Vol. 29 (1998) *ACTA PHYSICA POLONICA B* No 9

PROBLEMS IN HYPERNUCLEAR PHYSICS* , **

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(Received June 9, 1998)

The connection between the barion–barion interaction and the properties of hypernuclei, in particular Σ hypernuclei, is discussed. The inadequate accuracy of the so called low order Brueckner approximation is pointed out. The single particle Σ potential fitted to the pion spectrum measured in the (K^-, π^+) reactions appears to be repulsive. This eliminates some of the existing models of the barion–barion interaction.

PACS numbers: 21.80. +a

1. Introduction

I want to discuss the description of hypernuclei, especially Σ hypernuclei, which starts from realistic barion–barion interaction. Three models of such YN interaction have been worked out by the Nijmegen group (model D [1], model F [2], and the soft-core model [3]) and two models by the Jülich group (models A and B [4–6]). In applying these models of the YN interaction to the description of finite hypernuclei, one usually makes the local density approximation in which the hypernucleus in each point is approximated as a piece of nuclear matter. Thus the starting point is the theory of the system of nuclear matter plus a hyperon. A critical discussion of the present state of this theory is presented in Section 2.

The existing models of the YN interaction do not lead to identical predictions concerning the hyperon interaction in hypernuclei. In particular, they lead to different predictions of the single particle (s.p.) potential of the Σ hyperon in Σ hypernuclei, U_Σ (see [7–9]). In Section 3, I present an empirical estimate of U_Σ consisting of fitting it to the pion spectrum measured

* Presented at the NATO Advanced Research Workshop, Cracow, Poland, May 26–30, 1998.

** This research was partly supported by Komitet Badań Naukowych under Grant No. 2-P03B-048-12.

in the strangeness exchange reaction (K^-,π^+). This offers the possibility of finding the best model of the YN interaction, *i.e.*, the one compatible with the empirical U_Σ.

2. Hyperon binding in nuclear matter

I want to discuss the accuracy of the theoretical calculations of the properties of the system of nuclear matter and a hyperon. I shall consider the simplest case, namely that of a Λ hyperon, for which some results concerning the accuracy of the calculations are available.

The principal goal is here the calculation of the Λ binding energy in nuclear matter, B_Λ, which is equal to the depth of the Λ s.p. potential U_Λ. Its semi-empirical value $B_{\Lambda,\mathrm{se}} \simeq 28$ MeV. The early calculations resulted in values of B_Λ much bigger than $B_{\Lambda,\mathrm{se}}$. It was suggested by Bodmer [10] that most important in solving this overbinding problem is the suppression of the strong $\Lambda\Sigma$ conversion process $\Lambda N \leftrightarrow \Sigma N'$ in nuclear matter due to Pauli blocking and binding effects. Indeed, low order Brueckner (LOB) calculations with realistic barion–barion interactions, which include $\Lambda\Sigma$ coupling, like model D [1] and model F [2] potentials of the Nijmegen group, led to $B_\Lambda \simeq 30$ MeV [11]. The situation with other realistic barion–barion interaction models is similar (see, *e.g.*, [9]): the calculated B_Λ is in agreement with $B_{\Lambda,\mathrm{se}}$.

However, this conclusion might be premature because of the uncertain accuracy of the LOB approximation. In this approximation, the depth of the s.p. potential in nuclear matter of a Λ with zero momentum, $U_\Lambda = -B_\Lambda$, is given by:

$$U_\Lambda \simeq V_\Lambda = \sum_{\boldsymbol{k}_N}^{k_N<k_F} (\boldsymbol{k}_N \boldsymbol{k}_\Lambda{=}0|\mathcal{K}_{\Lambda N}|\boldsymbol{k}_N \boldsymbol{k}_\Lambda{=}0)\ , \tag{1}$$

where k_F is the Fermi momentum of nuclear matter, and the ΛN reaction matrix $\mathcal{K}_{\Lambda N}$ satisfies the equation

$$\mathcal{K}_{\Lambda N}|\boldsymbol{k}_N \boldsymbol{k}_\Lambda{=}0) = v_{\Lambda N}\left[1+\sum_{\boldsymbol{k}'_N \boldsymbol{k}'_\Lambda}^{k'_N>k_F} \frac{|\boldsymbol{k}'_N \boldsymbol{k}'_\Lambda)(\boldsymbol{k}'_N \boldsymbol{k}'_\Lambda|\mathcal{K}_{\Lambda N}}{e_N(k_N)+V_\Lambda-\varepsilon_N(k'_N)-\varepsilon_\Lambda(k'_\Lambda)}\right]|\boldsymbol{k}_N \boldsymbol{k}_\Lambda{=}0). \tag{2}$$

By ε_N and ε_Λ, we denote the kinetic energies of nucleon and Λ (*e.g.*, $\varepsilon_\Lambda(k'_\Lambda) = \hbar^2 k_\Lambda^2/2M_\Lambda$) and by e_N the s.p. nucleon energy ($e_N = \varepsilon_N + V_N$, where V_N is the nucleon s.p. potential). The computational convenience is the only justification for the choice of pure kinetic energies in the intermediate states in (2). In the original Brueckner theory the s.p. energies in the intermediate states contained also s.p. potentials [12, 13].

To test the accuracy of the LOB approximation, let us compare it with the variational Jastrow approach, in which we have:

$$B_\Lambda = \frac{< \Psi_{\rm NM}|H_{\rm NM}|\Psi_{\rm NM} >}{< \Psi_{\rm NM}|\Psi_{\rm NM} >} - \frac{< \Psi_{\rm NM+\Lambda}|H_{\rm NM+\Lambda}|\Psi_{\rm NM+\Lambda} >}{< \Psi_{\rm NM+\Lambda}|\Psi_{\rm NM+\Lambda} >} \ , \qquad (3)$$

where $H_{\rm NM}$ and $H_{\rm NM+\Lambda}$ are the Hamiltonians of nuclear matter and of the nuclear matter$+\Lambda$ system, and the respective wave functions are:

$$\Psi_{\rm NM} = \prod_{i<j} f_{NN}(r_{ij})\Phi_{\rm NM}, \quad \Psi_{\rm NM+\Lambda} = \prod_{i} f_{N\Lambda}(r_{0i})\varphi(0)\Psi_{NM} \ , \qquad (4)$$

where $\Phi_{\rm NM}$ is the wave function of noninteracting nucleons in the Fermi sea of nuclear matter, φ is the zero momentum plane wave of Λ (particle number 0), and f_{NN} and $f_{N\Lambda}$ are the NN and NΛ correlation functions.

Notice that although each of the expectation values in (4) has the variational upper bound property, their difference may be both bigger or smaller than the true value of B_Λ, when the expectation values are calculated with approximate wave functions. In the calculations described here [14], the best form of f_{NN}, determined for pure nuclear matter, was used, and $f_{N\Lambda}$ was determined by finding the maximum value of expression (4) for B_Λ.

As an example, let us consider the case of the central spin dependent ΛN interaction of Herndon and Tang [15] with no exchange ($x = 0$), denoted by HNX in [14], and the NN interaction OMY [16]. Both interactions have a relatively large hard core radius of 0.6 fm, which makes the test of accuracy of the B_Λ calculation more severe. In applying the Jastrow method, the Fermi-hypernetted-chain (FHNC) method [17] was used with the result: $B_{\Lambda,\rm FHNC} = 77.0$ MeV, which is much larger than the LOB result, $B_{\Lambda,\rm LOB} = 59.5$ MeV. Let us mention that the early Jastrow type calculations [18, 19] in the low order cluster (LOC) approximation led to the result, $B_{\Lambda,\rm LOC} = 75$ MeV, which is rather close to the FHNC result. This seems to suggest that a farther improvement of the Jastrow type calculation would not essentially change the resulting B_Λ, and that the discrepancy between the variational Jastrow and Brueckner theory results for B_Λ is caused by the inaccuracy of LOB.

The peculiar feature of LOB are the pure kinetic energies in the intermediate states in (2), which are hard to accept intuitively. They lead to a big energy gap in the s.p. spectrum, which reduces the calculating binding. In the case of the soft-core Nijmegen interaction [3], removing the gap with the help of a continuous s.p. spectrum increases B_Λ by 32 % [9].

The choice of the s.p. spectrum in $\mathcal{K}$-matrix equation (2) is irrelevant, provided higher order terms are calculated. The Brueckner theory is an expansion in the number of hole lines, and the small parameter is the "wound integral"

$$\kappa_{NN} = \varrho < \int d\boldsymbol{r} |\Psi_{\boldsymbol{k}}(\boldsymbol{r}) e^{i\boldsymbol{k}r}|^2 >_{Av} \quad , \tag{5}$$

where ϱ is the nuclear density, $\Psi_{\boldsymbol{k}}(\boldsymbol{r})$ is the wave function of the relative NN motion in nuclear matter with the relative momentum $\boldsymbol{k}$, and $\langle\,\rangle_{Av}$ denotes averaging over the Fermi sea. In the case of the nuclear matter + Λ system, there is a second small parameter $\kappa_{N\Lambda}$ connected with the "wound" in the relative $N\Lambda$ wave function. In the case of the OMY and HNX interactions the two wound integrals in the LOB theory are approximately equal $\sim 0.22 - 0.23$. The LOB theory is a two hole line approximation, and we expect that the three hole line corrections to be of the order of 22-23%, which could explain the difference between the LOB and HFNC results.

The two and three hole line diagrams in the Brueckner theory of B_Λ are shown in Fig.1. Diagram (a) represents $B_{\Lambda,\mathrm{LOB}}$, Eq. (1). Diagram (b) is the rearrangement contribution to B_Λ, which is equal $\kappa_{NN} V_\Lambda$ [20]. If we collect the contributions of diagrams $(a) + (b)$, we get

$$-B_\Lambda = U_\Lambda = (1 - \kappa_{NN}) V_\Lambda \ . \tag{6}$$

It became a standard procedure in the LOB calculations of B_Λ to apply expression (6). This of course is not correct, because together with diagram

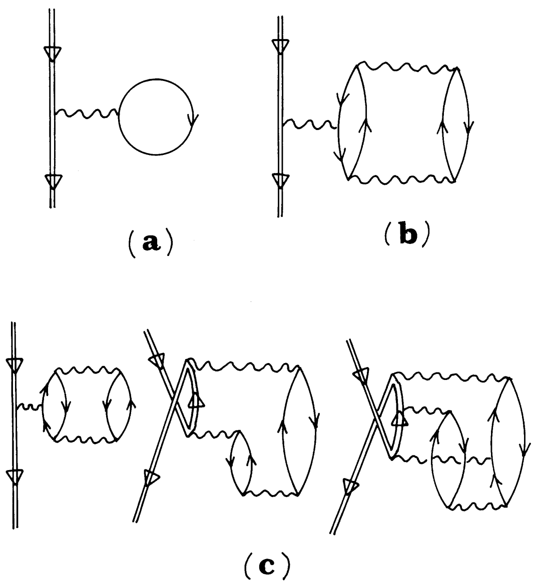

Fig. 1. Two and three hole line contributions to B_Λ. Single lines are nucleons, the double line denotes Λ.

(b) one should consider the remaining three hole diagrams (c), the genuine three body diagrams whose contribution to B_Λ, $B_{\Lambda 3}$, is expected to be of the same order of magnitude as the rearrangement contribution. Notice that there are infinitely many diagrams (c), and to sum them all one has to solve a Faddeev type equation (see [21]). No complete calculation of $B_{\Lambda 3}$ with realistic interactions has been performed so far. The few existing estimates [21], [22] are not conclusive because they are either approximate or/and apply simplified interactions.

To summarize, it appears that LOB may seriously underestimate B_Λ. It would be very important to calculate $B_{\Lambda 3}$, and also to apply the variational FHNC approach to the B_Λ problem with realistic barion–barion interactions.

3. Production of Σ hypernuclei in the (K^-, π^+) reactions

Now, we shall follow [23] and discuss a direct empirical estimate of U_Σ. The pion spectrum measured in the (K^-, π^+) reactions in the energy range of Σ production is sensitive to U_Σ. Thus by analyzing this spectrum in impulse approximation, we may estimate the strength of U_Σ. We consider the case of the (K^-, π^+) reaction because here only one direct elementary strangeness exchange process $K^- P \to \pi^+ \Sigma^-$ occurs, which leads to the formation of a definite hypernucleus, namely a Σ^- hypernucleus. (In the case of the (K^-, π^-) reaction both Σ^+ and Σ^0 hypernuclei may be produced.)

Similarly as in [24], we assume for the Σ s.p. potential the form of a square well (without the repulsive surface bump considered in [24]),

$$U_\Sigma(r) = -(V_{\Sigma 0} + iW_{\Sigma 0})\theta(R - r). \tag{7}$$

For the depth of the absorptive potential, we use the value $W_{\Sigma 0} = 2.5$ MeV, obtained in [25] and [26] from the $\Sigma^- P \to \Lambda n$ cross section. For the depth $V_{\Sigma 0}$ we assume values varying from -20 to 20 MeV. Notice that $V_{\Sigma 0}$ is positive for an attractive and negative for a repulsive potential.

Similarly as in [24], the state of the target proton which participates in the elementary strangeness exchange process $K^- P \to \pi^+ \Sigma^-$, is described by the s.p. wave function of the shell model with the potential

$$U_P(r) = -V_{P0}\theta(R - r) - V_{Pls}\boldsymbol{ls}\delta(R - r), \tag{8}$$

with $V_{P0} = 46$ MeV and $V_{Pls} = 15$ MeV fm. The radius R which is the same as in (7) is adjusted to the empirical s.p. proton energies in the target nucleus.

The early CERN measurements of the strangeness exchange reactions suggested an attractive U_Σ. One of these measurements, namely the measurement of the pion spectrum from the (K^-, π^+) reaction on the ^{16}O target is analyzed in [23]. Actually these early CERN measurements were the reason for using in problems of Σ hypernuclei the Nijmegen model D of the barion–barion interaction because it leads to an attractive U_Σ.

Recently, the (K^-, π) reaction on the ^{9}Be target has been investigated experimentally at $p_K = 600$ MeV/c at BNL [27] (see also [28]) with an order of magnitude better statistics than that reported in the early CERN experiments. In Fig. 2, the pion spectrum from the (K^-, π^+) reaction measured at BNL [27] is compared with our results obtained with four values of $V_{\Sigma 0}$: –20, –10, 10, and 20 MeV (curves A,B,C, and D respectively). The B_Σ on the abscissa is the separation (binding) energy of Σ^- from the hypernucleus produced. Since the data of [27] are only counting rates, our calculated results are normalized to match the overall magnitude of the data. We see that the A and B curves (obtained with repulsive U_Σ) show an overall agreement with the experimental data in contradistinction to the C and D curves (obtained with attractive U_Σ) which fail completely in reproducing the data at higher $-B_\Sigma$

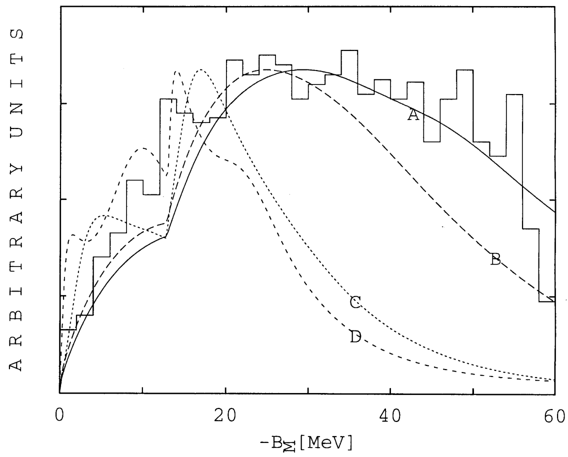

Fig. 2. Pion spectrum from (K^-, π^+) reaction on ^{9}Be at $p_K = 600$ MeV/c. See text for explanation.

Because of the much higher accuracy of the BNL results than that of the early CERN results, we shall restrict ourselves to the BNL results. Thus we conclude that U_Σ is repulsive with $V_{\Sigma 0} \sim -(10 - 20)$ MeV. A similar conclusion, that U_Σ is repulsive, has been drawn from the analysis of the energy levels of Σ^- atoms [29] (although the analysis is not very sensitive to the strength of U_Σ in the central part of the nuclei).

Starting with the two-body YN interaction, one may calculate the s.p. potential V_Σ in nuclear matter by applying the Brueckner theory (see [7–9]). In particular in [9], such calculations have been performed with the three models of the Nijmegen YN interaction (model D [1], model F [2], and the soft-core model [3]) mentioned in the Introduction. Although we have seen in the previous Section that the accuracy of these calculations is not well established, the sign of the resulting $V_{\Sigma 0}$ appears to be reliable. Among the three models, only model F leads to a repulsive V_Σ with a strength estimated in [9] to be of the order magnitude compatible with our present estimate. Thus we are led to the conclusion that among the Nijmegen barion–barion interactions, only model F is compatible with our analysis of the (K^-, π^+) reaction. (Notice that model F takes into account the exchange of the whole nonet of scalar mesons, and was introduced as an improvement of model D.)

Let us mention another approach to the problem of Σ hypernuclei based on the relativistic field model (see, *e.g.*, [30]), in which barions are described as Dirac particles coupled to mesons. Our conclusion that V_Σ is repulsive, should be helpful in constructing the proper relativistic field model. Namely, it imposes a restriction on the strength of the coupling between mesons and hyperons.

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