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CHIRAL SYMMETRY RESTORATION AND PARITY MIXING*

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We derive the expressions of the (isovector) vector and axial current from a chiral Lagrangian restricted to nucleons and pions. We show that in the heat bath certain terms can induce a mixing of the axial current into the vector one and vice versa. We generalize this concept to the case of a dense baryonic medium. We study the subsequent modifications of the axial nucleonic coupling constant and the pion decay one. They arise from a two pion exchange current of a new type. We discuss the link to the condensate evolution. The quenching of the axial coupling constant helps explaining the observed one of the Gamow–Teller sum rule.

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1. Introduction

Chiral symmetry which is spontaneously broken in the QCD vacuum is partly restored in a hot or dense medium.For independent particles the evolution of the quark condensate with density or temperature is governed by the sigma commutator of the particles present in the system with the simple following expression:

$$\frac{\langle\overline{q}q(\rho)\rangle}{\langle\overline{q}q(0)\rangle} = 1 - \sum_n \frac{\rho_n^s\ \Sigma_n}{f_\pi^2 m_\pi^2}\,, \tag{1}$$

where the sum extends over the species present in the medium, ρ^s is their scalar density and Σ their sigma commutator. In the heat bath pions play a crucial role in the restoration process as they enter as the lightest particles

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created by the thermal fluctuations. In the nuclear medium the main ingredients are the nucleons, with some corrections from the exchanged pions. At normal nuclear density the magnitude of the condensate has dropped by about 1/3, a large restoration effect. Such a large amount of restoration raises questions about possible manifestations directly linked to the symmetry. If there is no spontaneous violation of the symmetry, *i.e.*, if it is realized in the Wigner mode, the hadron masses vanish or there exist parity doublets, each hadronic state being degenerate with its chiral partner. It is therefore legitimate to believe that the large amount of restoration at normal density manifests itself either by a decrease of the hadron masses, or by effects linked to parity. The first question has been addressed [1]. On the other hand the significance of chiral symmetry restoration in the parity context was first established by Dey *et al.* [2] for the thermal case. They showed that in a pion gas a mixing occurs between the vector and axial correlators. It arises from the emission or absorption of s-wave thermal pions, which changes the parity of the system. Working in the low temperature and chiral limit they showed that the mixing is accompanied by a universal quenching of the correlators, which, to first order in the temperature, equals 4/3 of the quenching of the quark condensate. These points were also made by Steele *et al.* [3].

What I want to discuss in this talk is the implications, for the parity problem, of chiral symmetry restoration in the nuclear medium, in a world that we have for the moment restricted to nucleons and pions. In this case the only transitions allowed in the nucleus are nuclear transitions or pion production. This is a report of a work done in collaboration with Chanfray and Delorme [4]. Our starting point is a chiral Lagrangian from which we derive the explicit expressions of the axial and (isovector) vector currents. I will show how the concept of parity mixing translates in the nuclear medium. I will show that things are more subtle than in a heat bath but that the consequences are pretty much the same in the nucleus, namely the nuclear pions renormalize the coupling constants of the axial current and that the renormalizations can be expressed in terms of the pion scalar density. This last quantity also enters in the quark condensate evolution. However the complexity of the nuclear interactions bars a simple link between this evolution, which is an average concept, and the renormalizations. For instance, for the axial coupling constant g_A the detailed spatial structure of the quark condensate is needed.

I will start with the expressions of the axial and vector currents from the chiral Lagrangian, then use these expressions to study the renormalization of the nucleon axial coupling constant in the hot pion gas and in the nuclear medium. I study the thermal case as an illustration of the method since the results are already known.

2. The Lagrangian and the currents

My starting point is the chiral Lagrangian in the form introduced by Weinberg. I use the version of Lynn [5], which allows to obtain the nucleon sigma commutator in the tree approximation. The Lagrangian writes:

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{2}m_\pi^2\frac{\boldsymbol{\phi}^2}{1+\boldsymbol{\phi}^2/4f_\pi^2}+\frac{1}{2}\frac{\partial_\mu\boldsymbol{\phi}\cdot\partial^\mu\boldsymbol{\phi}}{(1+\boldsymbol{\phi}^2/4f_\pi^2)^2}\\ & +2\varSigma_N\overline{\psi}\psi\frac{\boldsymbol{\phi}^2/4f_\pi^2}{1+\boldsymbol{\phi}^2/4f_\pi^2}+\overline{\psi}(i\gamma_\mu\partial^\mu-M)\psi\\ & -\frac{1}{4f_\pi^2}\frac{\overline{\psi}\gamma_\mu(\boldsymbol{\tau}\times\boldsymbol{\phi})\cdot\partial^\mu\boldsymbol{\phi}\psi}{1+\boldsymbol{\phi}^2/4f_\pi^2}+\frac{g_A}{2f_\pi}\frac{\overline{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\phi}\psi}{1+\boldsymbol{\phi}^2/4f_\pi^2}\,. \end{aligned}\tag{2}$$

I have to specify the quantity $\varSigma_N$ associated with the nucleon density in Eq. (2). The free nucleon sigma commutator $\varSigma_N$ cannot be entirely attributed to the pion cloud. We define $\varSigma_N$ to be the difference between the total and pionic contributions:

$$\varSigma_N=\varSigma_N+\frac{1}{2}m_\pi^2\int d\boldsymbol{x}\langle N|\boldsymbol{\phi}^2(\boldsymbol{x})|N\rangle. \tag{3}$$

For instance in a description of the nucleon in terms of valence quarks and pions, the pionic contribution is approximately $1/2$ to $2/3$ of the total value [6, 7].

From the Lagrangian of Eq. (2) we derive the expressions of the axial and isovector vector currents:

$$\begin{aligned}\boldsymbol{\mathcal{A}}_\mu = & f_\pi\frac{\partial_\mu\boldsymbol{\phi}}{1+\boldsymbol{\phi}^2/4f_\pi^2}-\frac{1}{2f_\pi}\frac{[(\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi})\times\boldsymbol{\phi}]}{(1+\boldsymbol{\phi}^2/4f_\pi^2)^2}\\ & +\frac{g_A}{2}\overline{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi+\frac{g_A}{4f_\pi^2}\frac{\overline{\psi}\gamma_\mu\gamma_5[(\boldsymbol{\tau}\times\boldsymbol{\phi})\times\boldsymbol{\phi}]\psi}{1+\boldsymbol{\phi}^2/4f_\pi^2}-\frac{1}{2f_\pi}\frac{\overline{\psi}\gamma_\mu(\boldsymbol{\tau}\times\boldsymbol{\phi})\psi}{1+\boldsymbol{\phi}^2/4f_\pi^2}\,, \end{aligned}\tag{4}$$

$$\begin{aligned}\boldsymbol{\mathcal{V}}_\mu = & \frac{(\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi})}{(1+\boldsymbol{\phi}^2/4f_\pi^2)^2}\\ & +\frac{1}{2}\overline{\psi}\gamma_\mu\boldsymbol{\tau}\psi+\frac{1}{4f_\pi^2}\frac{\overline{\psi}\gamma_\mu[(\boldsymbol{\tau}\times\boldsymbol{\phi})\times\boldsymbol{\phi}]\psi}{1+\boldsymbol{\phi}^2/4f_\pi^2}-\frac{g_A}{2f_\pi}\frac{\overline{\psi}\gamma_\mu\gamma_5(\boldsymbol{\tau}\times\boldsymbol{\phi})\psi}{1+\boldsymbol{\phi}^2/4f_\pi^2}\,. \end{aligned}\tag{5}$$

With this expression the conservation of the vector current is fulfilled. These expressions are lengthy but some comments will help clarifying them. Let us first discuss the free case. We can recognise in some of the terms the usual expressions for the vector or axial current coupled to a free nucleon or pion. In addition the axial current can create one or more pions, either in free space (first terms of Eq. (4)), or when it acts on the nucleon via a term (last one of Eq. (4)) which is the equivalent for the axial current of the Weinberg–Tomozawa term of π-N scattering. Similarly the vector current acting on the nucleon can create one (or more) pion via the Kroll–Ruderman term, *i.e.* the contact piece of photoproduction (last term of Eq. (5)).

Let us now turn to the case of a hadronic medium. The expressions (4) and (5) illustrate the way in which the axial and vector current mixing occurs. Indeed, in the heat bath any of the pions can be a thermal one. As an example, consider the Kroll–Ruderman term of the vector current. The creation or annihilation of a thermal pion of momentum q in this term takes care of the pion field, leaving a factor $e^{\pm iqx}$ and we are left with a current of opposite parity, to be taken at the momentum transfer $k \pm q$ where k is the photon momentum. Similarly the pion production or annihilation by the Weinberg term of the axial current introduces the vector current nuclear matrix element. Notice in expressions (4) and (5) that the Kroll–Ruderman term itself can be obtained from the fourth term of the axial current by suppression of one of the pion fields (representing creation or annihilation of a thermal pion). Thus the three terms containing g_A in Eqs (4) and (5) are linked together by suppression or addition of one pion field. The same is true for the three purely pionic terms and for the three terms in γ_μ as well. Thus a grouping three by three of the various terms naturally emerges from our expressions. In the nuclear medium the virtual pions can

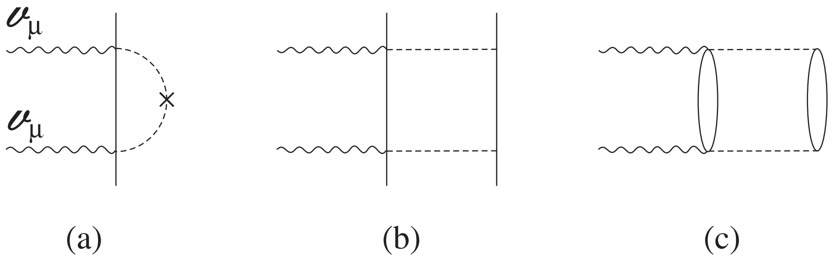

Fig. 1. Illustration of a mixing effect in the vector correlator (a) in the heat bath (denoted by a cross), (b) equivalent diagram in the nucleus with its translation (c) in many-body diagrams.

be seen as a pion bath and similar considerations about the mixing might apply. However the pions do not come from an external reservoir but fully belong to the nucleus. Strictly speaking there is no mixing. However the mixing terms of the currents can pick a pion from the cloud of a nucleon introducing a similarity with the heat bath as displayed in Fig. 1 in the case of the Kroll–Ruderman term. The corresponding process is the excitation of high lying nuclear states (2p–2h). In the case of the third isospin component of the current, it is part quasi-deuteron photoabsorption cross-section which is well known. This approach puts these effects, where the mixing terms of the currents pick a pion from the cloud, in a perspective linked to chiral symmetry. This has the merit that the concept of nuclear renormalization naturally follows. Indeed the mixing also means a renormalization of certain coupling constants, such as the axial one, that I now discuss.

3. The axial coupling constant

I start by the case of the heat bath. Although this case has been studied previously by Eletsky and Kogan [9] (with a totally different approach), it serves as an illustration of the method. The renormalization of the axial coupling constant is governed by the fourth term of Eq. (4). After rearrangement with the Gamow–Teller current (third term), we get:

$$\begin{aligned} &\frac{1}{2} g_A \overline{\psi} \gamma_\mu \gamma_5 \left(\boldsymbol{\tau} \; + \frac{1}{2f_\pi^2} \frac{\boldsymbol{\phi} \boldsymbol{\tau} \cdot \boldsymbol{\phi} - \boldsymbol{\tau} \boldsymbol{\phi}^{\mathbf{2}}}{1 + \boldsymbol{\phi}^2 / 4 f_\pi^2} \right) \psi \\ &= \frac{1}{2} g_A \overline{\psi} \gamma_\mu \gamma_5 \boldsymbol{\tau} \; \psi \left(1 - \frac{1}{3} \left\langle \frac{\boldsymbol{\phi}^2 / f_\pi^2}{1 + \boldsymbol{\phi}^2 / 4 f_\pi^2} \right\rangle \right) . \end{aligned} \tag{6}$$

On the other hand the condensate evolution is given by the expectation value of the chiral symmetry breaking part of the Lagrangian. It is equal to

$$\frac{\langle \overline{q} q \rangle_T}{\langle \overline{q} q \rangle_0} - 1 = \left(1 - \frac{1}{2} \left\langle \frac{\boldsymbol{\phi}^2 / f_\pi^2}{1 + \boldsymbol{\phi}^2 / 4 f_\pi^2} \right\rangle \right) . \tag{7}$$

Hence the axial coupling constant renormalized by the pion loops (Fig. 1a) follows, to all orders in the pion density, 2/3 of the quark condensate evolution (as long as the pions dominate the thermal excitations). The factor 2/3 is easily understood here: only two pion charges out of three contribute to the renormalization while all three charge states participate in the condensate evolution. The quenching of g_A is in agreement with the universal behaviour of Ref. [2] and with the former result of Ref. [9].

We now turn to the case of finite density. In symmetric nuclear matter the isospin dependence of the pion fields average in the same way as in the heat bath so that the expression of g_A^* in terms of the pion field squared is the same as in the previous case. However in the nuclear medium the pion originate from the other nucleons so that the nucleon-nucleon correlations cannot be ignored. Here it is useful to express this renormalization in the traditional picture of meson exchange currents. We keep only the two-body terms which are the dominant ones. The corresponding graph is that of Fig. 1b. This type of exchange graph with two pions is unusual in nuclear physics. Here it naturally follows from these chiral symmetry considerations.

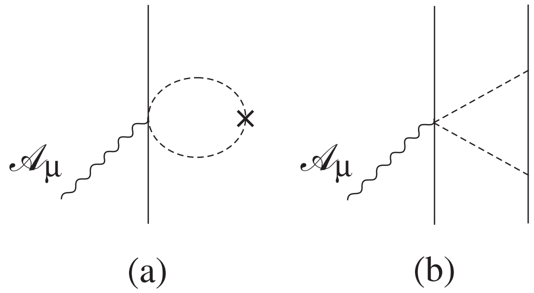

Fig. 2. Renormalization of the nucleonic axial coupling constant (a) — by a pion loop in the hot pion gas (the cross denotes the heat bath), (b) — by the virtual pion cloud in the nucleus.

A simplification occurs in the static approximation where the pions do not transfer energy to the nucleon line. For the two-body operator we are left with a simple form in x-space, expressed in terms of the squared pion field:

$$O_{12} = -\frac{1}{6f_\pi^2} g_A(\gamma_\mu\gamma_5)_1 \boldsymbol{\tau_1} \boldsymbol{\varphi^2(x_1, x_2)}\,, \tag{8}$$

where $\boldsymbol{\varphi(x_1, x_2)}$ is the Yukawa field, taken at the point $\boldsymbol{x_1}$, emitted by the nucleon located at the point $\boldsymbol{x_2}$. Sandwiching this operator between two-nucleon wave functions and summing over all the pion emitters, we obtain:

$$\delta g_A^{\mathrm{ex}}/g_A = -\frac{1}{3f_\pi^2}\int dx_2 \rho(x_2)[1 + G(x_1, x_2)]\boldsymbol{\varphi}^2(\boldsymbol{x_2}, \boldsymbol{x_1})\,, \tag{9}$$

where $\rho(x_2)$ is the nuclear density and $G(\boldsymbol{x_1}, \boldsymbol{x_2})$ is the nucleon-nucleon correlation function. It is clear on this expression that it is not the full pion field squared which acts in the renormalization of g_A, but only the part which extends beyond the range of the correlation hole.

In order to get an estimate for g_A^*, we assume a total exclusion of other nucleons in a sphere of radius $r_0 = 0.6fm$. In order to facilitate the comparison of the quenching effect of g_A to that of the condensate which is governed by the nucleon sigma term, we introduce a quantity $(\Sigma_N)_{\mathrm{eff}}$:

$$(\Sigma_N)_{\mathrm{eff}} = \frac{1}{2}m_\pi^2 \int d\boldsymbol{x}\theta(x - r_0)\boldsymbol{\varphi^2(x)}\ . \tag{10}$$

Numerically, for point-like pion emitters, we find an effective value $(\Sigma_N)_{\mathrm{eff}} \approx$ 21 MeV. This number do not include the Pauli blocking effect which removes the occupied states in the process of pion emission. This effect has been calculated in Refs. [10] but for the whole space integral (*i.e.* without a cut-off) of the quantity $\boldsymbol{\phi}^2$. Expressed in terms of a modification of the sigma commutator it amounts to a reduction $(\Delta\Sigma_N)_{\rm Pauli} = -2.6$ MeV . The blocking effect, which is moderate, should be even less pronounced with the cut-off. We can ignore it.

For the renormalized axial coupling constant, we have:

$$g_A^*/g_A = 1 - \frac{2}{3}\frac{\rho(\Sigma_N)_{\rm eff}}{f_\pi^2 m_\pi^2} \ . \tag{11}$$

This quenching applies to all the components, space or time, of the axial current. This represents a 10% quenching at normal nuclear density (for symmetric nuclear matter), while the condensate has dropped by 35%. The evolution of g_A is sizeably slower. Other renormalization effects have to be added. They act differently on the different components. In the case of the space component the nucleon polarization under the influence of the pion field $N \to \Delta$ leads to the Lorentz–Lorenz quenching [11]. The two renormalizations go in the same direction of a quenching. The extra reduction that I have introduced here can help to explain the large amount of quenching observed in Gamow–Teller transitions. To get an idea, we fictitiously translate the reduction by chiral symmetry into an equivalent Lorentz–Lorenz effect. We introduce an effective Landau–Migdal parameter $\delta g'_{N\Delta}$, to be added to the genuine one, so as to reproduce the 10% quenching. This corresponds to an increase $\delta g'_{N\Delta} \approx 0.16$, a significant increase. Indeed the observed quenching of the Gamow–Teller sum rule requires $g'_{N\Delta}$ to be as big as 0.6 while the favoured theoretical value is around 0.4 [12]. Hence the chiral induced quenching helps to account for the difference.

Independently of the role played by the short range NN correlation, which reduce the amount of quenching, a major difference between the axial coupling constant and condensate evolutions arises from the following effect: for g_A, the pion scalar density is the relevant parameter while the condensate evolution is only partly governed by this quantity. We have seen that there is a non pionic contribution to the nucleon sigma commutator, that we have denoted Σ_N, which does not influence g_A , at least in the present approach. This is a feature which distinguishes the dense case as compared to the heat bath. Another one emerges from the comparison between the renormalization of f_π and that of g_A which renormalize in the same way in the heat bath. This is no longer true in the medium. For the pion decay constant the short range NN correlations play no role as the pion born from the axial current can be anywhere in the nucleus. The full pionic scalar density then enters. The universality of the quenching, a striking feature of the heat bath situation, is lost in the nuclear medium.

In summary I have shown you how the concept of parity mixing between the axial and vector correlators, which exists in the hot pion gas can be extended to the nuclear medium. In the last case there is in fact no mixing since the pions responsible for the mixing are not part of an external system but they belong to the virtual pion cloud which is an integral part of the nucleus. I have shown that nevertheless certain consequences of the mixing survive. The nucleus behaves in certain respects as the pion reservoir of the heat bath. The mixing terms of the currents can pick a pion from this reservoir giving rise to what can be considered as a mixing cross-section (ex: the quasi-deuteron one). Associated with these mixing effects there occurs, as in the heat bath, a quenching of certain coupling constants. However this quenching is not universal. For instance g_A is sensitive to the short-range NN correlations while f_π is not.These renormalizations correspond, in the language of meson exchange currents to an unusual type, with two exchanged pions. In addition the renormalizations, although expressed in terms of the pion scalar density, cannot be simply linked to the condensate evolution because the last quantity also depends on non-pionic processes. This makes the comparison between the two evolutions model dependent. Nevertheless these renormalizations display in a striking way the consequence of chiral symmetry restoration which are most directly linked to parity.

This talk reports on a work done with Chanfray and Delorme. I take this opportunity to thank them for a most pleasant and continuous collaboration. I thank the organizers of this meeting and Prof. J. Speth for inviting me to participate.

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