PRODUCTION OF Σ HYPERNUCLEI IN THE (K^-, π^+) REACTION AND THE ΣN INTERACTION*

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Pion spectra from (K^-, π^+) reactions on ^{16}O and ^{9}Be targets in the region of Σ production are analyzed in impulse approximation for different strengths of the Σ single particle potential. It is concluded that this potential is repulsive. It is pointed out that among the Nijmegen models of the hyperon–nucleon interaction only model F is compatible with this conclusion.

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1. Introduction

Our knowledge of the hyperon (Y) nucleon (N) interaction is quite incomplete because the free space YN scattering data are extremely limited. In this situation, the construction of a realistic YN interaction usually relies on a combined analysis of the available NN and YN scattering data, in which SU symmetry relations are assumed. In this way, three models of the YN interaction have been worked out by the Nijmegen group: model D [1]. model F [2], and the soft-core model [3]. These models of the YN interactions do not lead to identical predictions concerning the hyperon interaction in hypernuclei. In particular, they lead to different predictions of the single particle (s.p.) potential of the Σ hyperon in Σ hypernuclei, U_Σ (see [4–6]).

In the present paper we discuss an empirical determination of U_Σ and the possibility of finding in this way the best model of the YN interaction, *i.e.*, the one compatible with the empirical U_Σ. Our empirical determination of U_Σ consists of fitting U_Σ to the pion spectrum measured in the strangeness exchange reaction (K^-, π^+). We simply calculate the cross section for the

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(K^-,π^+) reaction in the impulse approximation with different strengths of U_Σ, and compare it with experimental results.

We consider the case of the (K^-,π^+) reaction because here only one direct elementary strangeness exchange process $K^-P\to\pi^+\Sigma^-$ occurs, which leads to the formation of a definite hypernucleus, namely a Σ^- hypernucleus. (In the case of the (K^-,π^-) reaction both Σ^+ and Σ^0 hypernuclei may be produced.)

The paper is organized as follows. In Section 2, we calculate in the impulse approximation the cross section for the (K^-,π^+) reaction, and obtain final expressions for the spectrum of the produced pions. In Section 3, we present our results for the (K^-,π^+) reaction on the ^{16}O and ^{9}Be targets and compare them with the experimental data. Discussion of our results and conclusions are presented in Section 4.

2. Cross section for the (K^-,π^+) reaction in the impulse approximation

In the Σ s.p. model, the motion of Σ^- in the hypernucleus is described by the wave function $\psi_\Sigma(\boldsymbol{r})$ which is the solution of the s.p. Schrödinger equation with the s.p. potential $U_\Sigma(r)=V_\Sigma(r)+iW_\Sigma(r)$, where W_Σ represents the absorption due to the $\Sigma\Lambda$ conversion process $\Sigma^-P\to\Lambda N$. The target protons involved in the elementary process $K^-P\to\pi^+\Sigma^-$ are described by the wave functions $\psi_P(\boldsymbol{r})$ which are bound state solutions of the s.p. Schrödinger equation with the shell model potential $V_P(r)$.

We want to calculate the cross section for the (K^-,π^+) reaction in which the kaon with momentum $\boldsymbol{k}_K$ (in units of $\hbar$) and energy E_K transfers its strangeness to the target proton in the state ψ_P (with s.p. energy e_P) and emerges in the final state as pion in the direction $\hat{k}_\pi$ with energy E_π (both E_K and E_π are total energies including rest masses). We denote by $\boldsymbol{k}_\Sigma$ the momentum of the Σ^- hyperon in the final state. We apply the impulse approximation with K^- and π^+ plane waves, assume a zero-range spin-independent interaction for the elementary process $K^-P\to\pi^+\Sigma^-$ (with a constant transition matrix t) and obtain (with spins suppressed in the notation):

$$\frac{d^3\sigma}{d\hat{k}_\Sigma d\hat{k}_\pi dE_\pi}=\frac{E_KE_\pi M_\Sigma c^2k_\pi k_\Sigma}{(2\pi)^5(\hbar c)^6k_K}\left|t\int d\boldsymbol{r}\exp(-i\boldsymbol{qr})\psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)*}\psi_P(\boldsymbol{r})\right|^2, \tag{1}$$

where the momentum transfer $\boldsymbol{q}=\boldsymbol{k}_\pi-\boldsymbol{k}_K$, and $\psi_{\Sigma,\boldsymbol{k}_\Sigma}(\boldsymbol{r})^{(-)}$ is the Σ scattering wave function which behaves asymptotically as $\exp(i\boldsymbol{k}_\Sigma\boldsymbol{r})$ + incoming wave.

The energy conservation imposes the following relation between the energies of the particles involved in the (K^-,π^+) reaction:

$$\varepsilon_\Sigma = \frac{\hbar^2 k_\Sigma^2}{2M_\Sigma} = M_P c^2 - M_\Sigma c^2 + e_P + E_K - E_\pi\,. \tag{2}$$

Notice that the recoil of the hypernucleus is neglected here.

If only the energy spectrum of pions at fixed $\hat{k}_\pi$ is measured then this spectrum, $d^2\sigma/d\hat{k}_\pi dE_\pi$, is obtained by integrating cross section (1) over $\hat{k}_\Sigma$:

$$\frac{d^2\sigma(l_P j_P)}{d\hat{k}_\pi dE_\pi} = \int d\hat{k}_\Sigma \left\{ \frac{d^3\sigma(l_P j_P)}{d\hat{k}_\Sigma d\hat{k}_\pi dE_\pi} \right\}. \tag{3}$$

Here, we have indicated explicitly the quantum numbers $l_P j_P$ of the s.p. state ψ_P of the proton on which the elementary process $K^- P \to \pi^+ \Sigma^-$ takes place. To get the experimental pion spectrum, we have to sum expression (3) over all states occupied by target protons.

Let us consider the case when the $l_P j_P$ proton shell is closed, *i.e.*, when all the $2j_P+1$ magnetic substates of the $l_P j_P$ shell are occupied by protons (as is the case with the ^{16}O target considered in the next Section). When we introduce the spin coordinate ξ and the Σ and P magnetic quantum numbers μ_Σ and m_P, we may write the total contribution of the $l_P j_P$ proton shell to the pion spectrum in the form:

$$\frac{d^2\sigma(l_P j_P)}{d\hat{k}_\pi dE_\pi} = \frac{E_K E_\pi M_\Sigma c^2 k_\pi k_\Sigma}{(2\pi)^5 (\hbar c)^6 k_K} |t|^2 S(l_P j_P)\,, \tag{4}$$

$$S(l_P j_P) = \sum_{\mu_\Sigma m_P} \int d\hat{k}_\Sigma \left| \int d\tau \exp(-i\boldsymbol{q}r) \psi_{\Sigma,\boldsymbol{k}_\Sigma \mu_\Sigma}(\boldsymbol{r})^{(-)*} \psi_{P,l_P j_P m_P}(\boldsymbol{r}) \right|^2 , \tag{5}$$

where the Σ scattering wave function $\psi^{(-)}_{\Sigma,\boldsymbol{k}_\Sigma \mu_\Sigma}$ behaves asymptotically as a plane wave with momentum k_Σ and spin projection μ_Σ + incoming wave, $\psi_{P,l_P j_P m_P}$ is the normalized proton wave function in the s.p. state with quantum numbers $l_P j_P m_P$, and $\int d\tau$ denotes the $\boldsymbol{r}$ integration and the ξ summation.

A straightforward calculation (similar to that in [7]) leads to the following expression for $S(l_P j_P)$ in terms of Wigner 3-j and 6-j symbols:

$$S(l_P j_P) = (4\pi)^2 (2j_P+1)(2l_P+1) \sum_{Llj} (2j+1)(2L+1)(2l+1) \times \begin{pmatrix} L & l_P & l \\ 0 & 0 & 0 \end{pmatrix}^2 \begin{Bmatrix} j_P & 1/2 & l_P \\ l & L & j \end{Bmatrix}^2 |\langle lj | j_L(qr) | l_P j_P \rangle|^2 \tag{6}$$

with

$$\langle lj|j_L(qr)|l_Pj_P\rangle = \int dr u_{lj}(k_\Sigma;r)^{(-)*} j_l(qr) R_{l_Pj_P}(r)\,, \tag{7}$$

where $R_{l_Pj_P}(r)/r$ is the radial part of $\psi_{P,l_Pj_Pm_P}$ and $u_{lj}(k_\Sigma;r)^{(-)}/r$ is the radial part of the lj component of $\psi_{\Sigma,\boldsymbol{k}_\Sigma\mu_\Sigma}$, whose asymptotic behaviour is $u_{lj}(k_\Sigma;r)^{(-)}/r - j_l(k_\Sigma r) \sim h_l^{(2)}(k_\Sigma r)$.

In the case of the ^{9}Be target considered in the next Section, we assume a simple shell model in which there are only two $p_{3/2}$ protons (coupled to $J = 0$) whereas in a closed $p_{3/2}$ shell there would be four protons. A straightforward calculation leads to the not surprising conclusion that in this case $S(1, 3/2)$ in (4) should be replaced by $\frac{1}{2}S(1, 3/2)$.

3. Results for the ^{16}O and ^{9}Be targets

Let us start with the case of the ^{16}O target. Similarly as in [8], we assume for the Σ s.p. potential the form of a square well (without the repulsive surface bump considered in [8]),

$$U_\Sigma(r) = -(V_{\Sigma 0} + iW_{\Sigma 0})\theta(R - r)\,, \tag{8}$$

with $R = 3$ fm. For the depth of the absorptive potential, we use the value $W_{\Sigma 0} = 2.5$ MeV, obtained in [9] and [10] from the $\Sigma^- P \to \Lambda n$ cross section. For the depth $V_{\Sigma 0}$ we assume values varying from -20 to 20 MeV. Notice that $V_{\Sigma 0}$ is positive for an attractive and negative for a repulsive potential.

For the proton s.p. potential, we use — as in [8] — the form:

$$U_P(r) = -V_{P0}\theta(R - r) - V_{Pls}\boldsymbol{ls}\delta(R - r)\,, \tag{9}$$

with $V_{P0} = 46$ MeV and $V_{Pls} = 15$ MeV fm. This potential leads to the s.p. proton energies in the $p_{1/2}$ and $p_{3/2}$ states: $e_P(p_{1/2}) = -12.5$ MeV and $e_P(p_{3/2}) = -19.1$ MeV, which agree with the corresponding empirical proton energies in ^{16}O, -12.5 and -19 MeV [11].

The Coulomb interaction of Σ^- and the target proton is not taken into account explicitly. Its average value inside the nuclear core is ± 4 MeV, and we assume that it is included into $V_{\Sigma 0}$ and V_{P0}.

The only experimental results for the (K^-, π^+) reaction on ^{16}O have been obtained in the early CERN experiments [12] at $p_K = 450$ MeV/c ($\theta = 0°$). In Fig. 1, the data of [12] are compared with our results for $d^2\sigma/d\hat{k}_\pi dE_\pi$ obtained with four values of $V_{\Sigma 0}$: -20 MeV (curve A), -10 MeV (curve B), 10 MeV (curve C), and 20 MeV (curve D). The B_Σ on the abscissa is the separation (binding) energy of Σ^- from the hypernucleus produced (in the ground or excited state) with the nuclear core left in its ground state. (If the hypernucleus produced consists of the hyperon attached to the nuclear core in a state with an excitation energy E^*, we have $-B_\Sigma = \varepsilon_\Sigma + E^*$.) We restrict ourselves to energies $-B_\Sigma < 20$ MeV, because at higher energies the spectrum is dominated by the continuum due to the $K^- \to 3\pi$ decay. Since the data of [12] are only counting rates, our calculated results are normalized to match the overall magnitude of the data.

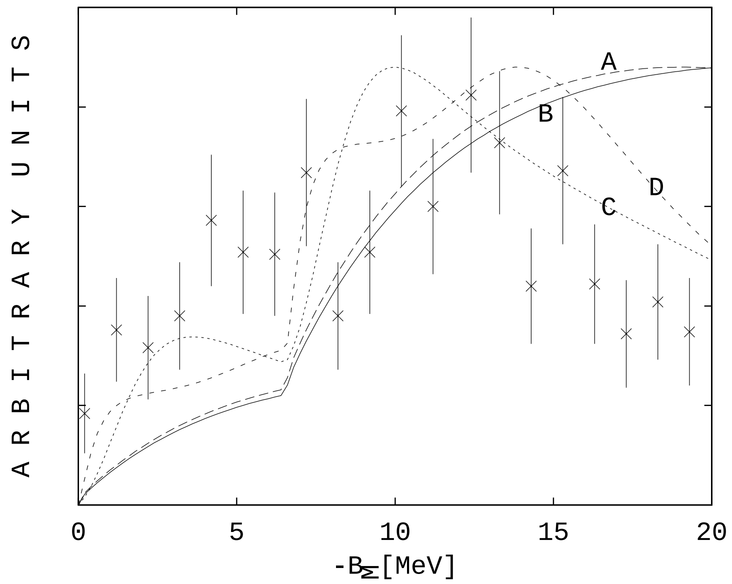

Fig. 1. Pion spectrum from (K^-,π^+) reaction on ^{16}O at $\theta = 0°$ at $p_K = 450$ MeV/c. See text for explanation.

In the energy range considered, only K^- interaction with $p_{1/2}$ and $p_{3/2}$ protons in ^{16}O contribute to the pion spectrum. This is best visible in the curve C, in which the maximum at lower (higher) energy $-B_\Sigma$ results from the K^- interaction with the $p_{1/2}$ ($p_{3/2}$) protons. The C and D curves (obtained with attractive V_Σ) show an overall better agreement with the experimental data than the A and B curves (obtained with repulsive V_Σ). {Actually the agreement may be further improved if one adds to V_Σ a repulsive surface bump [7, 8].}

In the case of the ^{9}Be target, we again assume expressions (8), (9) with the only difference that R is replaced by $\tilde{R} = 2.84$ fm. This leads to the s.p. proton energies in the $p_{3/2}$ and $s_{1/2}$ states: $\tilde{e}_P(p_{3/2}) = -16.888$ MeV and $\tilde{e}_P(s_{1/2}) = -29.8$ MeV, which should be compared with the corresponding empirical proton energies in ^{9}Be, -16.888 and $-(25–26)$ MeV [13].

Recently, the (K^-,π) reaction on the ^{9}Be target has been investigated experimentally at $p_K = 600$ MeV/c ($\theta = 4°$) at BNL [14] (see also [15]). In

Fig. 2, the data of [14] are compared with our results obtained with four values of V_Σ : -20, -10, 10, and 20 MeV (curves A,B,C, and D). Similarly as in Fig. 1, our curves in Fig. 2 contain normalization factors.

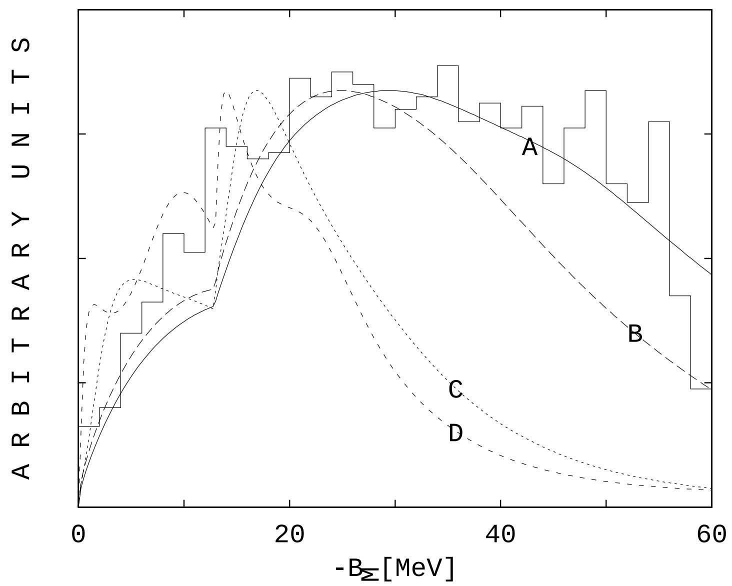

Fig. 2. Pion spectrum from (K^-, π^+) reaction on ^{9}Be at $\theta = 4°$ at $p_K = 600$ MeV/c. See text for explanation.

In the case of the ^{9}Be target, K^- may interact either with $p_{3/2}$ or with $s_{1/2}$ protons, and we have two contributions to the pion spectrum. This is best visible in the curve C, in which the maximum at lower (higher) energy results from the K^- interaction with the $p_{3/2}$ $(s_{1/2})$ protons. Contrary to the situation in Fig. 1, here the A and B curves (obtained with repulsive V_Σ) show an overall agreement with the experimental data in contradistinction to the C and D curves (obtained with attractive V_Σ) which fail completely in reproducing the data at higher $-B_\Sigma$.

4. Discussion and conclusions

Our analysis of the (K^-, π^+) reaction on the ^{16}O target and on the ^{9}Be target leads to different conclusions concerning the s.p. Σ potential V_Σ. However, the recent BNL results with the ^{9}Be target have been obtained with an order of magnitude better statistics than that reported in the early CERN experiments with the ^{16}O target. If consequently, we attribute more weight to the BNL data, we come to the conclusion that V_Σ is repulsive with $V_{\Sigma 0} \sim -(10–20)$ MeV. A similar conclusion, that V_Σ is repulsive, has been drawn from the analysis of the energy levels of Σ^- atoms [16] (although the analysis is not very sensitive to the strength of V_Σ in the central part of the nuclei).

Starting with the two-body YN interaction, one may calculate the s.p. potential V_Σ in nuclear matter by applying the Brueckner theory (see [4–6]). In particular in [6], such calculations have been performed with the three models of the Nijmegen YN interaction (model D [1], model F [2], and the soft-core model [3]) mentioned in the Introduction. Although the accuracy of these calculations is not well established (see, *e.g.*, [17]), the sign of the resulting V_Σ appears to be reliable. Among the three models, only model F leads to a repulsive V_Σ with a strength estimated in [6] to be of the order magnitude compatible with our present estimate (especially if one takes into account the ambiguity in the choice of the intermediate energy spectrum used in the G matrix equation).

Thus we are led to the conclusion that among the Nijmegen barion–barion interactions, only model F is compatible with our analysis of the (K^-,π^+) reaction. (Notice that model F takes into account the exchange of the whole nonet of scalar mesons, and was introduced as an improvement of model D.)

Let us mention another approach to the problem of Σ hypernuclei based on the relativistic field model (see, *e.g.*, [18]), in which barions are described as Dirac particles coupled to mesons. Our conclusion that V_Σ is repulsive, should be helpful in constructing the proper relativistic field model. Namely, it imposes a restriction on the strength of the coupling between mesons and hyperons.

The conclusion of the present paper should be considered as a tentative one. Strangeness exchange experiments with an improved statistics on heavier targets, especially on ^{16}O, would be most helpful in our analysis. On the other hand, several refinements of our simple approach are needed, especially taking into account the isospin structure of the Σ hypernuclei, which are different in the case of the ^{16}O and ^{9}Be targets.

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