

EFFECTIVE FIELD THEORIES
FOR NUCLEI AND DENSE MATTER*

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A recent development on the working of effective field theories in nuclei and in dense hadronic matter is discussed. We consider two extreme regimes: One, dilute regime for which fluctuations are made on top of the matter-free vacuum; two, dense systems for which fluctuations are treated on top of the “vacuum” defined at a given density, with masses and coupling constants varying as function of matter density (“Brown–Rho scaling”). Based on an intricate — as yet mostly conjectural — connection between the in-medium structure of chiral Lagrangian field theory which is a beautiful effective theory of QCD and that of Landau Fermi liquid theory which is an equally beautiful and highly successful effective theory of many-body systems, it is suggested that a chiral Lagrangian with Brown–Rho scaling in the mean field is equivalent to Fermi-liquid fixed point theory. I make this connection using electroweak and strong responses of nuclear matter up to nuclear matter density and then extrapolating to higher densities encountered in heavy-ion collisions and compact stars.

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1. Introduction

Effective Field Theories (EFTs) are a powerful tool not only in particle and condensed matter physics [1, 2] where they are more extensively studied but also more recently, in nuclear physics [3–9] where phenomenological approaches have traditionally been amply successful, thus drawing less attention to field-theory approaches. There are two superbly effective field theories that are quite relevant to nuclear physics. One is chiral Lagrangian field theory as a low-energy effective theory of QCD and the other is Landau

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Fermi liquid theory as a semi-phenomenological theory for nuclear matter. Both are beautiful examples of how effective field theory works in hadronic systems. For nuclear many-body systems and most of all for dense matter, both figure importantly. The first involves what I would call “chiral scale” with the chiral cutoff $\Lambda_\chi \sim 1$ GeV setting the scale below which the theory is useful and the second involves “Fermi-liquid scale” set by the Fermi momentum given by the density of the system.

In this talk, I would like to develop arguments that suggest that combining the two effective theories leads naturally to the notion of BR scaling [10] which has recently found a simple and striking application [11] in the heavy-ion data of the CERES collaboration [12]. If the arguments are correct, the implication is that what one usually attributes to change in the QCD vacuum — a quantity that is the focus of the present day nuclear and hadronic physics — may be related, albeit indirectly, to many-body interactions on top of the matter-free vacuum. This may be considered as a manifestation of how two apparently different dynamical pictures represent the same physical phenomenon or in the language of [9] a variant of the Cheshire-Cat phenomenon.

2. Strategy for effective theory

The idea of effective field theory is rather simple. Consider a generic field Φ which we would like to study at an energy scale less than a typical energy scale Λ_1 . Let us divide the field into the one we are interested in and the one we are not. In terms of energy scales, the former corresponds to Φ_L for $E < \Lambda_1$ and the latter to Φ_H for $E > \Lambda_1$, $\Phi = \Phi_L + \Phi_H$. We are interested in the Feynman integral

$$Z = \int [d\Phi] e^{iS[\Phi]} = \int [d\Phi_L][d\Phi_H] e^{iS[\Phi_L, \Phi_H]}.$$

Since we are not interested in the degrees of freedom represented by Φ_H , we will integrate it out of the Feynman integral. Define

$$e^{iS^{\text{eff}}[\Phi_L]} = \int [d\Phi_H] e^{iS[\Phi_L, \Phi_H]}, \quad (1)$$

then the generating functional (when sources are suitably incorporated) is $Z = \int [d\Phi_L] e^{iS^{\text{eff}}[\Phi_L]}$. This is an exact result since we have not done anything other than redefine things. Therefore we could have chosen the cutoff scale at $\Lambda_2 < \Lambda_1$. In fact we could define the effective action for any arbitrary scale by “decimating” the cutoff. *If everything is done correctly, physical quantities should not depend upon how the Λ_i ’s are chosen.* This statement is translated

into “renormalization-group invariance”. Now in our case, although we know what the correct theory is (that is, QCD), we do not yet know how to describe low-energy dynamics in terms of the QCD variables (quarks and gluons). What we see in nature are color-singlet hadrons. So the strategy is to write the effective action at a given cutoff Λ_i as an infinite series — and suitably truncate them — in terms of known variables

$$S_{\Lambda_i}^{\text{eff}} = \sum_{n=0}^{\infty} C_n Q_n, \quad (2)$$

where Q ’s are local operators involving (observable) hadron fields written in increasing power of momentum and/or of square of pion mass and C ’s are constants that are “natural”. In writing this expansion, one appeals to symmetries such as Lorentz (or Galilei) invariance, chiral invariance *etc.* In the usual chiral perturbation theory, the expansion involves the pion and baryon fields with the power $(\partial/\Lambda_\chi)^n$ and/or $(m_\pi^2/\Lambda_\chi^2)^n$.

How the effective action (2) changes under “decimation” is expressed through Wilson’s renormalization group-flow equation [2]. This implies that the Λ_i -dependent coefficients in (2) satisfy the Wilson equation $\frac{\partial C_i(\Lambda)}{\partial \Lambda} = \mathcal{F}_\Lambda(C_i)$ where $\mathcal{F}$ is a known function of C_i . In some cases, certain coefficients stay constant under the decimation due to the presence of “fixed points”. We shall see later that nuclear matter is described by a fixed-point theory, with the nucleon effective mass and the four-Fermi quasiparticle interactions being fixed-point quantities.

3. Two-nucleon systems

I shall now illustrate how the above effective theory strategy works in nuclear physics of two-body systems. All two-body systems at very low energy are accurately known in nonrelativistic phenomenological approach using two-body potentials. I propose that they can provide a precision check of the theory that we are developing.

Focusing on very low energy at an energy scale much less than the pion mass, $m_\pi \approx 140$ MeV, we can integrate out all degrees of freedom — including pions — other than the matter field, namely, the nucleon field. Pions will be introduced later to go higher order in the expansion. In the absence thereof, we can work up to the next-to-leading order (NLO). We choose the cutoff Λ of the order of the pion mass. Define the four-point vertex relevant to the process by

$$V(\mathbf{q}) = \frac{4\pi}{M} (C_0 + (C_2 \delta^{ij} + D_2 \sigma^{ij}) q^i q^j) + V_{\text{EM}}, \quad (3)$$

where V_{EM} is the electromagnetic interaction between two protons which is of course known, M is the nucleon mass and σ^{ij} is the rank-two tensor that is effective only in the spin-triplet channel. The coefficients $C_{0,2}$ are (spin) channel-dependent, and that D_2 is effective only in spin-triplet channel. Thus there are five parameters; two in 1S_0 and three in 3S_1 channel. In principle, these parameters are calculable from a fundamental Lagrangian (*i.e.*, QCD) but in practice, nobody knows how to do this. So in the spirit of EFTs, we shall fix them from experiments. Since the explicit form of the regulator should not matter [13], we shall choose the Gaussian form $S_A(\mathbf{p}) = \exp\left(-\frac{\mathbf{p}^2}{2\Lambda^2}\right)$ where Λ is the cutoff. As mentioned, the cutoff is not a parameter to be fine-tuned; physical quantities should not depend sensitively on it provided it is correctly chosen for the scale involved.

Given the four-point function (3), one can solve Lippman–Schwinger equation or Schrödinger equation with (3) inserted as the kernel. This is strictly speaking not an expansion in a rigorous accordance with the counting rule but one can show that it is correct up to the order we are considering.

To see how the strategy works, let us consider low-energy neutron-proton scattering. In Fig. 1 is shown the 1S_0 phase shift (in degrees) *vs.* cutoff for the scattering at a fixed CM momentum of $p = 68.5$ MeV. One sees that below $\Lambda \sim m_\pi$, the calculated phase shift varies rapidly and disagrees with the experiment but once the cutoff is chosen at about the pion mass, there is practically no cutoff dependence and the theory agrees very well with the experiment as one increases the cutoff. This therefore satisfies the condition for the consistency of an effective theory. The second condition can be seen

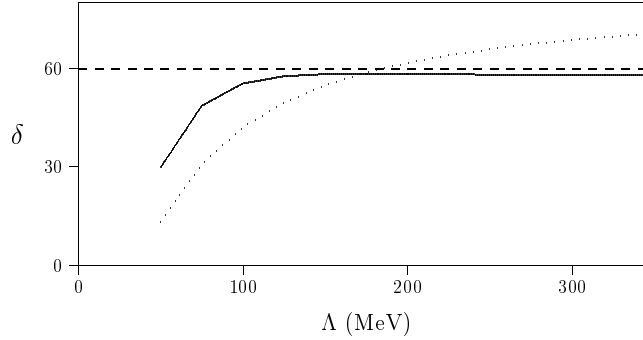


Fig. 1. np 1S_0 phase shift (degrees) *vs.* the cutoff Λ for a fixed CM momentum $p = 68.5$ MeV. The NLO result is given by the solid curve and the LO result by the dotted curve. The horizontal dashed line is the result from the v_{18} potential. Note the Λ independence for $\Lambda \gtrsim 150$ MeV.

in Fig. 2. For a given cutoff¹, here taken at $\Lambda = \Lambda_{Z=1} \simeq 170$ MeV, the theory agrees very well up to $p \lesssim 80$ MeV but beyond that it starts disagreeing. This indicates that the theory breaks down as the momentum approaches the cutoff. This may be due to the fact that higher order terms are needed or new physics enters into the picture. This feature is again required by the consistency of the effective theory.

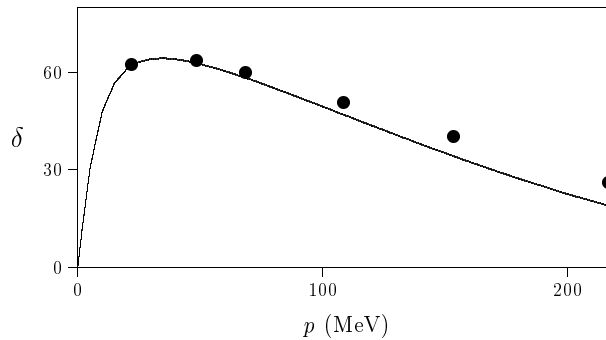


Fig. 2. np 1S_0 phase shift (degrees) *vs.* the center-of-mass (CM) momentum p . Our theory with $\Lambda = \Lambda_{Z=1} \simeq 172$ MeV is given by the solid line, and the results from the Argonne v_{18} potential [15] (“experiments”) by the solid dots. (See [14] for the precise definition of $\Lambda_{Z=1}$.) As expected the theory starts deviating as the cutoff scale is approached signalling that “new physics” is setting in.

This simple theory turns out to work extremely well for *all* two-nucleon properties [14], namely, the properties of the bound state deuteron, the radiative np capture

$$n + p \rightarrow d + \gamma \quad (4)$$

and the solar proton fusion process

$$p + p \rightarrow d + e^+ + \nu_e. \quad (5)$$

As one can see in Table I, the NLO calculation gives a remarkable agreement with all static properties of the deuteron, again with little dependence on the cutoff. A much more striking case is the radiative np capture process (4) for which the dominant contribution given by (3) with V_{EM} turned off is found to agree precisely with the result of the Argonne v_{18} potential [15]. The $\sim 10\%$ exchange current contributions that come at the next-to-next-to-leading order (NNLO) can also be accurately calculated [4, 14]. Taking into

¹ See [14] for the precise procedure of picking this cutoff. One should note that no fine-tuning is done here. The LO calculation the cutoff $\Lambda_{Z=1}$ corresponds to the NLO calculation with little dependence on cutoff in the sense of Fig. 1.

TABLE I

Deuteron properties and the $M1$ transition amplitude entering into the np capture for various values of Λ .

Λ (MeV)	198.8	216.1	250	Exp.	v_{18} [15]
B_d (MeV)	2.114	2.211	2.389	2.224	2.224
A_s (fm $^{-\frac{1}{2}}$)	0.877	0.878	0.878	0.8846(8)	0.885
r_d (fm)	1.960	1.963	1.969	1.966(7)	1.967
Q_d (fm 2)	0.277	0.288	0.305	0.286	0.270
P_D (%)	4.61	5.89	9.09	—	5.76
μ_d	0.854	0.846	0.828	0.8574	0.847
M_{1B} (fm)	4.01	3.99	3.96	—	3.98

account inherent uncertainty in short-distance physics which makes the main uncertainty in this process (in nuclear physics language, this has to do with what is called short-range correlation in the wavefunction), the calculated value for the cross-section $\sigma_{\text{ChPT}} = 334 \pm 3$ mb is in perfect agreement with the experimental value $\sigma_{\text{exp}} = 334.2 \pm 0.5$ mb. One could take this result as a “first-principle” calculation. This I believe is the first such calculation in nuclear physics.

The proton fusion process (5) plays a pivotal role for the stellar evolution of main-sequence stars of mass equal to or less than that of the Sun. The main contribution to the process comes from (3) (with the EM potential included) accounting for terms up to NNLO. Again exchange currents enter at NNLO which can be incorporated in the same way as in the np case, although the accuracy with which the NNLO terms can be calculated is not as good as in the np case. There are up to date no laboratory experimental data to check this prediction. The inverse process to (5) is however presently being measured and results will be forthcoming shortly. The only data so far available come from helioseismology in the Sun [16] which constrains the cross-section S factor to

$$3.25 \lesssim \frac{S(0)}{10^{-25} \text{MeV} - \text{b}} \lesssim 4.59. \quad (6)$$

The recent chiral perturbation calculation to NNLO [17] — which is an exact parallel to the np capture process — gives

$$S(0)_{\text{ChPT}} = 4.05(1 \pm 0.012) \times 10^{-25} \text{ MeV} - \text{b}. \quad (7)$$

This is consistent with the helioseismology (6) and agrees with the value used in the physics of solar neutrino by Bahcall and collaborators [18] using the Argonne v_{18} potential

$$S(0)_{\text{Bahcall}} = 4.00(1 \pm 0.007_{-0.011}^{+0.020}) \times 10^{-25} \text{ MeV} - \text{b}. \quad (8)$$

4. Infinite nuclear matter

4.1. Landau Fermi-liquid fixed points

Going to infinite matter bypassing all intermediate-mass nuclei, we encounter a new scale given by the Fermi sea occupied by nucleons. We are still far from deriving the Fermi sea from a chiral Lagrangian, not to mention from QCD. So I shall assume that nucleons form a Fermi sea and occupy up to Fermi momentum k_F . Consider excitations above and below the Fermi surface. Take a cutoff for such excitations at say $\tilde{\Lambda}_1/2$ below and above the Fermi sea and integrate out the excitations whose energy is greater than $\tilde{\Lambda}_1$ and write effective actions as described above. We may then proceed to do the “decimation” as above, but now around the Fermi surface. We shall call this “Fermi-surface decimation”. We learn from condensed matter systems [2] where Fermi-liquid theory plays a prominent role that as one scales down toward the Fermi surface, there are two families of fixed points. Transcribed to nuclear matter, one of the two is the nucleon effective mass m_N^* associated with the fixing of the density of the system and the other is the four-Fermi interaction that gives the Landau Fermi-liquid interaction $\mathcal{F}$. That is to say, *nuclear matter can be described by Landau Fermi-liquid fixed point theory.*

4.2. Landau parameters and BR scaling

It is possible to connect via BR scaling [10] the fixed points of Landau Fermi liquid matter to the parameters of effective chiral Lagrangians in dense medium. This can be done by looking at the response of a nucleon on the Fermi surface to electroweak fields [19, 20].

By gauge invariance, the convection current of a nucleon on top of the Fermi sea is given by the Landau–Migdal formula [21]

$$\mathbf{J} = g_l \frac{\mathbf{p}}{m_N}, \quad (9)$$

where g_l is the orbital gyromagnetic ratio given by

$$g_l = \frac{1 + \tau_3}{2} + \delta g_l \quad (10)$$

with δg_l expressed in terms of Landau parameters F_1 and F'_1 ,

$$\delta g_l = \frac{1}{6}(\tilde{F}'_1 - \tilde{F}_1)\tau_3 \quad (11)$$

with $\tilde{F} = \frac{m_N}{m_N^*} F$. On the other hand, chiral and scale invariance of QCD implies [10, 19]

$$\delta g_l = \frac{4}{9} \left[\Phi^{-1} - 1 - \frac{1}{2} \tilde{F}_1^\pi \right] \tau_3, \quad (12)$$

where $\tilde{F}_1^\pi$ is the pionic contribution to the Landau F_1 and Φ is the BR scaling parameter related to the ratio of the quark condensate $(\langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle_0)^n$ to some power n , the dependence of which is model-dependent. Φ is normalized such that at zero density it is equal to 1. Now the Landau fixed-point mass $\frac{m_N^*}{m_N} = (1 - \tilde{F}_1/3)^{-1}$ can also be expressed in terms of the BR scaling and the pionic contribution, $\frac{m_N^*}{m_N} = (\Phi^{-1} - \tilde{F}_1^\pi/3)^{-1}$. Comparing (11) and (12) for δg_l , we get

$$\tilde{F}_1 - \tilde{F}_1^\pi \approx \tilde{F}_1^\omega = 3(1 - 1/\Phi), \quad (13)$$

where the superscript ω indicates contributions from *all* massive isoscalar vector degrees of freedom, the most important of which is the familiar ω meson. (All higher energy mesons of the same quantum numbers are subsumed into that factor.) In this simplified picture, the relevant long-wavelength oscillation is given by the pion, $\tilde{F}_1^\pi$, and the short-range by the ω meson, $\tilde{F}_1^\omega$.

From giant dipole excitations in heavy nuclei, we know that $\delta g_l^p = 0.23 \pm 0.03$ for the proton [22]. From this we find that at normal density (F_1^π is known by chiral symmetry at any density)

$$\Phi(\rho_0) \approx 0.78. \quad (14)$$

We will see later that this can be connected to the dropping vector meson mass but for the moment we could simply relate it to the ratio f_π^*/f_π and get the ratio from Gell–Mann–Oakes–Renner mass formula applied to the mass of an in-medium pion. Assuming that the effective pion mass increases a bit in matter, one finds that the ratio at nuclear matter density from the in-medium GMOR relation is ~ 0.78 and agrees with (14). This relation has been checked with axial-charge transitions in heavy nuclei [20, 23, 24]

An immediate check of (14) is gotten by looking at the Landau mass of the nucleon. For (14), we get $m_N^*(\rho_0)/m_N \simeq 0.70$. This agrees with the QCD sum-rule result [25] $0.69^{+0.14}_{-0.07}$.

4.3. Evidence from nuclear matter

The next relation we need to establish is between the scaling of the meson masses and the BR scaling factor Φ . To do this it turns out to be most convenient to implement the scaling masses into a chiral Lagrangian

which in the mean field approximation gives the nuclear matter ground state correctly. For this, write the chiral Lagrangian truncated to the form of Walecka linear $\sigma - \omega$ model (that is, drop all the fields that do not enter in the mean field) as²

$$\begin{aligned} \mathcal{L}_{BR} = & \bar{\psi}[\gamma_\mu(i\partial^\mu - g_v^*(\rho)\omega^\mu) - M^*(\rho) + h\phi]\psi \\ & + \frac{1}{2}[(\partial\phi)^2 - m_s^{*2}(\rho)\phi^2] - \frac{1}{4}F_\omega^2 + \frac{1}{2}m_\omega^{*2}(\rho)\omega^2, \end{aligned} \quad (15)$$

where ψ is the nucleon field, ω_μ the isoscalar vector field, ϕ an isoscalar scalar field³ and the masses with asterisk are taken to be BR-scaling. It has been shown [26,27] that this Lagrangian in the mean field approximation gives *all* nuclear matter properties correctly (including a low compression modulus in contrast to the linear $\sigma - \omega$ Walecka model which differs from (15) in that the masses and coupling constants are non-scaling) for the canonical values of free-space masses for the hadrons provided the BR scaling

$$\Phi \approx m_V^*/m_V \approx M_N^*/m_N \approx m_\sigma^*/m_\sigma \approx f_\pi^*/f_\pi \quad (16)$$

holds with $\Phi(\rho) \approx (1 + 0.28\rho/\rho_0)^{-1}$ and the vector coupling scaling roughly the same way. As given, the scaling of Φ is consistent with what we found in the baryon sector (14). Although the connection is somewhat indirect, it is also possible to extract Φ from the QCD sum-rule calculation of the ρ meson in medium [28,29]. In fact Jin et al find $m_\rho^*(\rho_0)/m_\rho = 0.78 \pm 0.08$, entirely consistent with (14).

4.4. Evidence from kaon-nuclear interactions

There is yet another source for the scaling relation (16) that comes from the fluctuation of the BR scaling chiral Lagrangian into the strangeness flavor direction. As discussed in [27,30], the BR scaling Lagrangian at tree order predicts an attractive potential in the K^- -nuclear interaction which at nuclear matter density comes to ~ 190 MeV. This attraction has been seen in kaonic atom experiments. The recent analysis by Friedman, Gal and Mares [31] gives the attraction of 185 ± 15 MeV. This again supports the tree order calculation with BR scaling fluctuating around the matter ground state. As discussed in [32], the large attraction described in BR scaling can be attributed to the higher chiral order effects that are not taken into account in the conventional treatments.

² The quantity ρ that figures in the parameters of the Lagrangian is not a number but an operator whose mean field value is the matter density. How it is to be treated is a bit subtle. Naive interpretation of the density dependence of the mass leads to misleading results. See [26] for details.

³ Note that this scalar field is a chiral singlet — and not the fourth component of the chiral four-vector of the linear sigma model — to be consistent with chiral symmetry.

5. Dense matter

5.1. Dileptons in heavy-ion collisions

Fluctuating into non-strange directions, the effective Lagrangian with BR scaling has been successfully applied to the dilepton data of the CERES collaboration [12] by Li, Ko and Brown [11]. The heavy-ion process involves densities $\rho \sim 3\rho_0$, so a considerable extrapolation from nuclear matter is required. In an extremely simplified form, the masses of all hadrons drop linearly and become negligibly small at about $3\rho_0$. The picture is then that near the chiral phase transition the relevant degrees of freedom are the constituent quarks, that is, weakly interacting quasiquarks. Since as argued above, hadrons with BR scaling are quasiparticles at the density up to about ρ_0 , as density increases beyond ρ_0 , the effective degrees of freedom must crossover (possibly smoothly) in a manner described by the NJL model from the hadron quasiparticles to the quasiquarks forming the light-quark baryons and mesons up to the chiral phase transition. This was the argument given in [30]. How this picture emerges in understanding the CERES data will be discussed by Gerry Brown in the following talk.

5.2. Kaon condensation in compact stars

Fluctuated into the strangeness flavor direction, the dropping K^- mass discussed above leads in neutron star matter to condensation of kaons at about $\sim 3\rho_0$ with important consequences on the structure of compact stars [33]. Again the picture that emerges is that of the constituent quark.

6. Conclusions

In this talk, I argued that both dilute and dense hadronic systems can be described in effective field theories. For the former, the theory is defined in the matter-free vacuum and two-nucleon systems, bound and elastic and inelastic scattering at low energy, are accurately determined parameter-free when calculated up to NLO in the chiral counting. For the latter, the “decimation” at the Fermi-sea scale is introduced and BR scaling is identified as a means to *map* the mean-field chiral Lagrangian theory to Landau Fermi-liquid fixed-point theory. The BR scaling for the nucleon is checked with the electroweak responses of heavy nuclei and that for mesons is checked with the fluctuations built on top of the “vacuum” characterized by the density of the matter. The BR scaling parameter Φ is shown to be related to the Landau interaction parameter F_1^ω coming from massive isoscalar vector degrees of freedom that underlie *short-range interactions between nucleons*. This implies that if the BR scaling is indeed connected to the vacuum structure

of QCD as argued here, *the change of the QCD vacuum should be understandable in terms of interactions between hadrons, at least up to a certain density below that of the chiral phase transition*. This may be considered as a sort of Cheshire-Cat phenomenon [9]. It would be nice to quantify this statement.

Extrapolated into higher density regime in the most straightforward way, the theory can be applied to dense matter in heavy-ion collisions and in compact stars. As an effective theory, it is a mean-field theory. Going beyond the mean field approximation and calculating higher-order corrections remain to be formulated in a systematic way.

Finally it is argued that as density is raised above normal matter density, the correct degree of freedom should be the quasihadron and hence there must be a change-over from hadronic Fermi liquid to quark Fermi liquid of quasihadrons. Various phase transitions such as the chiral or color superconductivity could be addressed from the quark Fermi-liquid structure.

It is a pleasure to dedicate this paper to Josef Speth on the occasion of his 60th birthday. Josef and I had on various occasions — and long before Landau–Migdal theory was widely recognized by the nuclear physics community — exchanged our views on Fermi-liquid structure of nuclei and nuclear matter and the present paper is an unexpected and intriguing spin-off of the ideas in a modern context. This paper is based on work done in collaboration with Gerry Brown, Bengt Friman, Kuniharu Kubodera, Dong-Pil Min, Tae-Sun Park and Chaejun Song whom I would like to thank for discussions.

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