WEAK RADIATIVE DECAYS OF HYPERONS*

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Weak radiative hyperon decays are briefly reviewed. We discuss the conflict between expectations based on Hara's theorem on one side and experiment, quark, and VMD models on the other side. Two recent arguments against the interpretation of quark model results as a violation of Hara's theorem are presented and their shortcomings are indicated. Phenomenological success of the VMD prescription is emphasized. It is stressed that the predictions of the VMD model are clear-cut and different from those of other approaches. The decisive role of the soon-to-be-processed results of the KTeV experiment on $\Xi^0 \to \Lambda \gamma$ and $\Xi^0 \to \Sigma^0 \gamma$ asymmetries is pointed out.

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1. Introduction

In their review of low-energy weak interactions [1], Donoghue, Golowich and Holstein classify the problem of weak radiative decays of hyperons (WRHD's) among such puzzles as the origin of the $\Delta I = 1/2$ rule or CP violation. Today, 12 years after the publication of their review and despite much richer experimental data, there is still no consensus on how the puzzle should be solved. The most recent review on the subject is that of Ref. [2].

Dominant decays of hyperons are nonleptonic. WRHD's are interesting from the theoretical point of view because in these decays all three fundamental (weak, electromagnetic and strong) interactions are involved. The original problem with WRHD's concerned the large size of the $\Sigma^+ \to p\gamma$ asymmetry, which was in conflict with expectations based on Hara's theorem [3]. This conflict is still with us.

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2. Hara's theorem

According to Hara's theorem, the parity-violating amplitude $A(\Sigma^+ \rightarrow p\gamma)$ must vanish in the limit of exact *U*-spin symmetry. The three main assumptions upon which this theorem is based are: gauge invariance, CP conservation and exact U-spin symmetry.

Under $s \to d$ U-spin transformation the Σ^+ hyperon is replaced by the proton. If U-spin is exact, Σ^+ behaves essentially like a proton and we may consider the $pp\gamma$ parity-violating coupling instead of that of $\Sigma^+p\gamma$ (see Ref. [2] for a full argument). The most general such coupling is

$$\overline{\Psi}[g_1(q^2)(\gamma_\mu - \frac{\not q q_\mu}{q^2}) + g_2(q^2)i\sigma_{\mu\nu}q^\nu]\gamma_5\Psi A^\mu.$$
(1)

Since there cannot be a pole at $q^2 = 0$, we must have $g_1(0) = 0$. Furthermore, the g_2 term violates CP invariance. Hence, $g_2 = 0$ and the whole coupling vanishes. In the real world in which U-spin is not an exact symmetry and for a nonzero parity-conserving amplitude B, one then expects the $\Sigma^+ \to p\gamma$ decay asymmetry

$$\alpha = \frac{2Re(A^*B)}{|A|^2 + |B|^2} \tag{2}$$

to be small since SU(3) is usually broken weakly.

3. Experiment

Current experimental evidence, summarized in Fig.1, shows that the asymmetry is large. The most recent number, coming from the E761 experiment performed at Fermilab [4], is based on nearly 35 thousand events.



Fig. 1. History of measurement of $\Sigma^+ \to p\gamma$ asymmetry parameter.

The $\Sigma^+ \to p\gamma$ decay is not the only weak radiative hyperon decay. There are four other decays that can be studied experimentally: $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^- \to \Sigma^-\gamma$. Of these, the first three may be due both to $(su) \to (ud)\gamma$ processes involving W-exchange between the quarks, as well as to a single quark $s \to d\gamma$ transition. $\Xi^- \to \Sigma^-\gamma$ must be due to a singlequark transition only since there is no *u* quark in either initial or final baryon. >From the experimental branching ratio of the $\Xi^- \to \Sigma^-\gamma$ decay [5], one can estimate branching ratios of the remaining four WRHD's if the singlequark transition dominates. This leads to the estimate of the $\Sigma^+ \to p\gamma$ branching ratio which is two orders of magnitude below the experimental data. Thus, it is the $(su) \to (ud)\gamma$ transition that dominates in WRHD's.

4. Two asymmetry patterns

In order to describe WRHD's, theory must describe both parity-conserving and parity-violating amplitudes. Fortunately, there are no large qualitative differences between various approaches to parity-conserving amplitudes. Pole model or quark model approaches yield sets of parity-conserving amplitudes of the same relative signs and similar relative sizes. On the other hand, various models differ in their description of parity-violating amplitudes much more than just in details. It turns out that in the SU(3) limit one can group the models into two classes with different patterns of signs of parity-violating amplitudes. The resulting two asymmetry patterns are given in Table I.

TABLE I

	pattern I	pattern II
$\Sigma^+ \to p\gamma$	0(-)	_
$\Lambda \to n\gamma$	_	+
$\Xi^0 \to \Lambda \gamma$	_	+
$\Xi^0 \to \Sigma^0 \gamma$	_	_

Two asymmetry patterns

Pattern I is characteristic of the standard pole model [6] in which Hara's theorem is satisfied (the negative sign in parentheses in Table I gives the sign of asymmetry in Ref. [6] when SU(3) is broken). Asymmetry pattern II is obtained in the quark model [7] or in the pole model supplied with the VMD prescription [8] and — at least at first sight — it is characteristic of the violation of Hara's theorem. Other approaches, such as QCD sum rules, chiral perturbation theory, *etc.*, have less predictive power or disagree strongly with the data [2].

5. Quark diagrams

All possible quark diagrams are shown in Fig.2. From experiment we know that the contribution from diagram (a) is negligible. The contribution from diagram (c) vanishes in the SU(3) limit and is negligible in explicit calculations with broken SU(3) [2,6]. Diagram (d) is suppressed by the presence of two W propagators. Thus, it is the contribution from diagrams (b1) and (b2) only that may be significant. Calculation [9] shows that consideration of diagrams (b) in the quark model leads to the violation of Hara's theorem.



Fig. 2. Quark diagrams for weak radiative hyperon decays.

Upon inspection one can see that this surprising result has its roots in the intermediate quark entering its mass-shell. Namely, diagram (b1) leads to the expression

$$\frac{\overline{u} \not e(\not p + \not q + m_u) \Gamma_L^\mu s \overline{d} \Gamma_{L\mu} u}{(p+q)^2 - m_u^2}, \qquad (3)$$

(where $\Gamma_L^{\mu} = \gamma^{\mu}(1 - \gamma_5)$, ε is photon polarization vector, and q, p are final photon and u quark momenta), the denominator of which, for final u quark on its mass-shell, becomes equal to $2p \cdot q$ and contains a pole at zero photon momentum. The same pole is present in diagram (b2). A term proportional to q which appears in the numerator is cancelled by this 1/q factor. The resulting q-independent expression, when taken in between baryon SU(6) wave functions, yields a non-zero value, thus violating Hara's theorem.

6. Pole model

In the standard pole model the diagrams (b1) and (b2) correspond to the contribution from intermediate $\frac{1}{2}^-$ excited baryons B^* . Using the quark model one can calculate the $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$ weak transition elements and the $\frac{1}{2}^- \rightarrow \frac{1}{2}^+ + \gamma$ electromagnetic couplings. Their relative size is governed by group-theoretical spin-flavour factors. Consider $\Sigma^+ \rightarrow p\gamma$ decay. When one identifies the results of such quark model calculations with those hadron-level expressions that are allowed by gauge invariance (*i.e.* fixing parameters f and b for electromagnetic $(f \varepsilon^*_{\mu} \overline{u}_p \sigma^{\mu\nu} \gamma_5 u_{B^*} q_{\nu})$ and weak $(b \overline{u}_{B^*} u_{\Sigma^+})$ transitions), one finds that contributions from diagrams (b1) and (b2) enter with a relative minus sign [6]:

$$\sum_{B^*} \left(\frac{fb}{\Sigma^+ - B^*} - \frac{bf}{p - B^*_s} \right) \varepsilon^*_\mu \overline{u}_p \sigma^{\mu\nu} \gamma_5 u_{\Sigma^+} q_\nu \tag{4}$$

(symbols of particles in the denominators stand for their masses), thus ensuring cancellation of the corresponding contributions to the $\Sigma^+ \to p\gamma$ decay in the SU(3) limit when the masses of Σ^+ and p, as well as those of $B^*(uud)$ and $B^*_s(uus)$ become equal.

In explicit models SU(3) is broken in energy denominators with $B^* - \Sigma^+ = \Delta \omega - \delta s$, and $B_s^* - p = \Delta \omega + \delta s$ ($\Delta \omega \approx 0.57 \ GeV$ is the energy difference between excited and ground-state baryons, and $\delta s = m_s - m_{u,d} \approx 0.19 \ GeV$ is the strange-nonstrange quark mass difference). For $\delta s \neq 0$ the diagrams (b1) and (b2) - having different energy denominators - do not cancel exactly [6]. The corresponding formulae (up to an uninteresting normalization factor) are given in column 2 of Table II, where $x \equiv \frac{\delta s}{\Delta \omega} \approx \frac{1}{3}$. By construction the obtained $\Sigma^+ \to p\gamma$ parity-violating amplitude vanishes in the SU(3) limit ($x \to 0$).

TABLE II Parity-violating amplitudes with SU(3) breaking: (b1) - (b2) — standard pole model, (b1)+(b2) — VMD/quark model.

process	(b1)-(b2)	(b1)+(b2)
$\Sigma^+ \to p\gamma$	$-\frac{2x}{3\sqrt{2}}$	$-\frac{2}{3\sqrt{2}}$
$\Lambda \to n\gamma$	$+\frac{2x-1}{3\sqrt{2}}$	$+\frac{2-x}{3\sqrt{3}}$
$\Xi^0 \to \Lambda \gamma$	$+\frac{1-x}{2\sqrt{2}}$	$-\frac{1-x}{2\sqrt{2}}$
$\Xi^0 \to \Sigma^0 \gamma$	$+\frac{1+x}{2}$	$+\frac{1+x}{2}$

7. Vector meson dominance

One can couple photon to hadrons through intermediate vector mesons. If this is the only way photon couples to hadrons, one talks of strict VMD model. Such a model is known to describe the coupling of photon to hadrons very well, including for example a parameter-free prediction for baryon magnetic moments [10] *etc.* Thus, it is tempting to apply VMD to WRHD's. As a first step one has to evaluate the $\Delta S = 1$ parity-violating couplings of the ρ^0 , ω , and ϕ vector mesons to ground-state octet baryons. This has been done by Desplanques, Donoghue and Holstein [11] in an SU(6)_W-symmetric approach. The diagrams one has to consider are again (b)-type diagrams. The outgoing photon is replaced by a meson. By relating pseudoscalar and vector meson couplings through SU(6)_W, this approach fixes the size of all necessary vector meson couplings from well-known experimental data on nonleptonic hyperon decays. In the second step one replaces vector mesons by photons [8] according to the VMD prescription $V^{\mu} \rightarrow \frac{e}{g_V} A^{\mu}$ where e is electric charge and, for example, $g_{\rho} = 5.0$ etc.

According to the proposal of Kroll, Lee and Zumino [12, 13], VMD is a translation from quark to hadron level. One either couples a photon to quarks through minimal coupling and adds to it a coupling to a hadron as a whole through gauge-invariant photon-vector meson coupling $F^{\mu\nu}V_{\mu\nu}$ (that vanishes for real photons), or one couples a photon to hadrons through a vector meson using the $V^{\mu}A_{\mu}$ photon-vector meson coupling. Gaugeinvariance-violating photon mass induced by the $V \cdot A$ coupling may be removed through appropriate counter terms. Since, according to the KLZ scheme, VMD is equivalent to the quark model, one hopes that it should help in translating the quark model result to the hadron level.

Now, the authors of Ref. [11] identify the couplings of transverse vector mesons to p and Σ^+ with the $\overline{u}_p \gamma_\mu \gamma_5 u_{\Sigma^+} V^\mu$ coupling. In the pole model this coupling is generated by weak (eg. $b\overline{u}_{B^*}u_{\Sigma^+}$) and strong (eg. $g\overline{u}_p \gamma_\mu \gamma_5 u_{B^*} V^\mu$) couplings. Using symmetry properties of weak and strong couplings the pole model predicts then that the resulting $p\Sigma^+V$ coupling has the form:

$$\left(\frac{bg}{\Sigma^+ - B^*} + \frac{gb}{p - B^*_s}\right) \overline{u}_p \gamma_\mu \gamma_5 u_{\Sigma^+} V^\mu \,, \tag{5}$$

where the two terms come from diagrams (b1) and (b2).

From Eq. (5) we see that in the SU(3) limit there is no cancellation between the two diagrams (b1) and (b2). Using VMD one obtains photon coupling of the type $\overline{u}_p \gamma_\mu \gamma_5 u_{\Sigma^+} A^\mu$, which is a part of the gauge-invariant, current-conserving coupling $\overline{u}_p (\gamma_\mu - \frac{qq^\mu}{q^2}) \gamma_5 u_{\Sigma^+} A^\mu$. Through the presence of a nonzero g_1 coupling, VMD leads to the violation of Hara's theorem. Although we argued that VMD might be used to understand the origin of

Hara's theorem violation in the quark model at hadron level, a closer inspection reveals that the origins of this violation are different in the two models. Namely, in the quark model the violation comes from the intermediate quark entering its mass-shell, while in the VMD picture the intermediate excited baryon is off-shell. An obvious problem with the VMD approach is the presence of the pole at $q^2 = 0$. Such a term should be absent because there are no exactly massless hadrons. Still, VMD has worked well in so many places that it is interesting to see how it fares in WRHD's.

8. Comparison with experiment

When parity-violating amplitudes of Table II are supplemented with standard description of parity-conserving amplitudes, one obtains different signatures for standard pole model and VMD (quark model) predictions (see Table I). If Hara's theorem is satisfied, all four asymmetries are of the same sign. On the other hand, if the VMD (or quark model) model route is strictly followed, Hara's theorem is violated and the asymmetry sign of the $\Xi^0 \to \Lambda \gamma$ decay is opposite to that of $\Sigma^+ \to p\gamma$ and of $\Xi^0 \to \Sigma^0 \gamma$.

Table III

Asymmetries							
process	Ref. [6]	exp.	Ref. [2]	Ref. [7]			
	standard		VMD	quark			
	pole model			model			
$\Sigma^+ \to p\gamma$	$-0.80^{+0.32}_{-0.19}$	-0.76 ± 0.08	-0.95	-0.56			
$\Lambda ightarrow n\gamma$	-0.49		+0.80	-0.54			
$\Xi^0 \to \Lambda \gamma$	-0.78	$+0.43\pm0.44$	+0.80	+0.68			
$\Xi^0 ightarrow \Sigma^0 \gamma$	-0.96	$+0.20\pm0.32$	-0.45	-0.94			
Branching ratios (in units of 10^{-3})							
$\Sigma^+ \to p\gamma$	$0.92^{+0.26}_{-0.14}$	1.23 ± 0.06	1.3 - 1.4	1.24			
$\Lambda ightarrow n\gamma$	0.62	1.63 ± 0.14	1.4 - 1.7	1.62			
$\Xi^0 ightarrow \Lambda \gamma$	3.0	1.06 ± 0.16	0.9 - 1.0	0.5			
$\Xi^0 ightarrow \Sigma^0 \gamma$	7.2	3.56 ± 0.43	4.0 - 4.1	3.30			

Asymmetries and branching ratios - comparison of selected conflicting models and experiment.

Phenomenologically, the $\Xi^0 \to \Lambda \gamma$ decay is a much cleaner case than $\Lambda \to n\gamma$, where quark model also has a tendency to predict a positive asymmetry. However, depending on the details, a negative $\Lambda \to n\gamma$ asymmetry can be also obtained (see Ref. [2]). Measuring precisely the $\Xi^0 \to \Lambda \gamma$ asymmetry is therefore very important. Current data (Table III) on the $\Xi^0 \to \Lambda \gamma$

asymmetry reject Hara's theorem at an almost 3σ level. When other asymmetries and branching ratios are taken into account, the disagreement with Hara's theorem is even more significant (Table III, see also Ref. [2]). Data on the $\Xi^0 \to \Lambda \gamma$ asymmetry seem to corroborate VMD/quark model predictions. As Table III shows, there are small differences between the quark model and VMD. The data are better described by the latter. As discussed in [2], the VMD predictions are insensitive to model details.

9. Origin of quark model result

In a recent paper [14] it has been claimed that Hara's theorem can be proven using current conservation only. The argument is based on the standard definition of transverse electric dipole moment which is:

$$T_{1\lambda}^{el}(|\boldsymbol{q}|) = \frac{1}{i|\boldsymbol{q}|\sqrt{2}} \int d^3r \left\{ -\boldsymbol{q}^2 (\boldsymbol{J}_5 \cdot \boldsymbol{r}) + (\boldsymbol{\nabla} \cdot \boldsymbol{J}_5)(1 + \boldsymbol{r} \cdot \boldsymbol{\nabla}) \right\} j_1(|\boldsymbol{q}|r) Y_{1\lambda}(\boldsymbol{r}/r).$$
(6)

From current conservation $\nabla \cdot J_5 = 0$. Thus, the second term on the r.h.s. of Eq. (6) vanishes. When for small $|\mathbf{q}|$ one replaces $j_1(|\mathbf{q}|r)$ by its approximation $\mathbf{q}r/3$, the first term on the r.h.s. vanishes like \mathbf{q}^2 for small $|\mathbf{q}|$. The above replacement is justified when current J_5 is well localized. However, if it is not sufficiently well localized, the first term does not vanish [15].

A different way to understand the origin of the result of Kamal and Riazuddin [9] has been proposed by Azimov [16]. According to Azimov, when parity-violating weak interactions are included, the "bare" proton propagator $S_0^{-1} = \not p - m_0$ (*i.e.* with strong and electromagnetic interactions taken into account, but without any weak interactions) gets modified by a γ_5 dependent self-energy term $\Sigma(p)$ leading to a new propagator of the form $S'^{-1} = \not p(a + b\gamma_5) - m(c - is\gamma_5)$ with $b/a \ll 1$, $s/c \ll 1$. In order to be able to apply standard reasoning leading to Hara's theorem, one has to work with proton propagator without the γ_5 -dependent terms. Thus, one has to apply "chiral" renormalization, *i.e.* replace ψ' by $\psi = \exp(-\beta\gamma_5)\psi'$ with an appropriate β . By the same procedure, the general V - A current

$$J'_{\mu} = g'_1 \overline{\psi'} \gamma_{\mu} \gamma_5 \psi' + f'_1 \overline{\psi'} \gamma_{\mu} \psi' \tag{7}$$

obtained in Kamal–Riazuddin-like calculations is transformed into a perfectly legal vector current

$$J_{\mu} = f_1 \overline{\psi} \gamma_{\mu} \psi \,. \tag{8}$$

Thus, according to Azimov's proposal, the offending $\gamma_{\mu}\gamma_{5}$ term can be rotated away, which would mean that the Kamal–Riazuddin result was simply improperly interpreted as violating Hara's theorem: after an appropriate

rotation the contribution obtained in Ref. [9] gets hidden into the f_1 term. In order to be applicable, Azimov's mechanism requires the existence of the vector term $f_1\overline{\psi}\gamma_\mu\psi$ into which the $g'_1\overline{\psi'}\gamma_\mu\gamma_5\psi'$ term might be hidden. Such a vector term vanishes for the neutron since neutron charge is zero. Consequently, if quark model calculation gives a nonvanishing parity violating amplitude for $n \to n\gamma$, this amplitude cannot be rotated away by a γ_5 -dependent renormalization. Since in the $p \to p\gamma$ case the offending term comes from the $(ud) \to (ud) + \gamma$ transition and the wave function of the neutron is obtained from that of the proton by $u \leftrightarrow d$ interchange, it should be clear that the KR result is obtained for the neutron as well. Consequently, the origin of the quark model result is not related to the need for a possible γ_5 -dependent renormalization: Azimov's proposal is not the true universal cure for the KR disease [17].

As the analysis of paper [15] indicates, the origin of the quark model result seems therefore related to some kind of nonlocality. This should not be surprising since in the quark model calculation of Kamal and Riazuddin the external quarks are essentially free objects of definite momenta, *i.e.* plane waves located everywhere in position space.

10. Upcoming experimental results

As already stressed, important information will come from asymmetries of the $\Xi^0 \to \Sigma^0 \gamma$ and $\Xi^0 \to \Lambda \gamma$ decays. The relevant measurement has already been performed as part of the E832 KTeV experiment at Fermilab. The number of events for the two decays under consideration is approximately 5000 for $\Xi^0 \to \Sigma^0 \gamma$ and 700 for $\Xi^0 \to \Lambda \gamma$ [20]. The new data should allow a much better determination of relevant asymmetry parameters. It is therefore pertinent to stress that the (solid) predictions of the VMD are: -0.3 to -0.5 for $\Xi^0 \to \Sigma^0 \gamma$ and +0.65 to +0.80 for $\Xi^0 \to \Lambda \gamma$ (see [2] for more details). Quark model [7] (see also Table III) predicts large negative asymmetry (-0.94) for the $\Xi^0 \to \Sigma^0 \gamma$ decay and large positive asymmetry (+0.68) for the $\Xi^0 \to \Lambda \gamma$ asymmetry. Present data are $+0.60 \pm 0.96$ and $+0.43 \pm 0.44$ respectively. The first of these numbers differs from the one given in Table III by a factor of three since the experimental analysis of Ref. [18] (used in Table III and in Ref. [2]) did not take into account the depolarization due to intermediate $\Sigma^0 \to \Lambda \gamma$ decay [20]. The crucial measurement is that of the $\Xi^0 \to \Lambda \gamma$ asymmetry. Only Hara's-theorem-violating approaches yield positive sign for this asymmetry.

11. Conclusions

In summary, WRHD's expose in a particularly clean way the logical consequences of the very basic assumptions of the quark model (such as quark freedom) and confront them with other fundamental assumptions. If one tries to avoid using free quarks, one is led by the VMD approach to the violation of Hara's theorem again. In the VMD approach there is a possibility to satisfy Hara's theorem while obtaining a positive sign of the $\Xi^0 \to \Lambda \gamma$ asymmetry. This requires, however, a negligible size of weak meson-nucleon couplings, which is in disagreement with experiment [19]. With the results of the KTeV experiment becoming soon available, the issue of WRHD's should get even more interesting.

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