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THE RARE π^+ DECAY OF ${}^4_\Lambda$He*

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The π^+ emission from the weak decay of ${}^4_\Lambda$He has been a puzzle for more than 30 years. We discuss the significance of two contributions to the decay rate that are due to pion charge exchange and due to a virtual Σ^+ admixture of the initial Λ-hypernuclear state.

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1. Introduction

The π^+ emission in rare hypernuclear decay

$$ {}^4_\Lambda\text{He} \longrightarrow \pi^+ \ + \ (\text{nucleons}) \tag{1}$$

was observed in emulsion [1] and bubble chamber reactions [2]. The branching ratio with respect to the π^- decay process was established to be approximately

$$R(\pi^+/\pi^-) = 0.05 \pm 0.02 \quad . \tag{2}$$

The smallness of the relative rate (2) reflects the fact that the π^- emission process is driven by the $\Lambda \to p + \pi^-$ one-baryon decay interaction while the π^+ emission (1) occurs due to more complicated processes involving at least two baryons.

Until now, a proper explanation of the observed π^+ decay rate has been missing. In sixties, the issue was explored by Dalitz and Von Hippel [3, 4], who considered various processes contributing to the decay (1). Their conclusion was that only two of them can contribute significantly to the π^+ emission: (A) $\Lambda \to n + \pi^0$ decay followed by pion charge-exchange, and (B) $\Sigma^+ \to n + \pi^+$ decay following the $\Lambda + p \to \Sigma^+ + n$ conversion. However,

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neither of the processes was found to account for more than a 1% π^+ decay rate. In fact, Von Hippel's calculations [4] suggested that the combined processes (A) and (B) might yield the branching ratio $R(\pi^+/\pi^-)$ as large as about 3% provided that the Σ^+ is s-wave. The π^+/π^- ratio calculated for p-wave Σ^+ decay was found to be only slightly larger than the one due to single charge exchange process. Unfortunately, the later experimental observation established the $\Sigma^+ \to n + \pi^+$ decay as a p-wave process which ruled out the promising result of Ref. [4].

Recently, the contribution due to pion charge-exchange was re-examined by Cieplý and Gal [5], and the significance of the $\Lambda + p \to \Sigma^+ + n$ conversion was analyzed by Gibson and Timmermans [6]. The results of those two papers are the basis of our discussion.

2. Pion charge exchange process

The authors of Ref. [5] used s-wave effective Lagrangians [7,8] to derive the T-matrix of charge exchange process (A). In their notation the T-matrix reads as

$$\begin{aligned} \langle \pi^+(\boldsymbol{q}) \mid T_{\rm fi} \mid 0 \rangle &= -\sqrt{48\pi}\, s_{\pi_0} a_{\rm CEX} f(q^2) \\ &\times \langle \Psi_f(n,n,n;p) \mid \frac{e^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}_3}}{(2\pi)^{3/2}(2\omega_q)^{1/2}} \frac{e^{\mathrm{i}k_E r_{13}}}{r_{13}} \mid \Psi_i(\Lambda;n;p,p) \rangle \,, \end{aligned} \tag{3}$$

where $\boldsymbol{q}$ and $\omega_q = (m^2+q^2)^{1/2}$ denote the final momentum and energy of the emitted pion, and k_E stands for the momentum of the pion propagating in the intermediate state. The coupling constant s_{π^0} is assigned to the $\Lambda p \pi^0$ vertex and $a_{\rm CEX}$ stands for the charge-exchange πN scattering length. The radial vectors $\boldsymbol{r}_k$ are the baryon space coordinates, $\boldsymbol{r}_{kl} = \boldsymbol{r}_k - \boldsymbol{r}_l$, and we have assumed that the intermediate pion propagates from the first particle to the third one. The initial and final state baryon wave functions $\Psi_i(\Lambda;n;p,p)$ and $\Psi_f(n,n,n;p)$ are to be antisymmetrized for identical particles separated by commas. Finally, the energy dependent factor $f(q^2)$ is

$$f(q^2) = \frac{E+\omega_q}{2m}\,(1+\frac{m}{M}) \gtrsim 1 \tag{4}$$

with $E^2 = k_E^2 + m^2$. The factor $f(q^2)$ appears in Eq.(3) as a direct consequence of the lagrangian form adopted for the $\pi NN\pi$ vertex [5].

Following closely the approach of Dalitz and Von Hippel [3], under the closure approximation the decay rate can be written as

$$\Gamma(\pi^+) = 4\left[s_{\pi_0}\, a_{\rm CEX}\, f(q_f^2)\right]^2 \frac{q_f}{1+\omega_f/M_R} \sum_{P_j} (-1)^{P_j}\, S_j\, I_j \,, \tag{5}$$

where q_f is an effective pion momentum, ω_f is the corresponding pion energy and M_R denotes the recoil mass. S_j are spin factors resulting from the separation of spin and coordinate degrees of freedom in the initial state wave functions. The integrals I_j are given by

$$I_j = \int \frac{d\hat{\boldsymbol{q}}}{4\pi} dV \, \varphi^*(1';2';3',4) \, e^{-\mathrm{i}\boldsymbol{q}_f \cdot (\boldsymbol{r}_3 - \boldsymbol{r}_{3'})} \frac{e^{\mathrm{i}k_E(r_{13} - r_{1'3'})}}{r_{13} \, r_{1'3'}} \varphi(1;2;3,4) \,, \tag{6}$$

where dV symbolizes the integration over the space coordinates and φ stands for the space part of the initial hypernuclear wave function. The labels 1,2,3 (and 1',2',3') refer to the spin and space coordinates of the final neutrons and the summation runs over the six permutations P_j of these labels.

The physical meaning of Eq.(5) is clear. According to the discussion in Ref. [3], the "direct" term

$$I_1 = \int dV \mid \varphi(1;2;3,4) \mid^2 \frac{1}{r_{13}^2} = \langle \frac{1}{r_{13}^2} \rangle \tag{7}$$

dominates for large momenta q_f and k_E, corresponding to a semi-classical estimate in which the Pauli blocking is ineffective. The "exchange" integrals I_j, $j = 2-6$, represent effects arising due to identity of the final state neutrons and their contribution tends to lower the total branching ratio with respect to the semi-classical one.

It was already shown in Ref. [3] that the integrals (6) depend only moderately on the momenta q_f and k_E. In our calculations we adopted $T_\pi(q_f) = 20$ MeV and $T_\pi(k_E) = 20$ MeV which corresponds to the mean kinetic energy of the observed emitted π^+ and to an assumed approximate average kinetic energy of the intermediate pion, respectively.

The number of integrations in Eq.(6) can be reduced dramatically by a suitable parametrization of the ${}^4_\Lambda$He wave function. In a first step one separates the Λ-core relative motion by using the product form

$$\varphi(1;2;3,4) = \varphi_\Lambda(\boldsymbol{r}_1 - \frac{\boldsymbol{r}_2 + \boldsymbol{r}_3 + \boldsymbol{r}_4}{3}) \, \varphi_C(2,3,4) \,, \tag{8}$$

where the core nucleus ^{3}He wavefunction φ_C is represented by the Gaussian form

$$\varphi_C(2,3,4) = \exp[-b(r_{23}^2 + r_{24}^2 + r_{34}^2)] \tag{9}$$

with the parameter b related to the nuclear core rms radius R, $b = 1/(6R^2)$. The $\Lambda - {}^3$He relative motion wave function was taken in the approximate two-gaussian form

$$\varphi_\Lambda(\boldsymbol{r}) = C_1 \exp(-a_1 r^2) + C_2 \exp(-a_2 r^2) \tag{10}$$

for which the parameter values a_1, a_2 and C_2/C_1 were obtained by fitting the binding energy $B_\Lambda = 2.33$ MeV. The coefficient C_1 was chosen to normalize the space wavefunction (8) to unity. Using the parametrization defined by Eqs. (8-10) the integrals I_j can be reduced to the form in which only three integrations remain to be done numerically. The parameter sets used in Ref. [5] are listed in Table I:

(i) The parameter set used by the authors of Ref. [3].

(ii) The parameter b changed to a value derived from the updated value of the rms charge radius of the ^{3}He nuclear core, $R_{ch} = 1.88$ fm.

(iii) The parameters a_1, a_2 and C_2/C_1 obtained by a variational calculation to fit the lambda binding energy using the Λ-nuclear core potential with a central repulsion (so called Isle potential [10]).

TABLE I

Parametrization of the initial hypernuclear wave function.

Parameter set	a_1 [fm^{-2}]	a_2 [fm^{-2}]	C_2/C_1	b [fm^{-2}]
(i)	0.0384	0.214	2.95	0.0694
(ii)	0.0384	0.214	2.95	0.0597
(iii)	0.1069	1.491	-0.20	0.0597

TABLE II

Results. We have used the experimental value $\Gamma(\pi^-) = (1.3 \pm 0.2) \cdot 10^9\, s^{-1}$ [11] to calculate the relative ratio $R(\pi^+/\pi^-)$ for $^4_\Lambda$He decay.

j	P_j	S_j	I_j/m^2 *(i)*	I_j/m^2 *(ii)*	I_j/m^2 *(iii)*
1	(123)	1	0.6251	0.5955	0.5926
2	(321)	1/2	0.5727	0.5455	0.5404
3	(231)	$-1/2$	0.3323	0.3034	0.3093
4	(312)	$-1/2$	0.3323	0.3034	0.3093
5	(132)	1/2	0.3290	0.3003	0.2976
6	(213)	-1	0.3777	0.3447	0.3640
$\sum_j (-1)^{P_j} S_j I_j/m^2$			0.2197	0.2139	0.2283
$\Gamma(\pi^+)$ [$10^7\, s^{-1}$]			1.61	1.56	1.67
$R(\pi^+/\pi^-)$ [%]			1.22	1.20	1.29

The results obtained by Cieplý and Gal are presented in Table II. The sum of the integrals $\sum_j (-1)^{P_j} S_j I_j = 0.220\, m^2$ is slightly smaller than that given in Table II of Ref. [3]. However, the calculated branching ratio is enhanced partly due to the charge-exchange amplitude $a_{\rm CEX}$ which is by about 10% larger than the old value used in [3]. Since the energy dependent factor is $f(q_f) \approx 1.31$, the resulting decay rate $\Gamma(\pi^+)$ is about twice as large as the one reported by Dalitz and Von Hippel.

The comparison of results obtained with different parameter sets shows that even though the separate integrals depend on the parametrization of the hypernuclear wave function, the resulting decay branching ratios remain about the same for all three choices. It should be noted that the calculated rate does not drop due to Λ-^{3}He hard-core repulsion which is taken into account in the parametrization *(iii)*. This phenomenon was discussed in detail in Ref. [5]. Unfortunately, the improved calculation still fails to explain the observed π^+/π^- branching ratio. This means that the pion charge-exchange mechanism by itself cannot account fully for the measured π^+ decay rate and one has to consider contributions due to some other processes.

3. Σ^+ admixture

The strong coupling of the ΛN and ΣN channels seems to play an important role in few-body hypernuclear physics. Therefore, it is tempting to relate the π^+ emission process (1) to the decay of virtual Σ^+ that is induced by the $\Lambda N \to \Sigma N$ conversion. Gibson and Timmermans [6] assumed that the π^+ decay of ${}^4_\Lambda$He is driven by the s-wave three-body transition $\Sigma^+ + N \to \pi^+ + n + N$. The three-body nature of this process allows to explain the observed π^+ energy spectrum which is practically flat. This is in contrast with a picture suggested originally by Von Hippel [4] who considered a two-body Σ^+ decay.

Gibson and Timmermans further assume that, apart from the reduction in phase space, the Σ^+ decay rate is not modified in the medium; that is, the Σ^+ in-medium three-body decay rate is taken to be approximately equal to the two-body free decay rate, except for the phase space difference due to the Σ^+ being highly virtual. The relevant decay ratio is then crudely estimated as

$$R(\pi^+/\pi^-) \simeq \frac{1}{2} \times \frac{70}{185} \times \frac{\Gamma(\Sigma^+ \to n\pi^+)}{\Gamma(\Lambda \to p\pi^-)} \times P(\Sigma^+) \simeq 0.5\, P(\Sigma^+)\,, \qquad (11)$$

where $P(\Sigma^+)$ stands for the probability of Σ^+ admixture in the ${}^4_\Lambda$He wave function. The factor 1/2 takes into account approximately the relative difference in Pauli suppression of π^+ and π^- emission processes [4], and the phase space gives an additional factor 70/185, being the ratio of the average π^+ momenta for in medium and free Σ^+ decays.

If the Σ^+ admixture was large enough (on the level of 10%), the estimated ratio (11) would agree with the observed π^+/π^- branching ratio. However, the available estimates of the magnitude of Σ admixture in strangeness -1 hypernuclear systems are strongly model dependent and range from $P(\Sigma)$ well below 1% [12,13] to the values as high as the desired 10% [14]. In our opinion, it would be surprising if the probability of virtual Σ^+ in ${}^4_\Lambda$He was larger then few per cent. Nevertheless, the effect on the discussed π^+ emission may be stronger than the rough estimate (11) suggests, especially if it combines coherently with the pion charge-exchange amplitude.

4. Summary

We have discussed the present theoretical status on the puzzling π^+ emission from weak decay of ${}^4_\Lambda$He. Since the simple pion charge-exchange process fails to account fully for the measured ratio $R(\pi^+/\pi^-)$, the contribution due to Σ^+ admixture should be considered seriously. Clearly, a realistic model calculation which includes the charge exchange channel [5] and the three-body $\Sigma^+ + N \to \pi^+ + n + N$ decay mechanism [6] is called for.

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