not necessarily so narrow.

Since Γ as given in Eq. (46) is pure time component of a Lorentz four-vector, it is consistent with the special relativity, while inserted into Eq. (15). We prefer, therefore, this choice, though it is not so close to Eq. (14) as the choice (17).

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[6] For a general formalism of projecting out an interacting subsystem from a larger quantum system *cf.* W. Królikowski, J. Rzewuski, *Nuovo Cim.* **25B**, 739 (1975), and references therein. Such a projection leads generally to a nonzero energy-width operator for the remaining quantum subsystem, in particular, to a decay-width operator. In the case of chronodynamics, the remaining quantum subsystem is the whole matter system containing particles of all sorts, while the subsystem projected out is the physical spacetime (the possibility of constructing a larger quantum system including the spacetime as its subsystem is anticipated in this argument: this inclusion should lead to quantum gravity).