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TRANSVERSE POLARIZATION OF QUARK PAIRS CREATED IN STRING FRAGMENTATION *

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Classical arguments predict that the quark and the antiquark of a pair created during string fragmentation are both transversely polarized in the direction of $\hat{\boldsymbol{z}} \times \boldsymbol{q}_\perp$, where $\hat{\boldsymbol{z}}$ is the direction of the pull exerted by the string on the antiquark and $\boldsymbol{q}_\perp$ ($-\boldsymbol{q}_\perp$) is the transverse momentum of the quark (antiquark). The existence of this effect at the quantum-mechanical level is investigated by considering two analogous processes involving the tunnel effect in a strong field: (1) dissociation of the positronium atom (2) electron pair creation. In case (1) the positronium is taken in the $^3P_0$ state to simulate the vacuum quantum numbers $J^{\rm PC} = 0^{++}$. Using the nonrelativistic WKB method, the final electron and positron are indeed found to be transversely polarized along $\hat{\boldsymbol{z}} \times \boldsymbol{p}_\perp$. On the contrary, case (2), treated with the Dirac equation, shows no correlation between transverse polarization and transverse momentum both when the field is uniform and when it depends on z and t. The pair is nevertheless produced in a triplet spin state. The difference between these two results and their relevance to transverse spin asymmetry in inclusive reactions is discussed.

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1. Introduction

The experimental observation of single-spin asymmetries in inclusive hadron production at high energy [1] have been tentatively explained by various models: Thomas precession [2], P-wave orbitals [3], Regge exchange [4,5],

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semi-classical string mechanism [6, 7]. The asymmetric part of the cross-section is of the form $A\ \boldsymbol{P}_\perp \cdot (\hat{\boldsymbol{z}} \times \hat{\boldsymbol{p}}_\perp)$ where $\hat{\boldsymbol{z}}$ is the collision axis, $\boldsymbol{p}_\perp$ the transverse momentum of the produced particle, $\boldsymbol{P}_\perp$ its transverse polarization (*transversity*) or that of one of the colliding baryons. The "hat" denotes a unitary vector: $\hat{\boldsymbol{p}}_\perp \equiv \boldsymbol{p}_\perp/|\boldsymbol{p}_\perp|$. A similar effect correlating the transversity of the leading quark of a jet with the transverse momentum of one of the fastest particles was predicted by Collins [8]. This effect, if confirmed experimentally, would serve as a transverse "quark polarimeter". It was used in Ref. [7] to explain single spin asymmetry in inclusive meson production.

In this paper we start from a popular string hadronization picture [9–11], in which quark–antiquark pairs are produced from the string by a tunneling mechanism analogous to the Schwinger mechanism for pair creation in a strong homogeneous electric field [12–20]. This picture accounts rather well

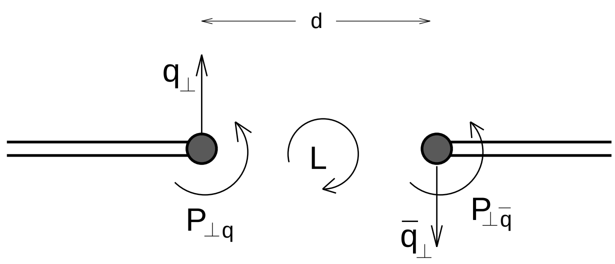

Fig. 1. Semi-classical string mechanism of quark polarization. The orbital angular momentum of the $\bar{q}q$ pair is compensated by the spin of q and $\bar{q}$, thereby causing the correlation between spin and transverse momentum of the quark and the antiquark.

for the exponential cut–off in $\boldsymbol{p}_\perp$, the relative suppression of strange quarks and the almost complete suppression of heavy quarks. Let us recall the semi-classical arguments [6] for a transverse polarization of the quark and the antiquark (see Fig. 1):

— the quark and the antiquark come from a pair fluctuation like those which occur in ordinary vacuum. At zero separation, the pair has zero total energy-momentum. In particular, quark and antiquark have opposite transverse momentum $\boldsymbol{q}_\perp$ and $\bar{\boldsymbol{q}}_\perp$. In the vacuum case, the pair stays virtual and disappears after a time of the order of the quark Compton wave length. In the string case, the linear mass density $\kappa \simeq 1$ GeV/fm of the string is converted into energy of the pair, which becomes real at a longitudinal separation $d = 2E_\perp/\kappa$, where $E_\perp = (m^2 + \boldsymbol{q}_\perp^2)^{1/2}$ is the quark (or antiquark) transverse energy.

— The orbital angular momentum

$$\boldsymbol{L} = d\ \hat{\boldsymbol{z}} \times \bar{\boldsymbol{q}}_\perp\,, \tag{1}$$

is compensated, at least partly, by the spins of the quark and the antiquark. Assuming equal polarization $\boldsymbol{P} \equiv 2\,\langle \boldsymbol{s}_q \rangle = 2\langle \boldsymbol{s}_{\bar{q}} \rangle$ for the quark and the antiquark, we have therefore

$$\boldsymbol{P} = -\boldsymbol{L}\; f(L)\,, \tag{2}$$

where $f(L)$ is a reduction factor insuring that $|\boldsymbol{P}|$ is smaller than unity, *e.g.*, $f(L) = (1+L)^{-1}$.

To summarize, the polarizations of the quark and of the antiquark are of the form

$$\boldsymbol{P}^q = \boldsymbol{P}^{\bar{q}} = -A(q_\perp)\; \hat{\boldsymbol{z}} \times \hat{\boldsymbol{q}}_\perp\,, \tag{3}$$

where

$$A(q_\perp) = L\, f(L) \;\leq \min\{L, 1\} \tag{4}$$

is the *analysing power* of the mechanism and

$$L = 2\kappa^{-1}\, q_\perp\, (m^2 + q_\perp^2)^{1/2}\,. \tag{5}$$

The compensation between $\boldsymbol{L}$ and $\boldsymbol{s}_q + \boldsymbol{s}_{\bar{q}}$ is further motivated by the phenomenological success of the "$^3P_0$" model of quark pair creation in hadronic decay [21]. This model assumes that the $(q\,\bar{q})$ pair is created in the $^3P_0$ spin state, therefore having the vacuum quantum numbers $J^{\rm PC} = 0^{++}$ (by contrast, a model in which the pair comes from one gluon gives $J^{\rm PC} = 1^{--}$).

The string, as well as the constant electric field, is not rotationally invariant and therefore the total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{s}_q + \boldsymbol{s}_{\bar{q}}$ is not conserved during tunneling. This is a weak point of the classical model reviewed above. C- and P- quantum numbers are also not separately conserved. Therefore it is desirable to check the existence of the transverse polarization effect in a true quantum-mechanical model. Although too difficult at present in the string theory, this is quite possible for the problem of pair creation in strong homogeneous field. To begin with (in Section 2) we will consider the nonrelativistic process of the dissociation of a positronium atom, which we assume to be in the $^3P_0$ state. The relativistic case of pair creation in a field $\vec{\mathcal{E}}(t,z)$ parallel to the $\hat{\boldsymbol{z}}$ axis (independent of x and y but not necessarily on t and z) will be considered in Section 3, using the Dirac hole theory for the positron. The different results obtained in these two cases will be discussed in Section 4.

2. Positronium dissociation

The nonrelativistic $e^+\,e^-$ system in a constant electric field is governed by the Hamiltonian

$$H = \frac{\boldsymbol{p}_+^2}{2m} + \frac{\boldsymbol{p}_-^2}{2m} - \frac{\alpha}{|\boldsymbol{r}_+ - \boldsymbol{r}_-|} + e\,\vec{\mathcal{E}}\cdot(\boldsymbol{r}_- - \boldsymbol{r}_+)$$

$$= \frac{\boldsymbol{p}_{\rm tot}^2}{4m} + \frac{\boldsymbol{p}^2}{m} - \frac{\alpha}{r} - Fz \equiv K_{\rm barycentre} + K_r + V_c - Fz\,. \qquad (6)$$

The motion of the barycentre can be separated from the relative motion and from now on we will consider only the latter, governed by the three last terms of Eq. (6). $\boldsymbol{r} = \boldsymbol{r}_+ - \boldsymbol{r}_-$ and $\boldsymbol{p} = (\boldsymbol{p}_+ - \boldsymbol{p}_-)/2$ are the relative position and momentum. We take $F \equiv e\mathcal{E}_z > 0$. The potential $V_c - Fz$ is shown in Fig. 2. Note that the Hamiltonian is spin-independent, therefore cannot produce spin effects by itself. However, we will assume that the initial state of the pair is a ${}^3P_0$ positronium (corresponding to the vacuum quantum numbers):

$$\varPhi(\boldsymbol{r}) = f(r)\ \left[\, Y_1^{-1}(\hat{\boldsymbol{r}})\ |+1\rangle\ - Y_1^0(\hat{\boldsymbol{r}})\ |0\rangle\ + Y_1^{+1}(\hat{\boldsymbol{r}})\ |-1\rangle\,\right], \qquad (7)$$

where the kets denote the three different triplet spin states,

$$|+1\rangle = |\uparrow\uparrow\rangle\,, \qquad |0\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}\,, \qquad |-1\rangle = |\downarrow\downarrow\rangle\,. \qquad (8)$$

In this way, the orbital motion and the spin are entangled. This state is a bound eigenstate of $K_r + V_c$ with energy $-B$. After turning on the external electric field, the relative wave function will eventually migrate toward $z = +\infty$ by tunnel effect, which means that the positron runs toward $z = +\infty$ and the electron runs toward $z = -\infty$. The pair remains in the spin-triplet subspace. We chose $\hat{\boldsymbol{x}}$ as the spin quantization axis and are interested in the relative probabilities to obtain the different final spin states $|S_x\rangle$, with $S_x \equiv s_x^+ + s_x^-$, for a given final transverse momentum $\boldsymbol{q}_\perp = q\,\hat{\boldsymbol{y}}$. The corresponding asymptotic state is an eigenstate of $K_r - Fz$, again with energy $-B$, and its wave function is

$$\varPsi(\boldsymbol{r}) = \psi(\boldsymbol{r})\ |S_x\rangle = \mathrm{e}^{iqy}\ g(z - z_t)\ |S_x\rangle\,, \qquad (9)$$

where g is the Airy function, solution of the one-dimensional Schrödinger equation

$$\left[\frac{p_z^2}{m} - F\,(z - z_t)\right]\ g(z) = 0 \qquad (10)$$

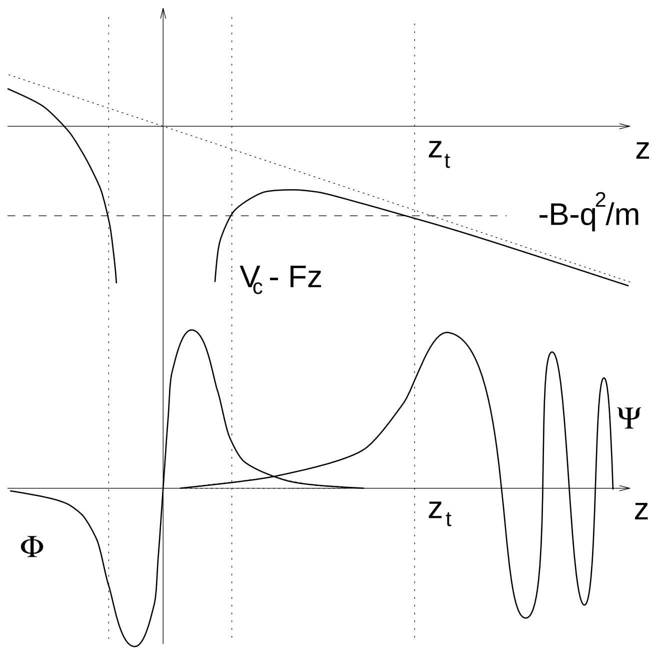

Fig. 2. Positronium dissociation in constant electric field. The top curve is the superposition of the Coulomb potential and the external electric potential. The classically allowed region for a given transverse momentum q is limited by the horizontal dashed line. The bottom curves are the p wave function of the positronium, Φ, and the wave function of the free solution, Ψ (restricted to the z axis). The overlap of Φ and Ψ is responsible for the tunneling.

and $z_t = (B + q^2/m)/F$ is the classical turning point (see Fig. 2). Here we give a heuristic proof and an estimation [1] of the spin asymmetry:

— We assume that the tunneling length $z_t \sim m\alpha^2/(16F)$ is much larger than the radius $\sim 8/(m\alpha)$ of the bound state. Near the bound state, we can use the WKB approximation for g:

$$g(z - z_t) \sim \mathrm{e}^{\lambda(z-z_t)} \tag{11}$$

with $\lambda = (mB + q^2)^{1/2}$.

— Near the origin, $\psi(\boldsymbol{r})$ can be expanded in partial waves:

$$\psi(\boldsymbol{r}) \simeq \mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{r}} = 4\pi \sum_{l,m} i^l\ j_l(pr)\ (-1)^m\ Y_l^{-m}(\hat{\boldsymbol{p}})\ Y_l^m(\hat{\boldsymbol{r}}) \tag{12}$$

[1] A more rigourous treatment could be done using the method of Landau & Lifshitz (*Quantum Mechanics* [22]), for hydrogen dissociation in a strong electric field.

with $\boldsymbol{p} = q\hat{\boldsymbol{y}} - i\lambda\hat{\boldsymbol{z}},\ p = i(\lambda^2 - q^2)^{1/2},\ \hat{\boldsymbol{p}} = \boldsymbol{p}/p$. We assume that tunneling couples mainly the components of Φ and Ψ with the same $Y_l^m(\hat{\boldsymbol{r}})$, and the tunneling amplitude is proportional to the coefficient of this harmonic (this is intuitive if we consider the inverse process of trapping an initially free particle into the Coulomb potential well). The $l = 1$ terms of ψ are proportional to

$$j_1(pr)\ |\boldsymbol{p}^2|^{-1/2}\ [\ -(p_y - ip_z)\, Y_1^{+1}(\hat{\boldsymbol{r}}) + (p_y + ip_z)\, Y_1^{-1}(\hat{\boldsymbol{r}})\ ]$$

$$= j_1(pr)\ (\lambda^2 - q^2)^{-1/2}\ [\ (\lambda - q)\, Y_1^{+1}(\hat{\boldsymbol{r}}) + (\lambda + q)\, Y_1^{-1}(\hat{\boldsymbol{r}})\ ]\,. \quad (13)$$

Comparing with Eq. (7) we find that the tunneling amplitudes squared are in the ratio

$$|T(S_x = +1)|^2\ :\ |T(S_x = 0)|^2\ :\ |T(S_x = -1)|^2\ =\ |\lambda + q|^2\ :\ 0\ :\ |\lambda - q|^2\,. \quad (14)$$

Note the vanishing of $T(S_x = 0)$. It happens because the second term of Φ is odd in x and cannot tunnel to Ψ, which is even in x (orbital x-parity is a symmetry of the problem). The polarization of the electron and the positron are equal and given by

$$\boldsymbol{P} = \frac{|T(+1)|^2 - |T(-1)|^2}{|T(+1)|^2 + |T(-1)|^2}\ \hat{\boldsymbol{x}} = -\frac{2\sqrt{mB + q^2}}{mB + 2q^2}\ \hat{\boldsymbol{z}} \times \boldsymbol{q}_\perp\,. \quad (15)$$

We see that the polarization of the created particle is of the form (3), (4) and has the same sign as predicted by the classical string arguments. Classical trajectories leading to the positronium dissociation shown in Fig. 3 explain this fact intuitively.

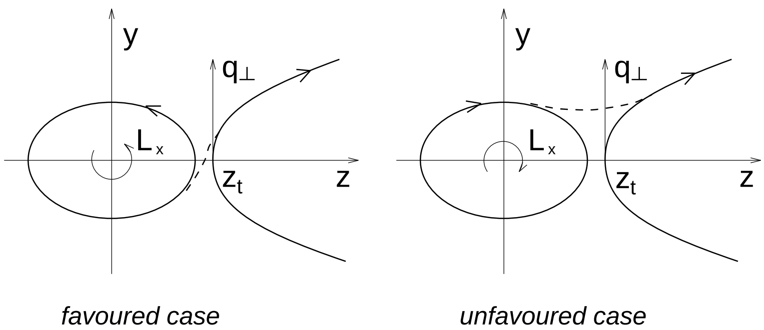

Fig. 3. Classical trajectories of the positronium dissociation for the two cases $L_x = +1$ and $L_x = -1$. The dashed lines are (classically forbidden) tunneling trajectories.

3. Pair creation in strong field

In a static constant electric field, electron–positron pairs are created spontaneously in vacuum, the positron running in the field direction and the electron in the opposite direction. In the Dirac hole theory, this process is interpreted as tunneling of the electron from the Dirac sea in one half-space to the upper continuum in the opposite half-space, without changing its total energy, as shown in Fig. 4 [12]. We will study the more general

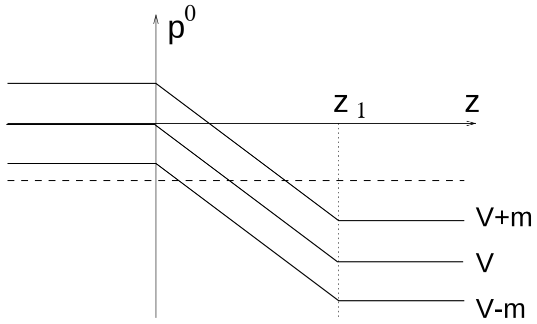

Fig. 4. The Dirac sea distorted by constant electric field between 0 and z_1. A negative-energy electron on the left-hand side can reach the upper continuum of the right-hand side by tunneling trough the forbidden band, becoming physical electron. The hole created on the left-hand side is the physical positron. The dashed line represent the energy of the tunneling wave function.

case of a time- and z-dependent field $\vec{\mathcal{E}}(t,z)$ parallel to $\hat{\boldsymbol{z}}$ and consider Dirac wave functions of definite transverse momentum parallel to the $\hat{\boldsymbol{y}}$ axis: $\boldsymbol{p}_\perp \equiv (p_x, p_y) = (0, q)$. Discarding the trivial y-dependence in $\exp(iqy)$, the Dirac equation reads

$$[\, i\partial_t + eA_0 - \alpha_z(i\partial_z + eA_z) - q\alpha_y - m\beta \,]\ \psi(t,z) = 0\,. \tag{16}$$

Calculations will be simpler using the light-cone coordinates

$$\eta \;=\; \frac{t+z}{2}\,, \qquad \xi = \frac{t-z}{2}\,, \tag{17}$$

$$\partial_\eta \;=\; \partial_t + \partial_z\,, \qquad \partial_\xi = \partial_t - \partial_z\,, \tag{18}$$

$$A_\eta = A_0 + A_3 = A^0 - A^3\,, \qquad A_\xi = A_0 - A_3 = A^0 + A^3\,. \tag{19}$$

Furthermore we choose the spinorial representations of the Dirac matrices

$$\alpha^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix}\,, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\,, \quad \Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}\,, \tag{20}$$

where σ^i are the Pauli matrices. The Dirac equation becomes

$$[\,\alpha^\eta\,(i\partial_\eta + eA_\eta) + \alpha^\xi\,(i\partial_\xi + eA_\xi) - q\alpha_y - m\beta\,]\;\psi(\eta,\xi)$$

$$\equiv \begin{pmatrix} i\partial_\eta + eA_\eta & iq & -m & 0 \\ -iq & i\partial_\xi + eA_\xi & 0 & -m \\ -m & 0 & i\partial_\xi + eA_\xi & -iq \\ 0 & -m & iq & i\partial_\eta + eA_\eta \end{pmatrix} \psi(\eta,\xi) = 0\,. \qquad (21)$$

This equation is invariant under the transformation $\psi_1 \leftrightarrow \psi_4,\ \psi_2 \leftrightarrow \psi_3$, which is performed by the matrix $\beta\,\Sigma_x = \gamma^x\gamma^5$. This matrix commutes with the Hamiltonian and has eigenvalues ± 1. For a particle at rest, $\beta\,\Sigma_x = 2s_x$. For a particle with nonzero p_y and p_z, it is the x-component of the transversity operator. For the states under consideration, the $\beta\,\Sigma_x$ transformation is equivalent to the parity about the (y,z) plane,

$$P_{yz} = \mathrm{e}^{-i\pi J_x}\,P = -i\,\Sigma_x \times \beta \times P_{\text{intrinsic}} \times P_{yz}^{\text{orbital}}\,. \qquad (22)$$

Since $p_x = 0$, $P_{yz}^{\text{orbital}} = 1$ and $\beta\,\Sigma_x$ is equivalent to P_{yz} (up to a phase factor, depending on the choice of the intrinsic parity). The $\beta\,\Sigma_x$ invariance comes therefore from the symmetry of the problem about the (y,z) plane.

Taking the transversity eigenstates

$$\psi^\uparrow = \frac{1}{\sqrt{2}}\begin{pmatrix} F \\ G \\ G \\ F \end{pmatrix}, \qquad \psi^\downarrow = \frac{1}{\sqrt{2}}\begin{pmatrix} F \\ G \\ -G \\ -F \end{pmatrix}, \qquad (23)$$

we come to coupled differential equations

$$(i\partial_\eta + eA_\eta)\;F = (\pm m - iq)\;G\,, \qquad (24)$$

$$(i\partial_\xi + eA_\xi)\;G = (\pm m + iq)\;F\,, \qquad (25)$$

where $\pm$ is the sign of the transversity.

From these we get the second order differential equations

$$[\,(i\partial_\xi + eA_\xi)\,(i\partial_\eta + eA_\eta) - M^2\,]\;F = 0\,, \qquad (26)$$

$$[\,(i\partial_\eta + eA_\eta)\,(i\partial_\xi + eA_\xi) - M^2\,]\;G = 0\,, \qquad (27)$$

where $M = (m^2 + q^2)^{1/2}$ is the "transverse energy" of the electron. Note that Eqs (26) and (27) depend only on the transverse energy, not on the transversity. We can infer that there is no correlation between transverse spin and transverse momentum, contrarily to the case of positronium dissociation. Let us check this result more carefully. Denoting by $F_{m,q}$ and $G_{m,q}$ the solution of Eqs (24), (25) and setting $m + iq = M\,\mathrm{e}^{i\alpha}$, we have

$$F_{m,q} = F_{M,0}\mathrm{e}^{\mp i\alpha/2}\,, \qquad G_{m,q} = G_{M,0}\mathrm{e}^{\pm i\alpha/2}\,. \tag{28}$$

These equations tell how the solution transforms under a "rotation" in the (m,q) plane (which leaves M invariant). The current 4-vector $(J^0, J^i) = (\psi^\dagger\psi, \psi^\dagger\alpha^i\psi)$ is given by

$$J^0 = |F|^2 + |G|^2\,, \qquad J^x = 0\,, \qquad J^z = |F|^2 - |G|^2\,, \tag{29}$$

$$J^y = \ 2\ \mathrm{Im}\,(F^* G) = \ \frac{2q}{M^2}\mathrm{Re}\,[\,F^*\,(i\partial_\eta + eA_\eta)\,F\,] \pm \frac{m}{M^2}\ \partial_\eta |F|^2\,. \tag{30}$$

The last form of J^y has been obtained using Eq. (24). The first part is the "convection" term, the second part is the "magnetization" term. J^0, J^z and the convection part of J^y do not depend on the transversity. This confirms the absence of transverse spin effect in the Schwinger mechanism of pair creation. The magnetization term of J^y depends on transversity but is located at the edges of the wave packet (we can replace $\partial_\eta|F|^2$ by $(\partial_0\, J^z + \partial_z\, J^0)/2$, using current conservation) and is not observable by a macroscopic $e^\pm$ detector.

To fix the idea, let us consider a homogeneous field $\vec{\mathcal{E}} = \mathcal{E}\,\hat{\boldsymbol{z}}$ confined in the region $0 < \xi < \xi_1$. This field corresponds to a capacitor moving with light velocity. We use the null-plane gauge

$$A_\xi = 0\,, \quad A_\eta = \begin{cases} 0 & \text{if} \quad \xi < 0; \\ 2\,\xi\,\mathcal{E} & \text{if} \quad 0 < \xi < \xi_1; \\ 2\,\xi_1\,\mathcal{E} & \text{if} \quad \xi_1 < \xi. \end{cases} \tag{31}$$

At fixed light-cone momentum $p_\eta \equiv p_0 + p_3 \equiv p^0 - p^z$, we have the following solutions:

$$G \ = \ \mathrm{e}^{-ip_\eta\eta - iM^2\,\xi/p_\eta}\,, \quad F = \frac{\pm m - iq}{p_\eta}\,G\,, \quad (\xi < 0)\,, \tag{32}$$

$$G \ = \ \mathrm{e}^{-ip_\eta\eta}\,\left(\frac{2\,\kappa\xi}{p_\eta} + 1\right)^{-iM^2/2\kappa}\,, \quad F = \frac{\pm m - iq}{p_\eta + 2\,\kappa\xi}\,G\,, \quad (0 < \xi < \xi_1)\,, \tag{33}$$

$$G \ = \ \left(\frac{P'_\eta}{P_\eta} + 1\right)^{-iM^2/2\kappa} \exp\left[-ip_\eta\eta - iM^2\frac{\xi - \xi_1}{p_\eta + 2\kappa\xi_1}\right],\ F = \frac{\pm m - iq}{p_\eta + 2\kappa\xi_1}G,\ (\xi_1 < \xi). \tag{34}$$

$\kappa = e\mathcal{E}$ is the electric force and $P_\eta = p_\eta$ and $P'_\eta = p_\eta + 2\,\kappa\xi_1$ are the initial and final "mechanical" (gauge invariant) light-cone momenta ($P_\mu \equiv p_\mu + eA_\mu$).

Electrons from the Dirac sea at $\xi < 0$ having light-cone momentum P_η in the range $[-2\kappa\xi_1, 0]$ become physical electrons ($P'_\eta > 0$) at

$$\xi_c \equiv -\frac{P_\eta}{2\kappa}\,. \tag{35}$$

The electron flux going through the hyperplane $\xi =$ constant being proportional to $J^\xi = (J^0 - J^z)/2 = |G|^2$, the tunneling probability is

$$\frac{J^\xi(\eta, \xi > \xi_c)}{J^\xi(\eta, \xi < \xi_c)} = \mathrm{e}^{-\pi\, M^2/\kappa}\,. \tag{36}$$

This result is clearly independent on spin.

Some remarks have to be made concerning the above calculation:

— The last result is obtained giving a small positive imaginary part to P_η. This corresponds to the physical condition that the field does not interact with the wave at $t = -\infty$.

— In such a field, the created electron escape the field region (at $\xi = \xi_1$) but not the positron. This can be seen from their classical trajectories in the field region shown in Fig. 5:

$$\eta = \eta_0 \pm \frac{M}{2\kappa}\mathrm{e}^{r}\,, \quad \xi = \xi_c \mp \frac{M}{2\kappa}\mathrm{e}^{-r}\,, \quad y = y_0 + q\,\frac{r}{\kappa}\,, \tag{37}$$

where η_0 and y_0 are free parameters and $r = \pm\,(\kappa/M)\times$(proper time) is the rapidity of the $e^\pm$.

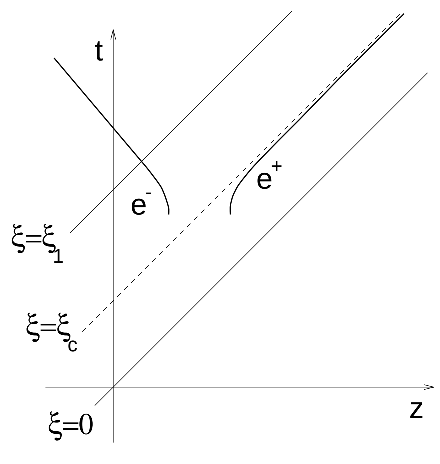

Fig. 5. Classical trajectories of the electron and the positron described by Eq. (37) created in the field Eq. (31) confined in the region bounded by the two solid diagonal lines. The electron escapes from the field region while the positron remains in the field forever.

— $J^\eta = |F|^2$ becomes infinite at $\xi = \xi_c$. The current looks like a "jet stream". It is due to the deflection of the incoming flux by the field during infinite time.

4. Discussion

After obtaining the positive result with the positronium model, the absence of transverse polarization in the Schwinger mechanism was rather unexpected. This absence does not happen due to standard discrete symmetries like C, P and T but due to invariance with respect to the particular transformation (28).

In spite of the difference between the positronium dissociation and the Schwinger mechanism, both models predict that the electron and the positron have equal spin components s_x^+ and s_x^- along the $\hat{x}$ axis:

$$s_x^+ s_x^- = +\tfrac{1}{4} \,. \tag{38}$$

This property is built-in in the $^3P_0$ positronium model. For the Schwinger mechanism, both s_x^+ and s_x^- are equal to $\frac{1}{2}\beta\Sigma_x$. Indeed, a positron at rest has $s_x^+ = -\frac{1}{2}\Sigma_x$, since it is a "hole", and $\beta = -1$ because the corresponding (unoccupied) state has negative energy. Eq. (38) imply that the pair is in a triplet state also for the Schwinger mechanism.

The difference between the two models may be connected with their different chiral properties, which appear in the $m \to 0$ limit: $^3P_0$ positronium dissociation is similar to the decay of a 0^{++} particle into two fermions, in which the fermions necessarily have opposite chirality (equal helicity). On the contrary, the Schwinger mechanism involves only vector interactions, therefore e^+ and e^- must have the same chirality (opposite helicity). The difference is particularly important at zero transverse momentum, where conservation of angular momentum J_z requires $s_z^- = -s_z^+$. Positronium dissociation is allowed for $m = q_\perp = 0$, whereas pair creation (with back-to-back e^+ and e^-) is forbidden [2].

The absence of correlation between spin and transverse momentum in the Schwinger mechanism does not preclude such correlations for $q\bar{q}$ pairs created during string breaking. There are many effects of string breaking

[2] Eq. (36) seems to allow pair creation at $m = q_\perp = 0$. However, this is a too special case where the field (31) occupies an infinite domain of the (z,t) plane. In fact, for $m = q_\perp = 0$, the functions F and G decouple (see Eqs (26), (27) and the left- and right-moving currents $J^\xi = |G|^2$ and $J^\eta = |F|^2$ are separately conserved. If the field domain is finite in the (z,t) plane, e^+e^- pairs are produced at $m = q_\perp = 0$, but with e^+ and e^- going in the *same* direction. e^+ and e^- going in opposite direction belong to independent pairs.

not included in the Schwinger mechanism. First of all, the chromoelectric field between the quark and the antiquark is totally screened after their creation, unlike in the Schwinger process where the field extends everywhere all the time [3]. Secondly, the field of the QCD string is confined to a thin tube. One way to simulate this fact in QED is to impose the MIT-bag boundary conditions for the electron in the transverse coordinates [17, 18]. In our problem, we cannot use this method because we need a well-defined transverse momentum. Important effects may also come from the "transverse inertia" of the string, because part of it must follow the transverse motion of the quark. Finally, the Schwinger mechanism does not include the final state interactions which recombine the quarks from different pairs to form hadrons and resonances. Resonances probably play a major role in single spin asymmetries [5, 23, 24], because the latter come from the interference between different spin amplitudes having different phases.

To summarize, the interplay of spin and transverse momentum in pair creation is a subtle phenomenon and we cannot conclude from the simple model presented above whether the correlation exists or not.

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