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CHIRAL SYMMETRY IN MATTER*

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Soft-pion theorems are used to show how chiral symmetry constrains the contributions of low-momentum pions to the quark condensate, the pion decay constant and hadron masses, all of which have been proposed as signals of partial restoration of chiral symmetry in matter. These have contributions of order T^2 for a pion gas or of order m_π for cold nuclear matter, which have different coefficients in all three cases, showing that there are no simple relations between the changes to these quantities in matter. In particular, such contributions are absent from the masses of vector mesons and nucleons and so these masses cannot scale as any simple function of the quark condensate. More generally, pieces of the quark condensate that arise from low-momentum pions should not be associated with partial restoration of chiral symmetry.

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1. Introduction

One of the main topics being addressed by this workshop is the partial restoration of chiral symmetry in nuclear or hadronic matter and the possibility that signals of this restoration have been seen in relativistic heavy-ion collisions. Although a rather unified picture seemed to be emerging from some of the previous contributions, what I want to do here is to unravel some of the threads in this picture by examining whether changes in quantities like the quark condensate, the pion decay constant and hadron masses are in fact related to each other, and the extent to which they can be interpreted in terms of symmetry restoration.

Central to these questions are the soft-pion theorems that embody the consequences of chiral symmetry for interactions of low-momentum pions [1].

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The focus of this contribution will be the constraints that these place on the properties of hadrons in matter. I will devote most of my discussion to the case of a warm gas of pions, where the calculations are somewhat cleaner than for nuclear matter. In fact the role of chiral symmetry in the pion gas is rather an old topic [2–12] and it is rather embarrassing to admit how long it has taken me learn the lessons from it. However I want to stress that these lessons are general ones and apply equally to the case of cold nuclear matter [13–16].

The chiral isospin symmetry SU(2)×SU(2) is a good approximate symmetry of the QCD Lagrangian, broken only by the small current masses of the up and down quarks. This symmetry is realised in the hidden ("spontaneously broken") mode since the QCD vacuum is not chirally invariant. This can be pictured in terms of a Mexican-hat potential for the vacuum with a "chiral circle" of degenerate vacuum states running round its brim. The pions are, approximately, the corresponding massless Goldstone bosons. The nonzero quark condensate (the scalar density of quarks in the vacuum $\langle\overline{\psi}\psi\rangle$) is an example of an order parameter for the hidden symmetry. Other indications of the hidden nature of this symmetry include the nonzero pion decay constant f_π, the absence of degenerate parity doublets in the hadronic spectrum, and the masses of constituent quarks.

In dense hadronic matter, which could be either a gas of pions or nuclear matter, we expect the vacuum to move towards the phase with manifest chiral symmetry. Signals of such a partial restoration of the symmetry that have been suggested include:

- a decrease in the magnitude of the quark condensate,
- a smaller pion decay constant,
- smaller splittings between states of opposite parity in the hadronic spectrum, for example the ρ and a_1 mesons
- decreased hadron masses.

Although much attention has focussed on the last of these, this effect has been demonstrated only in mean-field approaches such as the linear sigma and Nambu–Jona-Lasinio models. The general arguments outlined here show that pionic fluctuations make significant, but very different, contributions to these quantities and so it is important to go beyond the mean-field approximation in studying hadron properties in matter.

2. Soft-pion theorems

The basic tools for elucidating the consequences of chiral symmetry for the interactions of pions are the "soft-pion theorems" [1]. These are obtained from physical matrix elements involving pions with zero 3-momentum by extrapolating them to zero energy. This (off-shell) extrapolation is done using the PCAC pion field

$$\boldsymbol{\phi} = \frac{\partial_\mu \mathbf{A}^\mu}{f_\pi m_\pi^2}, \tag{1}$$

which connects pions to chiral symmetry. For example, the soft-pion limit of the pion propagator defined using this field gives the Gell-Mann–Oakes–Renner (GOR) relation,

$$\bar{m}\langle 0|\overline{\psi}\psi|0\rangle \simeq -m_\pi^2 f_\pi^2, \tag{2}$$

which relates the pion mass to the expectation value of the explicit symmetry breaking piece of the QCD Hamiltonian, $\mathcal{H}_{\rm SB} = \bar{m}\overline{\psi}\psi$. Taking a typical value of $\bar{m} \simeq 7$ MeV for the average of the up- and down-quark current masses, we find that the quark condensate in vacuum is $\langle\overline{\psi}\psi\rangle \sim -3$ fm^{-3}.

For a more general matrix element we have

$$\langle\alpha|\mathcal{O}|\beta\pi(q)\rangle \simeq -\frac{i}{f_\pi}\langle\alpha|[Q_5, \mathcal{O}]|\beta\rangle, \tag{3}$$

up to corrections involving the pion momentum q or the explicit symmetry breaking strength m_π^2. The connection between a commutator with an axial charge operator and creation or annihilation of a pion shows that a low-momentum pion can be thought of as acting like a chiral rotation. This implies that any pion scattering amplitude should vanish as $q \to 0$ in the chiral limit ($m_\pi^2 = 0$). A central role in the discussion here will be played by the isospin-averaged amplitude for scattering of a pion from some other hadron. The leading terms in the chiral expansion of this amplitude are of order q^2 and m_π^2.

In studying changes to the quark condensate, we shall need the scalar densities of quarks in hadrons. For the pion this can be evaluated with the help of a soft-pion theorem:

$$\langle\pi|\bar{m}\overline{\psi}\psi|\pi\rangle \simeq -\frac{1}{f_\pi^2}\langle 0|[Q_5, [Q_5, \mathcal{H}_{\rm SB}]]|0\rangle \simeq m_\pi^2, \tag{4}$$

where the GOR relation (2) has been used to express the matrix element in terms of m_π^2. Factoring out the covariant normalisation of $2m_\pi$, we are left with the pion-pion sigma commutator $\sigma_{\pi\pi} = m_\pi/2 = 70$ MeV. This corresponds to an integrated scalar density of quarks in a pion of $\sigma_{\pi\pi}/\bar{m} \sim 10$. This large number shows that pions make important contributions to the scalar density of quarks in hadrons or matter. These contributions are proportional to the scalar density of pions in the state of interest, $\langle\alpha|\frac{1}{2}\boldsymbol{\phi}^2|\alpha\rangle$.

The corresponding matrix element for a nucleon can be found from the pion-nucleon sigma commutator, $\sigma_{\pi N} = \langle N|\bar{m}\overline{\psi}\psi|N\rangle \simeq 45 \pm 7$ MeV [17]. This corresponds to an integrated scalar density of quarks in a nucleon of ~ 6. In contrast, simple relativistic quark models would give values of 2–3. The difference arises from the pion cloud of the nucleon, which typically contributes about 25 MeV to $\sigma_{\pi N}$ in chiral bag or soliton models [18].

3. Warm pion gas

To illustrate how chiral symmetry constrains hadron properties in matter, I consider first the case of matter at finite temperature in the chiral limit. At low temperatures and zero baryon density, hadronic matter is just a gas of weakly interacting, massless pions. This has been studied for some time [2–12] and the results are cleaner than those for cold nuclear matter.

The scalar density of quarks in a pion in the chiral limit is, from (4),

$$\langle\pi|\overline{\psi}\psi|\pi\rangle = \left.\frac{m_\pi^2}{\bar{m}}\right|_{\bar{m}\to 0} = -\frac{1}{f_\pi^2}\langle 0|\overline{\psi}\psi|0\rangle, \tag{5}$$

and the scalar density of pions in the gas, obtained by averaging over the Bose-Einstein distribution, is

$$\langle\tfrac{1}{2}\boldsymbol{\phi}^2\rangle_T = \frac{T^2}{8}. \tag{6}$$

With these we can calculate the change in the quark condensate to order T^2:

$$\langle\overline{\psi}\psi\rangle \simeq \langle 0|\overline{\psi}\psi|0\rangle + \langle\tfrac{1}{2}\boldsymbol{\phi}^2\rangle_T\,\langle\pi|\overline{\psi}\psi|\pi\rangle = \langle 0|\overline{\psi}\psi|0\rangle\left(1 - \frac{1}{8}\frac{T^2}{f_\pi^2}\right). \tag{7}$$

In the presence of matter, the leading change in the mass of any particle is given by a sum over (the scalar parts of) the amplitudes for the scattering of the "probe" particle from the various particles in the medium. For a heavy hadron, such as a vector meson or nucleon, chiral symmetry requires that the isoscalar hadron-pion scattering amplitude be of order q^2 (where q is the pion momentum) in the chiral limit. As a result the change in the mass of such a particle is proportional to an integral over the Bose-Einstein distribution of pions weighted with an extra factor of momentum-squared relative to (6). Since the typical momenta of the pions in the gas are of order T, the changes in hadron masses are of order T^4 instead of T^2. This shows that hadron masses in the gas cannot scale like the the quark condensate (or any simple function of $\langle\overline{\psi}\psi\rangle$).

In fact, by analogy with the behaviour of a superfluid [19], we should not have expected any simple relation between these quantities. There the condensate density (the order parameter) changes at order T^2 while the superfluid density (which is a response function) changes at order T^4. Hadron masses can be defined in terms of response functions of the QCD vacuum and so behave in a similar way to the superfluid density, and quite differently from the condensate.

One of the lessons to be learned from this is the changes in the quark condensate from low-momentum pions do *not* necessarily signal partial restoration of the symmetry. Large-amplitude, low-momentum fluctuations of the pion fields around the chiral circle (or indeed a pion condensate) can significantly reduce the average value of the condensate without moving the system off the chiral circle. In such a case the hadron spectrum remains unchanged and so the chiral symmetry is still hidden. One should remember that $\langle\overline{\psi}\psi\rangle$ is only one possible order parameter; others can become important if $\langle\overline{\psi}\psi\rangle$ is small[1].

We can excite a superfluid just by pouring it. Since this action is invariant under phase rotations of the condensate, any changes in the response function reflect changes in the excitation spectrum. In contrast the easiest ways to study the response of the QCD vacuum involve operators that are not chirally invariant. As a result the corresponding response functions can contain temperature-dependent pieces that reflect the chiral transformation properties of the operators and have nothing to do with any changes in the spectrum. A typical example is the correlator of two isovector vector currents,

$$C^V_{\mu\nu}(p,T) = (2\pi)^{-4}\int d^4x\, e^{ip\cdot x}\langle \mathrm{T}[V_\mu(x), V_\nu(0)]\rangle_T. \qquad (8)$$

The pion gas changes the couplings of the currents to physical hadrons and mixes them with the axial currents [3, 4, 8], so that to leading order in T^2 the correlator is

$$C^V_{\mu\nu}(p,T) \simeq \left(1 - \frac{1}{6}\frac{T^2}{f_\pi^2}\right) C^V_{\mu\nu}(p,0) + \frac{1}{6}\frac{T^2}{f_\pi^2} C^A_{\mu\nu}(p,0), \qquad (9)$$

where, after averaging over isospin, the leading effects of the pion gas can be expressed in terms of $\frac{4}{3}\langle\alpha|\frac{1}{2}\boldsymbol{\phi}^2|\alpha\rangle$. The effect on the correlator of axial

[1] The pion would then be anomalously light [20], with a mass proportional to $\bar{m}$ rather than its square root. There is a school of chiral perturbation theory that suggests that this is already the situation in the normal vacuum [21].

currents is very similar, with the pion decay constant becoming [2]

$$f_\pi(T) = f_\pi \left(1 - \frac{1}{12}\frac{T^2}{f_\pi^2}\right). \tag{10}$$

However, even though the couplings change and the correlators mix at order T^2, the spectrum of states excited by the currents in (9) is still that of the zero-temperature correlators $C^{V,A}_{\mu,\nu}(p,0)$. Hence to this order the ρ-a_1 splitting is unchanged and there is no restoration of chiral symmetry in the spectrum.

These contributions to correlators in matter arising from the chiral transformation properties of the operators mean that we need to be rather careful when using QCD sum rules to study hadrons in matter. These sum rules are obtained by using the operator product expansion (OPE) to relate correlators to condensates (expectation values of various local operators).

A particularly simple case is the correlator of two isoscalar vector currents, which can be used to derive a sum for the mass of the ω meson. The OPE of this correlator involves only chirally invariant operators such as $[\overline{\psi}\gamma^\mu(1\pm\gamma_5)\lambda_a\psi]^2$. The expectation values of these are known as four-quark condensates. They are often estimated by Fiertz rearranging to write them in terms of $\langle(\overline{\psi}\psi)^2\rangle$ and then assuming a factorised form, $\kappa\langle\overline{\psi}\psi\rangle^2$. However for a chirally invariant operator $\mathcal{O}$ a soft-pion theorem gives

$$\langle\pi(q)|\mathcal{O}|\pi(q)\rangle \simeq -\frac{1}{f_\pi^2}\langle 0|[Q_5,[Q_5,\mathcal{O}]]|0\rangle = 0, \tag{11}$$

and so there should be no contributions to the condensates that are proportional to the scalar density of pions. The leading temperature dependence of the OPE representation of the correlator is thus of order T^4. This matches with the order-T^4 change in the ω mass, but it is not consistent with the factorised ansatz often used for the four-quark condensates.

The correlators used to derive sum rules for the ρ meson and nucleon masses are more complicated. The OPE's of these include operators that are not chirally invariant and so give rise to terms of order T^2 but a careful treatment of low-momentum pion terms in the spectral representations of the correlators shows that these exactly cancel the pieces of order T^2 from the OPE [4, 5, 8, 10]. This leaves leading changes in the masses that are of order T^4.

4. Nuclear matter

The discussion of nuclear matter is somewhat more complicated because the pions involved are virtual rather than real, and because the nucleons are strongly interacting. Nonetheless the same basic features are present [13].

Let me start with the simple additive estimate of the change in the quark condensate in nuclear matter with a scalar density ρ_s of nucleons [22]:

$$
\begin{aligned}
\langle\overline{\psi}\psi\rangle_\rho &\simeq \langle 0|\overline{\psi}\psi|0\rangle + \rho_s\langle N|\overline{\psi}\psi|N\rangle \\
&\simeq \langle 0|\overline{\psi}\psi|0\rangle \left(1 - \frac{\rho_s\sigma_{\pi N}}{f_\pi^2 m_\pi^2}\right).
\end{aligned} \tag{12}
$$

This suggests a $\sim 30\%$ reduction in the quark condensate in nuclear matter with a density of $\rho = 0.17\ \mathrm{fm}^{-3}$.

This change in the condensate contains a significant contribution from low-momentum pions, in this case virtual particles in the pion clouds of the nucleons. Like the real pions in the gas, these cannot contribute to hadron masses in matter. To pick out their contribution we can make use of the methods of chiral perturbation theory (ChPT) [23].

The relevant piece is proportional to the scalar density of pions in a nucleon. The chiral expansion of this quantity in powers of m_π has the form

$$
\langle N|\frac{1}{2}\boldsymbol{\phi}^2|N\rangle \simeq A_\pi - \frac{9}{16\pi}\left(\frac{g_{\pi NN}}{2M_N}\right)^2 m_\pi + \cdots. \tag{13}
$$

One can write the sigma commutator as a sum of core and cloud contributions,

$$
\begin{aligned}
\sigma_{\pi N} &= A_{\mathrm{core}} m_\pi^2 + \langle N|\tfrac{1}{2}\boldsymbol{\phi}^2|N\rangle\langle\pi|\bar{m}\overline{\psi}\psi|\pi\rangle \\
&\simeq \left[A - \frac{9}{16\pi}\left(\frac{g_{\pi NN}}{2M_N}\right)^2 m_\pi\right] m_\pi^2.
\end{aligned} \tag{14}
$$

The constant A ($= A_{\mathrm{core}} + A_\pi$) is a "counterterm" in ChPT. It contains both short-distance (core) and long-distance (cloud) contributions, which cannot be separated in a model-independent way. In contrast the second term, which has a nonanalytic dependence on m_π^2, is a purely long-distance effect. It arises from the lowest-momentum pions in the tail of the pion cloud. It is model-independent, with a coefficient given entirely in terms of the πN coupling and nucleon mass. Moreover, being a long-distance effect, it is unaffected by short-range correlations between the nucleons. We can therefore use nonanalytic terms like this as markers for the contributions of low-momentum pions.

From (13) we see that there is a contribution from these pions to the quark condensate in matter that is of order $m_\pi\rho$. For hadron masses to scale with $\langle\overline{\psi}\psi\rangle_\rho$, there would need to be a term of order m_π in the hadron-nucleon scattering amplitude. However Weinberg's power counting shows that no such term in present [24][2]. Instead the leading nonanalytic contribution from isoscalar two-pion exchange to hadron-nucleon scattering is of order m_π^3, which gives rise to a leading change in the hadron mass of order $m_\pi^3\rho$. This suppression by two chiral powers relative to the scalar density of pions has exactly the same origin as in the pion gas: the leading terms of the isoscalar pion-hadron scattering amplitude are of order q^2 and m_π, as required by chiral symmetry. Hence we again see that hadron masses cannot scale like any simple function of $\langle\overline{\psi}\psi\rangle_\rho$ in matter [13].

There are also similar contributions from low-momentum pions to the couplings of vector and axial currents to hadrons. These can lead to changes in quantities like f_π and g_A which are proportional to $\langle\frac{1}{2}\boldsymbol{\phi}^2\rangle$ in isospin symmetric nuclear matter. The latter can show up as a two-pion exchange contribution to the quenching of g_A [16]. There is also an analogue of the mixing of vector and axial correlators [15] which can, in principle, affect pion photoproduction [16].

5. Conclusions

We have seen that there are no simple relations between changes to the quark condensate, the pion decay constant and hadron masses in matter. Specifically, low-momentum real or virtual pions generate terms in the condensate of order T^2 in a pion gas or $m_\pi\rho$ in nuclear matter that are absent from the masses. As a result, hadron masses cannot scale like any simple function of the condensate (or of f_π) in matter. This is a consequence of the chiral suppression of the interactions between low-momentum pions and other hadrons, which means that contributions to the condensate from the scalar density of pions cannot lead to changes in the masses of heavy hadrons like vector mesons and nucleons.

Operators like the vector and axial currents are not chirally invariant and so low-momentum pions can contribute to observables like f_π and g_A. In isospin symmetric matter, the lowest order pieces of the changes in such quantities can be expressed in terms of the scalar density of pions. However the operators have different isospin structures from the quark condensate and so these terms have different coefficients compared with the similar term in $\langle\overline{\psi}\psi\rangle$. More generally there is no simple relation between such couplings and the condensate in matter.

[2] Strictly, Weinberg's counting rules apply to the two-particle irreducible scattering amplitude. However this is the relevant amplitude for the definition of a mass that could appear in, for example, a Dirac equation for a nucleon in matter. It avoids the problem of strong energy dependence of the full scattering amplitude produced by poles close to threshold.

These results show the importance of including pionic fluctuations in any study of hadron properties in matter. Pionic fluctuations can change the average value of the quark condensate with moving the system off the chiral circle, and so a decrease in one order parameter, such as the condensate, on its own need not be a signal of partial symmetry restoration. One needs to look at the response functions of the vacuum as well, and in particular the hadron spectrum.

Let me finish with a question: Can we find some other order parameter or similar quantity with a more direct relation to hadron masses than the quark condensate?

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