

QUARK MODEL, CHIRAL SYMMETRY,
DEFORMATION OF THE NUCLEON AND THE
NUCLEON–NUCLEON INTERACTION*

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A property of Quantum Chromodynamics (QCD) which should be included into effective models describing QCD at low energies is chiral symmetry. It is conserved if one assumes that the quark masses are zero. This symmetry is spontaneously broken, which leads to constituent quark and as the Goldstone Boson one obtains the pion and its chiral partner the σ meson. We use the linear σ model which has compared to the non-linear one the advantage, that one treats pions π and σ mesons not only on the chiral circle, but allows also fluctuations around it. The scale of this fluctuations is the σ meson mass. If one eliminates gluons in second order, one obtains the “Tuebingen chiral quark model” with effective gluon exchange between the quarks and with pions and sigma mesons. A confinement potential is added. With this model we describe the photo and electro-excitation of the nucleon into the delta resonance and the decay of this resonance into a nucleon and a pion. The angular distribution gives information about the $C2/E2$ admixture into the $M1$ transition from the nucleon to the delta resonance. The quadrupole contribution of this transition has been described in the past by d state admixture due to tensor forces from the gluon and the pion exchange. This yields values which are more than a factor 10 too small compared with recent data for the $C2/E2$ Sachs transition form factor. We show that meson and gluon pair exchange currents can explain the data without the need of a large nucleon or delta deformation. The same model is then used to describe the nucleon-nucleon phase shifts. An essential ingredient for the good agreement is to include to all orders couplings to Δ channels. The 1S_0 phase shift can only be described in agreement with the data if the coupling to the 5D_0 nucleon delta channel is included.

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1. Introduction

In this talk I want to ask the question: Is the nucleon deformed? And in a second part we describe within the same “Tuebingen chiral quark model” the nucleon-nucleon interaction. The starting point are symmetries of the QCD Lagrangian. For zero quark masses, left-handed quarks stay always left-handed and right-handed quarks always right-handed. In this approximation QCD is chiral invariant.

Due to the strong gluon quark interaction, quark-antiquark pairs form in the vacuum a quark condensate. This breaks chiral symmetry by giving the quarks a constituent mass of the order of 300 MeV. The Goldstone Boson of this symmetry breaking is the π meson with the chiral partner, the σ scalar, isoscalar meson. For describing the π and σ meson, which couple to the quarks, we are using the linear σ model. The non-linear σ model eliminates the σ meson on the chiral circle, while the linear sigma model allows fluctuations around the chiral circle for which the scale is the σ meson mass which is determined by the partially conserved axial vector current (PCAC).

$$m_\sigma^2 = (2m_q)^2 + m_\pi^2 = (675 \text{ MeV})^2. \quad (1)$$

Chiral symmetry dictates the coupling of the quarks to the σ and π mesons.

$$\begin{aligned} L_{\pi\sigma,q} &= -g\bar{q}[\sigma + i\gamma_5\boldsymbol{\pi}\cdot\mathbf{T}]q \\ g &= g_0\left(\frac{\Lambda^2}{\Lambda^2+k^2}\right)^2. \end{aligned} \quad (2)$$

The quark π, σ coupling constant g has a form factor with the cut-off parameter Λ which is due to the internal structure and the finite size of the constituent quarks.

$$\begin{aligned} \langle r^2 \rangle^{1/2} &= \frac{3}{\Lambda^2} = 0.41 \text{ fm}, \\ \Lambda &= 4.2 \text{ fm}^{-1} \hat{=} 0.828 \text{ GeV}/c, \\ \frac{g_0^2}{4\pi} &= \frac{g_{\sigma q}^2}{4\pi} = \frac{g_{\pi q}^2}{4\pi} = \left(\frac{3}{5} \frac{m_q}{m_N}\right)^2 \frac{g_{\pi N}^2}{4\pi}. \end{aligned} \quad (3)$$

The value of the quark σ and π coupling constant is derived from the pion-nucleon coupling constant.

Eliminating the gluons in second order, one obtains a Hamiltonian for the $n = 3$ and $n = 6$ quark system.

$$\begin{aligned}
 \mathbf{H}_{nq} = & \sum_{i=1}^n \left[m_i + \frac{\mathbf{p}_i^2}{2m_{qi}} \right] \\
 & + \sum_{i < j=1}^n [V_g^{qq}(i, j) - \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j ar_{ij}^2 \\
 & + V_\pi^{qq}(i, j) + V_\sigma^{qq}(i, j)] . \tag{4}
 \end{aligned}$$

This Hamiltonian eliminates the gluons, the pions and the σ mesons in a second order and adds a quadratic confinement potential. The relativistic Hamiltonian is expanded up to the order of v^2/c^2 . The expectation value of this quantity in the nucleon wave function is about 0.46 and the square which is neglected therefore 0.20. The Hamiltonian should therefore not be more accurate than 20 %. But the results show a better agreement with experiment in most observables. This probably is connected with the fact that the four free parameters of the model are adjusted including this truncation. The four quantities are determined by the nucleon mass $m_N = 938$ MeV, the Δ mass $m_\Delta = 1232$ MeV, the charge root mean square radius of the proton and the pion-nucleon coupling constant. The cut-off parameter Λ is taken from the size of the constituent quarks, estimated in the literature (see above) and the oscillator length b for the harmonic oscillator basis used is determined by the stability condition: The mass of the nucleon must have a minimum at the right root mean square charge radius of the proton including the pion cloud at the right mass of 938 MeV.

The Hamiltonian (4) is now used to calculate the photo production of the Δ resonance and the decay of this resonance into a pion and a nucleon.

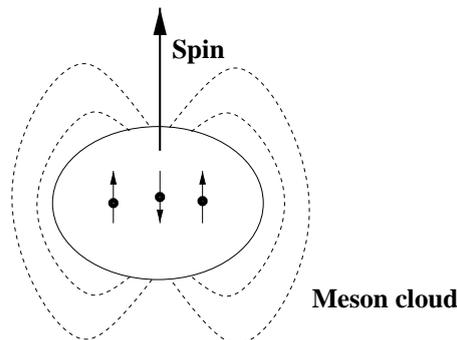


Fig. 1. The figure shows the coupling of the pion cloud to the nucleon with three valence quarks.

The meson cloud couples preferentially along the spin axis of the nucleon (see Fig. 1). Since the pion cloud exerts pressure, one can expect that the nucleon is deformed. The microscopic reason for a possible deformation of the nucleon is a tensor force component in the pion and gluon exchange part of the Hamiltonian (4). The d state admixture in the single quark wave function lies between 0.5 to 1.0 % including the tensor forces from pion and gluon exchange. Experimentally the quadrupole deformation is measured according to figure 2 by photo excitation of the delta resonance and the angular distribution of the decay pion. The quadrupole deformation is described by the C2 longitudinal Coulomb and the transversal electric quadrupole (E2) Sachs form factor at the photon point (three momentum transfer $\mathbf{q}^2 = 0$ and energy transfer $\omega = m_\Delta - m_N$).

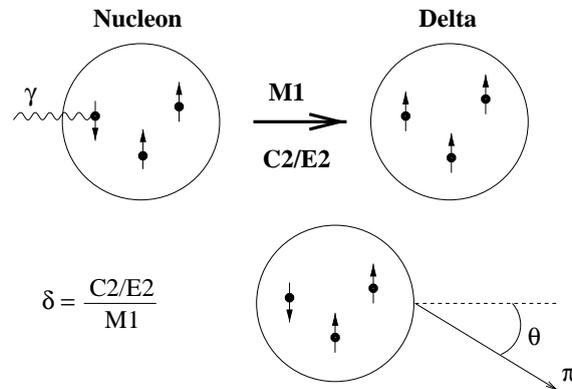


Fig. 2. The quadrupole deformation of the nucleon is determined by photo excitation of the nucleon into the delta resonance. The $(C2, E2)/M1$ ratio is determined by the angular distribution of the pion.

TABLE I

Experimental E2 Sachs form factors [9] of the $N - \Delta$ transition by photoexcitation.

$G_{E2}(\mathbf{q}^2 = 0; w = m_\Delta - m_N)$	Ref.
0.066 (18)	4
0.133 (20)	5
0.110 (20)	6
0.141	7
0.14	8

The theoretical values of the E2 Sachs form factor at the pseudo threshold are listed in Table II.

TABLE II

Sachs form factor G_{E2} at the photo point and at an energy transfer $\omega = m_{\Delta} - m_N$ calculated by Gershstein *et al.* [10], by Isgur and Karl [11] by Drechsel and Gianinni [12] and by Capstick and Karl [13]. Reference [9] gives the value calculated by the Tuebingen group including two-body currents. The last line shows an average of the experimental values (mainly references [5, 6]). The values of Refs. [10–13] are calculated as a one quark transition. Our value [9] includes two body currents.

$G_{C2/E2}(\mathbf{q}^2 = 0)$	Ref.
0.0055	10
0.0042	11
0.0076	12
0.0084	13
0.124	9
≈ 0.120 (20)	Exp.

Figure 3 shows the diagrams included in the two-body currents. The pair currents by pion exchange and by gluon exchange are the most important ones.

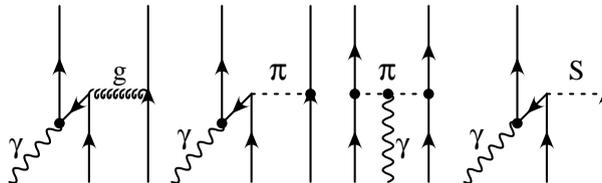


Fig. 3. Meson exchange diagrams which allow that a photon is absorbed by two quarks. The main contributions are the pair currents from pion and gluon exchange. The σ pair current is zero. In our approximation the pion in flight does contribute only little to the E2 strength [9].

The absorption of a photon on two quarks due to exchange currents allows for a double spin flip (see figure 4). This can produce an electric quadrupole transition from the nucleon into the Δ resonance without any d-state contribution of the valence quark wave functions.

Table II shows a comparison of our theoretical value including two-body currents (second last line) and the experimental data of reference [5–7]. The agreement is remarkably good, only a very small percentage of the transition

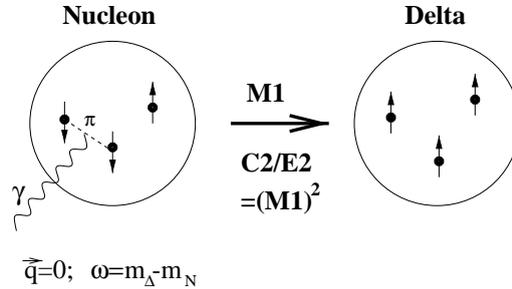


Fig. 4. The figure shows how the absorption of a photon due to exchange currents can flip the spin of two quarks and induce a transition from the nucleon to the delta resonance without any deformation of the valence part of the nucleon or the delta wave functions.

described by the Sachs form factor is due to the impulse approximation (single quark transition) with deformation. By far the largest part is due to the two-body currents. Especially due to the pair currents with pion and gluon exchange.

2. The nucleon-nucleon interaction

In this chapter I want to present results concerning the nucleon-nucleon phase shifts calculated on the six-quark level with π and σ meson clouds. The parameters are determined as indicated in the introduction. To describe the deuteron properties, it is essential to have the right deuteron binding energy of 2.22 MeV. In the Tuebingen chiral quark model this quantities depends sensitively on the charge radius of the proton including the pion cloud, which determines the oscillator length $b = 0.5$ to 0.6 fm for the oscillator basis wave functions. $b = 0.5$ fm yields the total mass of the deuteron by about 1 MeV too large. This is a deviation less than 1 per thousand. But the binding energy is by a factor 2 wrong. Thus we do a fine tuning of the oscillator length to reproduce the correct binding energy. This requests $b = 0.518$ fm, which does not modify the charge radius of the proton out of the range of the experimental uncertainty.

The phase shifts are calculated on the six-quark level with pion and σ meson exchange, using the resonating group method [1, 2]. We include excitations into the delta resonance. For example in the 1S_0 channel the coupling of the six-quarks into a nucleon and a delta configuration with spin $S = 2$ and orbital angular momentum $L = 2$ (5D_0 configuration) plays an important role [14]. The experimental phase shifts are taken from Arndt and coworkers [15]. The comparison between the different phase shifts $^{2S+1}L_J$ up to D -waves are given in figures 5–8.

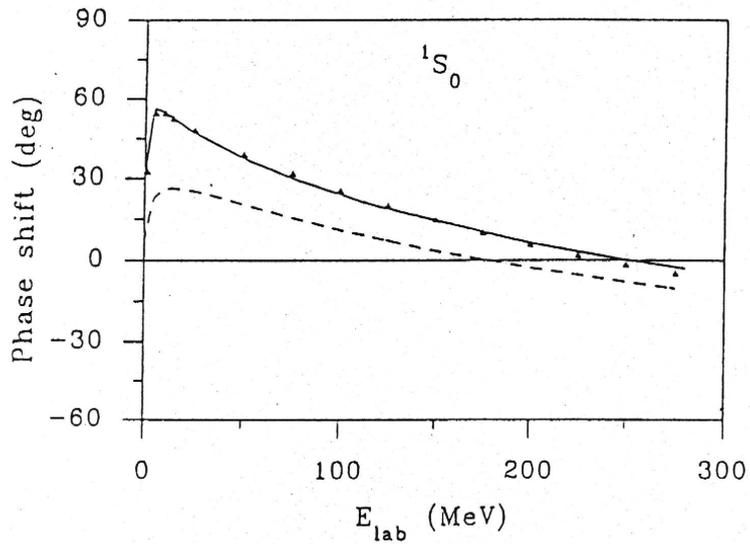


Fig. 5. Singlet $^1S_0 NN$ phase shift including chiral symmetry and coupling to the $^5D_0(N\Delta)$ channel (solid line). The dots are the experimental phase shifts. The dashed line is the result without the inclusion of the $^5D_0 N\Delta$ channel.

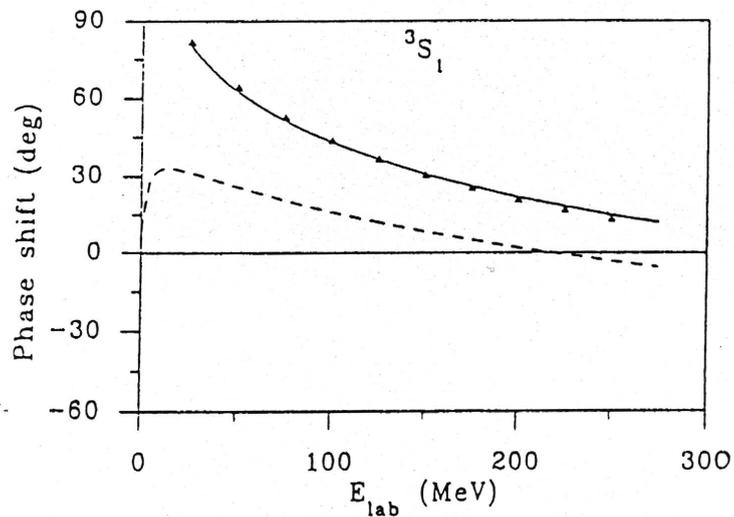


Fig. 6. $^3S_1 NN$ phase shift. The dots are the experimental values. The solid line is the result of the present model including chiral symmetry. The dashed curve is the result without coupling to the $^3D_1 NN$ partial wave.

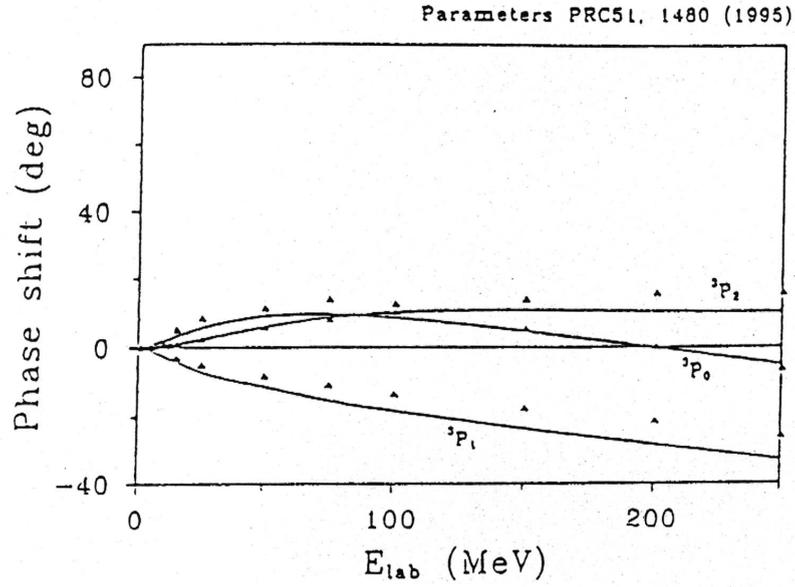


Fig. 7. Phase shifts of the 3P_0 , 3P_1 and 3P_2 partial waves of the nucleon-nucleon interaction as a function to the laboratory bombarding energy compared with the data.

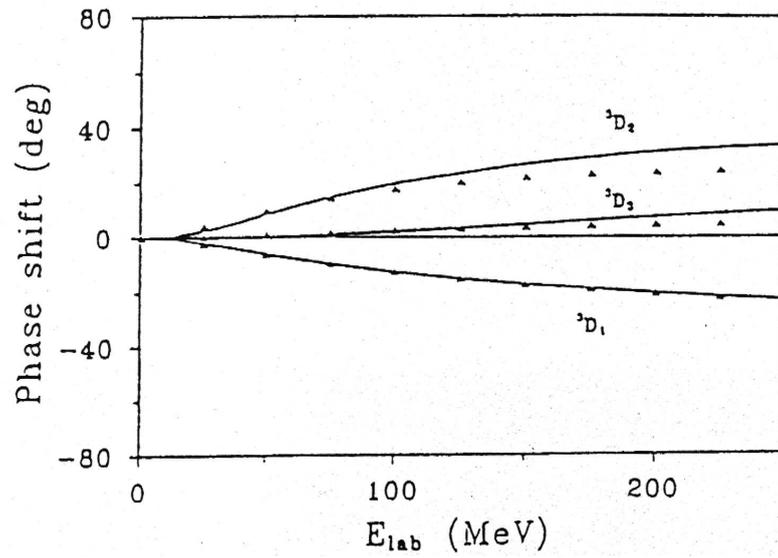


Fig. 8. Phase shifts of the 3D_J ($J = 1, 2, 3$) partial waves.

TABLE III

Parameters of the Tuebingen chiral quark model fitted by the mass of the nucleon, the mass of the delta resonance, the pion nucleon coupling constant, the charge radius of the proton, the stability condition for the proton (minimum of the proton mass at the right radius and the right mass). The σ meson mass is determined by the partially conserved axial vector current and the cut-off parameter Λ for the size of the constituent quarks is taken from the literature.

m_q [MeV]	b [fm]	α_s	$g_{\pi NN}^2/4\pi$	a_c [MeV fm ⁻²]	m_σ [fm ⁻¹]	Λ [fm ⁻¹]
313	0.518	0.485	13.7	46.94	3.42	4.2

3. Conclusions

In this contribution I wanted to essentially communicate two messages:

- (i) The electric quadrupole contribution to the photo induced transition from the nucleon into the delta resonance does not request a deformation of the nucleon or/and the delta resonance. Two-body exchange currents allow for a double spin flip, which is essentially an $E2 = (M1)^2$ transition. The one-body impulse approximation for the nucleon-delta transition works only with a d-state admixture, due to the tensor force from gluon and pion exchange. But the deformation produced by the tensor force is by far too small to explain the data. The two-body exchange currents explain in a very natural way a measured Sachs form factor at the photo point for the quadrupole transition from the nucleon to the delta resonance.
- (ii) The same quark model (“Tuebingen chiral quark model”) can also describe the nucleon-nucleon phase shifts in agreement with the data. For the 1S_0 phase shift the $^5D_0(N\Delta)$ admixture plays an important role. The two-body spin-orbit force from gluon and σ exchange may add or subtract in different partial waves. This improved the description of the 3L_J phase shifts without fitting the two-body spin-orbit strength.

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