Vol. 29 (1998) *ACTA PHYSICA POLONICA B* No 9

DUAL GINZBURG–LANDAU THEORY AND QUARK NUCLEAR PHYSICS *

H. Toki, H. Suganuma, M. Fukushima, K. Amemiya
A. Tanaka, S. Umisedo and T. Sakai

Research Center for Nuclear Physics (RCNP), Osaka University
Ibaraki, Osaka 567, Japan

(Received July 3, 1998)

The fundamental building blocks of matter are quarks. Hence, it is fundamental to describe hadrons and nuclei in terms of quarks and gluons, the subject of which is called Quark Nuclear Physics. The quark–gluon dynamics is described by quantum chromodynamics (QCD). Our interest is the non-perturbative aspect of QCD as confinement, chiral symmetry breaking, hadronization *etc.* We introduce the dual Ginzburg–Landau theory (DGL), where the color monopole fields and their condensation in the QCD vacuum, play essential roles in describing these non-perturbative phenomena. We apply the DGL theory to various observables. We discuss then the connection of the monopole fields with instantons, which are the classical solutions of the non-abelian gauge theory.

PACS numbers: 12.38.–t

1. Introduction

In Quark Nuclear Physics (QNP), we describe hadrons and nuclei in terms of quarks and gluons. The most essential phenomena in QNP are confinement of quarks and gluons and chiral symmetry breaking. It is difficult to describe confinement in a clear manner, while chiral symmetry breaking is described very nicely in the NJL model [1].

An interesting picture of color confinement was proposed around 1975 [2–4]. When we insert a superconductor into a magnetic field, the superconductor does not allow the magnetic field to pass through it. When the magnetic field is strong and the superconductor is of second kind, the magnetic field should go through the superconducting material in a small vortex-like

* Presented at the NATO Advanced Research Workshop, Cracow, Poland, May 26–30, 1998.

configuration by breaking the Cooper pairs by the minimum amount. This is known as the Meissner effect. The idea is to take its dual version for quark confinement. If the vacuum is normal, the color electric field should look like the one of the Coulomb potential between a positive and a negative color charge. If the vacuum is superconductor-like (dual superconductor), then the vacuum inhibits the electric field from passing through it and hence the color electric flux ought to be confined in a vortex-like configuration. This is called the dual Meissner effect. For this picture to be accepted, however, we should verify the abelian dominance assumption and the appearance of color magnetic monopole from QCD.

In 1981 't Hooft demonstrated the natural appearance of color magnetic monopoles in QCD [5]. In the non-abelian gauge theory like QCD, he introduced a particular gauge called abelian gauge, to reduce it to the abelian gauge theory like QED.

From a topological argument, color magnetic monopoles appear in the abelian space when some condition is fulfilled. This finding then supports the idea of the above picture for confinement. Hence, QCD naturally reduces to QED with magnetic monopoles, which is the Maxwell equation with magnetic charges and currents. This Maxwell equation has the duality symmetry, which naturally arises in the abelian gauge of QCD.

In this paper, we would like to describe how the dual Ginzburg–Landau (DGL) theory may be derived from QCD in Sect. 2. We apply the DGL theory to various phenomena as the static potential, chiral symmetry breaking, the phase transition at finite temperature *etc.* in Sect. 3. In Sect. 4, we discuss the connection of the monopoles to the instantons. Sect. 5 is devoted to the conclusion.

2. The dual Ginzburg–Landau theory

The QCD Lagrangian consists of the quark field, q, and the gluon field, A_μ,

$$\mathcal{L}_{\rm QCD} = \bar{q}\left(i\gamma_\mu\partial^\mu - m - e\gamma_\mu A^\mu\right)q + \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}\,. \tag{2.1}$$

Here $G^a_{\mu\nu}$ is the anti-symmetric field tensor of the gluon fields with the non-linear coupling terms due to the non-abelian nature. We make now the following three assumptions to arrive at the DGL Lagrangian;

1. the color monopoles appear in the abelian space due to the choice of the abelian gauge.

2. we introduce the Higgs term for the color monopole fields in order to make monopole condensation.

3. we neglect the non-abelian gluon fields for the low energy non-perturbative phenomena.

These three assumptions are supported by the recent lattice QCD calculations. The appearance of monopoles is noticed by the monopole trajectories in the abelian gauge, particularly in the maximal abelian (MA) gauge [6]. There appears long and complicated monopole loops below the critical temperature, which indicates monopole condensation [6–9]. The abelian dominance was first conjectured by Ezawa and Iwasaki [10] for long range phenomena. The lattice QCD calculations for the string tension and the chiral condensate demonstrated this abelian dominance [11, 12].

With these three assumptions, we arrive at the dual Ginzburg–Landau (DGL) Lagrangian [13–15],

$$\begin{aligned}\mathcal{L}_{\rm DGL} &= \mathcal{L}_{\rm dual} + \bar{q}\left(i\gamma_\mu\partial^\mu - m - e\gamma_\mu\vec{A}^\mu\cdot\vec{H}\right)q \\ &\quad + \sum_{a=1}^{3}\left[\mid(i\partial_\mu - g\vec{\alpha}_a\cdot\vec{B}_\mu)\chi_a\mid^2 - \lambda(\mid\chi_a\mid^2 - v^2)^2\right]. \qquad (2.2)\end{aligned}$$

Here $\mathcal{L}_{\rm dual}$ denotes the gluon dynamics in the Zwanziger form to treat the electric and the magnetic currents [16],

$$\begin{aligned}\mathcal{L}_{\rm dual} &= -\frac{1}{2n^2}\left[n\cdot(\partial\wedge\vec{A})\right]^\nu\left[n\cdot^*(\partial\wedge\vec{B})\right]_\nu \\ &\quad + \frac{1}{2n^2}\left[n\cdot(\partial\wedge\vec{B})\right]^\nu\left[n\cdot^*(\partial\wedge\vec{A})\right]_\nu \\ &\quad - \frac{1}{2n^2}\left[n\cdot(\partial\wedge\vec{A})\right]^2 - \frac{1}{2n^2}\left[n\cdot(\partial\wedge\vec{B})\right]^2 \qquad (2.3)\end{aligned}$$

with the dot product $(n\cdot G)_\mu = n^\nu G_{\nu\mu}$ and the cross product $(A\wedge B)_{\mu\nu} = A_\mu B_\nu - A_\nu B_\mu$. n_μ denotes a constant four vector. q is the quark field with mass m and χ_a is the monopole field with the monopole charge $g\vec{\alpha}_a$, where $\vec{\alpha}_a$ is the root vector of SU(3). The dual gluon coupling constant, g, satisfies the Dirac condition, $eg = 4\pi$. $\vec{B}_\mu$ is the dual gauge field and $\vec{A}_\mu$ is the gauge field with 3rd and 8th components. $\vec{H} = (\lambda_3/2, \lambda_8/2)$ is the diagonal part of the SU(3) generators. The last term is the Higgs term to cause monopole condensation, where λ and v are the parameters of the DGL Lagrangian. The monopole fields satisfy the condition, $\sum_{a=1}^{3}\arg\chi_a = 0$. We note that the DGL Lagrangian has $\mathrm{U}(1)_e^3\times\mathrm{U}(1)_e^8$ gauge symmetry and $\mathrm{U}(1)_m^3\times\mathrm{U}(1)_m^8$ dual gauge symmetry [15].

We shall see now the dual Meissner effect be caused by the monopole condensation in the DGL theory. We separate the monopole field χ_a into the mean field $\langle\chi_a\rangle = v$ and its fluctuation $\tilde{\chi}_a$ as $\chi_a = (v+\tilde{\chi}_a)e^{i\xi_a}$. Here the angles ξ_a satisfies the condition $\sum_{a=1}^{3} \xi_a = 0$. The monopole condensate does not depend on a due to the Weyl symmetry. With this separation the DGL Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\mathrm{DGL}} &= \mathcal{L}_{\mathrm{dual}} + \bar{q}\left(i\gamma_\mu \partial^\mu - m - e\gamma_\mu \vec{A}^\mu \cdot \vec{H}\right) q + \frac{1}{2} m_B^2 \vec{B}_\mu^2 + \sum_{a=1}^{3} \left[(\partial_\mu \tilde{\chi}_a)^2 \right. \\ &- \left. m_\chi^2 \tilde{\chi}_a^2\right] + \sum_{a=1}^{3} \left[g^2 (\vec{\alpha}_a \cdot \vec{B}_\mu)^2 (\tilde{\chi}_a^2 + 2\tilde{\chi}_a v)^2 - \lambda (4v\tilde{\chi}_a^3 + \tilde{\chi}_a^4) \right] , \quad (2.4) \end{aligned}$$

where $m_B = \sqrt{3} g v$ and $m_\chi = 2\sqrt{\lambda} v$ are the masses of the dual gauge field B_μ and the monopole field $\tilde{\chi}_a$. Hence, the monopole condensation makes the dual gauge field massive and the two phases of the monopole field are changed into the longitudinal degrees of freedom of the dual gauge field (dual Higgs mechanism). As a conseqence the color electric field $\vec{E}$ cannot propagate long distance than $1/m_B$, which indicates the dual Meissner effect [17].

3. Application of the DGL theory

3.1. Static $q\bar{q}$ potential

We apply the DGL theory first on the $q\bar{q}$ static potential. We place a quark and an anti-quark with a distance r as an external source. After taking the mean field approximation for the monopole field χ, we take integration over A_μ and B_μ and get the current–current interaction term, $\mathcal{L}_{j-j} = -\frac{1}{2}\vec{j}_\mu D^{\mu\nu}\vec{j}_\nu$. Here the gluon propagator is expressed as

$$D_{\mu\nu} = D_{\mu\nu}^{(0)} + \frac{1}{q^2}\frac{m_B^2}{q^2 - m_B^2}\frac{n^2}{(n \cdot q)^2} X_{\mu\nu} \,. \quad (3.1)$$

$D_{\mu\nu}^{(0)}$ denotes the free gluon propagator. The anti-symmetric tensor is made of the constant four vector n_μ and q_μ as $X_{\mu\nu} = \frac{1}{n^2}\varepsilon_{\lambda\mu\alpha\beta}\varepsilon^{\lambda}_{\nu\gamma\delta} n^\alpha n^\gamma q^\beta q^\delta$. We can work out the static potential [15]

$$V(r) = \frac{e^2}{12\pi}\frac{\mathrm{e}^{-m_B r}}{r} + \frac{e^2}{24\pi} m_B^2 \ln\left(\frac{m_\chi^2 + m_B^2}{m_B^2}\right) \cdot r \,. \quad (3.2)$$

The potential comes out to have a Yukawa term and a linear confining term. The coefficient of the linear confining potential agrees with the energy per unit length of the Abrikosov vortex of the superconductor.

We may fix the parameters in the DGL Lagrangian so as to reproduce the interquark potential of heavy quarkonium; $e = 5.5$, the monopole condensate

$v = 0.126$ GeV and the interaction strength of the monopoles $\lambda = 25$. These parameters lead to the magnetic charge $g = 2.3$, the mass of the dual gauge field $m_B = 0.5$ GeV, the monopole field mass $m_\chi = 1.26$ GeV and the string tension 1.0 GeV/fm. We show the result in Fig. 1 together with the phenomenological potential. The appearance of linear potential is not surprising, since it is modelled in the DGL theory. It is worthwhile to stress, however, that there are no other models, which are able to realize confinement of colors and at the same time have a strong link with QCD as described above.

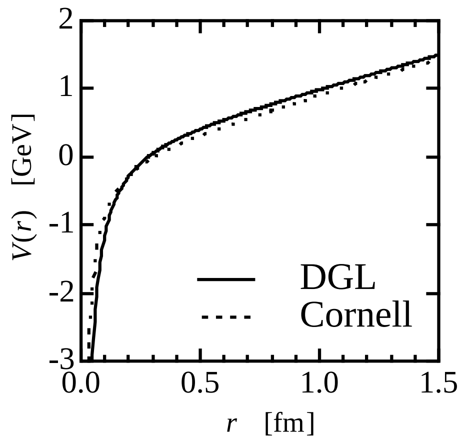

Fig. 1. The static potential between a quark and antiquark pair calculated within the DGL theory to compare with the phenomenological potential of Cornell group. Taken from Suganuma *et al.* . [15]

We have taken the abelian dominance assumption in the construction of the DGL Lagrangian. The validity of the abelian dominance is demonstrated by the recent lattice QCD calculations for the SU(2) case. The potential with full space compares very well with the one of only the abelian space. They agree in the long range part. [7] While in the short range part, $r < 0.2$ fm, we see some difference. This result indicates that the abelian dominance assumption is valid only in the long distance or in the infrared region.

We may get the information of the abelian dominance from another view point. We can measure the gluon correlation, $\langle A^a_\mu(r) A^a_\mu(0)\rangle$, on the lattice for the diagonal gluon and the charged gluon for the pure SU(2) gauge. The results are shown in Fig. 3. The correlation diminishes already about 0.2 fm for the charged gluon, while it reaches long distance for the diagonal gluon. If we extract the mass from this correlation function, it is about 1 GeV for the abelian case [19]. Hence, the long range physics is dominated by the abelian gluon in the MA gauge.

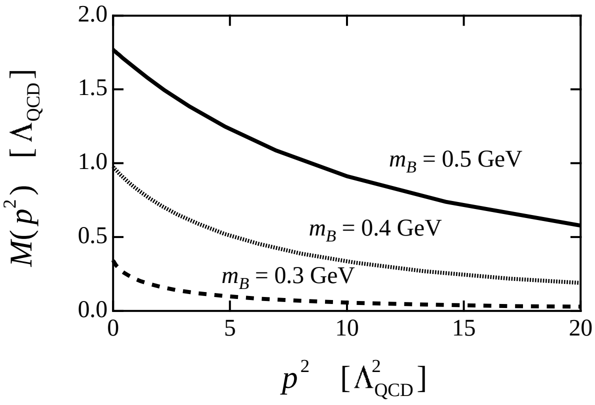

Fig. 2. The constituent quark mass calculated within the DGL theory with various values of the dual gluon mass, which indicates the strength of monopole condensation, as a function of the Euclidean momentum square. The unit $\lambda_{\rm QCD}$ is 200MeV. Taken from Suganuma *et al.* [15]

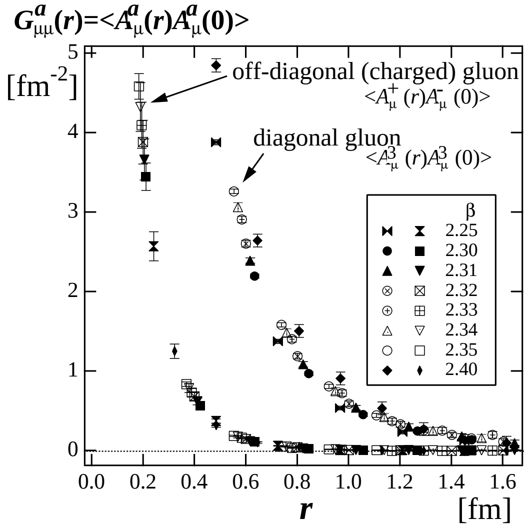

Fig. 3. The gluon correlation, $\langle A^a_\mu(r)A^a_\mu(0)\rangle$, on the lattice for the diagonal gluon and the charged gluon for the SU(2) gauge as a function of the four dimensional distance, r. Taken from Amemiya and Suganuma [19].

3.2. Spontaneous chiral symmetry breaking

Chiral symmetry breaking is directly related with the quark mass generation in the QCD vacuum. How quarks behave in monopole condensed vacuum? It corresponds to solving the Schwinger–Dyson equation, where quarks get the self-energy corrections due to the non-perturbative interaction with gluons. The Schwinger–Dyson equation under the rainbow approximation is written as,

$$S^{-1} = S_0^{-1} + \mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} Q^2 \gamma_\mu S \gamma_\nu D^{\mu\nu} \,. \tag{3.3}$$

We take the Landau gauge and assume that the full quark propagator is given as $S^{-1} = i\gamma_\mu p^\mu - M(p^2)$. We take the average over the angle of the quark momenta with respect to the string direction n due to the quark motion in the confining region and introduce the infrared cutoff a of the hadronic scale.

$$\frac{1}{(n\cdot q)^2} \to \left\langle \frac{1}{(n\cdot q)^2 + a^2} \right\rangle_{\mathrm{av}} \,. \tag{3.4}$$

This introduction of the infrared cutoff a is made by considering the natural weight of reducing the contribution of the long range propagation of gluons due to confinement, which is naturally present in the DGL Lagrangian. In order to make full order calculations without the introduction of the infrared cutoff, it would be very interesting to perform the lattice calculations of quark condensate with the use of the DGL Lagrangian.

With the above formulations we can perform numerical calculations for the quark mass. The results are shown in Fig. 2, where the quark mass is plotted as a function of the Euclidean four momentum. The quark mass, $M(q^2)$, becomes finite at $m_B \sim 200$ MeV and increases with m_B. We find also the pion decay constant and the quark condensate to have the values close to the semi-experimental values. This calculation demonstrates that monopole condensation is the source of both the confinement and the chiral symmetry breaking.

It is very interesting to see the results of lattice QCD calculations on the chiral condensate, which were performed by Miyamura and his collaborators [12]. The chiral condensate obtained with the full space is compared very nicely with the one of the abelian space. The further reduction of the abelian degrees of freedom to only the monopole field do not change the chiral condensate largely. These results confirm the results of the DGL theory on chiral symmetry breaking, on which the monopole condensation plays the dominant role.

We have worked out also the recovery of these symmetries at finite temperature in several publications [17, 18].

3.3. Meson spectrum in the pion channel

We can apply the DGL theory to the mesons. We formulate the Bethe–Salpeter equation for mesons using the full gluon propagator and the quark propagator obtained by solving the Schwinger–Dyson equation. Due to its simplicity we first work out the pion channel; the parity is $\Pi = -(-)^J$ and the charge conjugation is $C = -\Pi$. The results are shown in Fig. 4. The experimental masses are shown by the crosses with experimental widths of the corresponding states and the theoretical ones by the black dots connected by dashed straight lines as a function of the spin J. We find clearly the Regge behavior; $M^2 \propto J$.

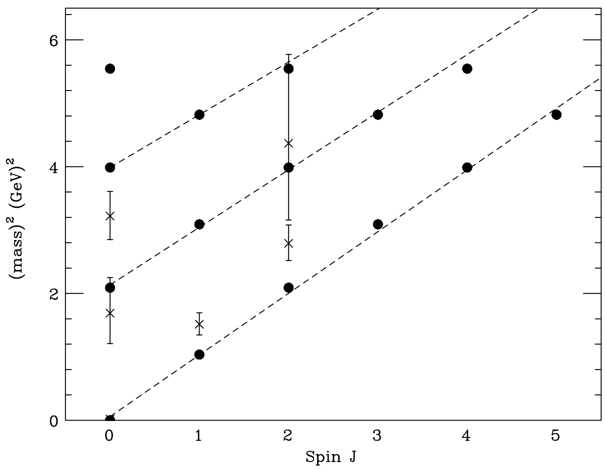

Fig. 4. The meson masses square, M^2, in the pion channel are plotted as a function of their spins, J. The experimental masses are shown by the crosses with experimental widths of the corresponding states and the theoretical ones by the black dots connected by dashed straight lines.

4. Instantons and monopoles

We have been talking that color monopoles are the essential degrees of freedom for non-perturbative QCD phenomena. What is the physical origin of color monopoles? Are they merely the mathematical objects related with the abelian gauge fixing? What is the gauge independent objects producing this important degrees of freedom? In order to answer these questions, we ought to go back to the condition of the appearance of color monopoles by choosing some $\mathrm{SU}(N_c)$ variables, $X(x)$, to diagonalize. We can show that the color monopole appears at the point where X becomes the hedgehog configuration in the SU(2) subspace. What then causes $X(x)$ to have the hedgehog configurations?

Instantons are the classical solutions of the non-abelian gauge theory in the Euclidean R^4 space. It is also well known that the instantons provide the

$U_A(1)$ anomaly and explain the η' mass problem. We observe also clearly instantons after cooling in lattice QCD simulations in the QCD vacuum. In this connection, it is very interesting to see if the instantons are connected with the color monopoles.

The instanton solution is written in the singular gauge in SU(2) gauge theory as

$$A_\mu(x;z,\rho) = i\tau^a \bar{\eta}^a_{\mu\nu} \frac{(x-z)_\nu \rho}{(x-z)^2\left[(x-z)^2+\rho^2\right]}\,, \tag{4.1}$$

with $\bar{\eta}^a_{\mu\nu}$ being the 't Hooft symbol. z and ρ denote the center and the size of the instanton. It is interesting to find that the A_4 has the hedgehog structure; $A_4(x; z=0,\rho) \propto \tau^a x^a$ Hence, if we choose A_4 for $X(x)$, which is called the Polyakov gauge, for the Abelian projection, we find the monopole at the center of the instanton. This simple consideration clearly demonstrates the connection of the monopoles with the instantons.

In the lattice QCD study it has been demonstrated that the Abelian dominance assumption is fulfilled in the maximal abelian (MA) gauge [6]. In the MA gauge $X(x) = \sum_\mu \left[U_\mu \tau_3 U_\mu^{-1}\right]$ is diagonalized on the lattice, where $U_\mu = \mathrm{e}^{iaA_\mu}$ is the link variable. This amounts to maximize the quantity

$$R = \sum_{s,\mu} \mathrm{Tr}\,\left[U_\mu(s)\tau_3 U_\mu^{-1}(s)\tau_3\right]\,. \tag{4.2}$$

In this case the monopole trajectory appears around the instanton and its radius is proportional to the instanton size [20]. These works establish the relation between instantons and monopoles. It is then very interesting to make an assumption that the QCD vacuum consists of multi-instantons.

We take the sum ansatz for the multi-instanton configuration [21],

$$A_\mu(x) = \sum_k \left[A^I_\mu(x; z_k, \rho_k, O_k) + A^{\bar{I}}_\mu(x; z_k, \rho_k, O_k)\right]\,. \tag{4.3}$$

Here $\bar{I}$ denotes the anti-instanton solution, which is written in the similar form as the instanton by replacing $\bar{\eta}$ by η of the 't Hooft symbol. In this configuration, the positions and the orientations of the instantons are chosen randomly. As for the size of instantons we take the form,

$$f(\rho) = \frac{1}{\left(\frac{\rho}{\rho_{\mathrm{IR}}}\right)^n + \left(\frac{\rho_{\mathrm{UV}}}{\rho}\right)^{11N_c/3-5}}. \tag{4.4}$$

The power of the ultra violet distribution is obtained by the perturbative QCD, while the power of the infrared distribution is not known. We choose it as $n=3$ and $n=5$. The parameters, ρ_{IR} and ρ_{UV}, of the distribution are fixed to provide the peak of the distribution function is 0.4 fm. We perform the MA projection on the lattice. We see clearly the linear relation between the instanton size and the monopole loop length for the case of very small instanton density.

We then calculate the monopole length distributions by increasing the instanton density, the results of which are shown in Fig. 5. At low density the distribution falls off as the power law. On the other hand, at high density there appears long complicated monopole trajectories. This appearance of the long monopole loop indicates that the whole space is strongly correlated and the monopole condensation takes place. It is very interesting to compare this result with the one of the lattice QCD. This is done in Fig. 5. There are very strong one-to-one correspondence between the two cases.

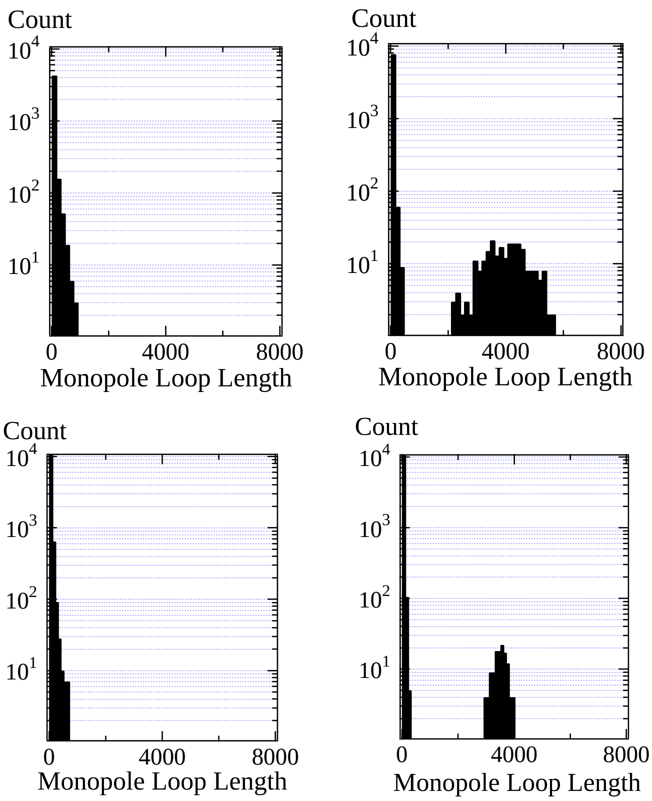

Fig. 5. The monopole length distribution. The upper two figures are the results of the multi-instanton gas with small density (left) and with large density (right). The lower two figures correspond to the case of pure SU(2) gauge theory on 163×4 at high temperature (left) and at low temperature (right), which correspond to the confined phase. Taken from Fukushima *et al.* [21]

5. Conclusion

Quark Nuclear Physics (QNP) is the subject to describe nucleons, mesons and nuclei in terms of quarks and gluons. The fundamental theory of QNP is QCD. Non perturbative phenomena are, however, difficult to describe directly in terms of QCD. Hence, we introduce the effective theory as the workable effective theory of QNP. QCD is a non-abelian gauge theory. The abelian gauge fixing and the abelian dominance assumption, which are demonstrated by the recent lattice QCD, supports the DGL theory. Here, the Lagrangian is written in terms of the abelian gluons and the color monopoles. The monopole condensation is able to describe confinement of quarks and even chiral symmetry breaking. Since they are the most essential phenomena in QNP, the DGL theory could be said as the workable effective theory as the shell model for Nuclear Physics. Now it is a problem of time for theoreticians to work out mesons and baryons and their interactions and even their properties at finite temperature. Theory is ready for exciting experimental phenomena of QNP.

It is now an important task to derive the DGL theory from QCD. In particular, we believe that condensation of color monopoles is the most important phenomenon to be derived from QCD. In this direction, we are now considering the importance of instantons, which are the classical solutions of QCD, for the derivation of the DGL theory. In fact, both the lattice QCD calculations and an analytic study on the instanton configurations provide strong correlations between instantons and color monopoles. Furthermore, these studies strongly suggest that the multi-instanton configuration leads to the highly complicated color monopole world lines, which are the signals of monopole condensation. We stress that there remain many fruitful phenomena and problems to be solved in the confinement physics. They are the very central tasks for the Quark Nuclear Physics.

The authors thank S. Sasaki, H. Ichie and K. Kusaka for fruitful collaborations and many illuminating discussions on the DGL theory and nonperturbative phenomena of QCD.

REFERENCES

[1] Y. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); *Phys. Rev.* **124**, 245 (1961).

[2] Y. Nambu, *Phys. Rev.* **D10**, 4262 (1974).

[3] G. 't Hooft, *High Energy Physics*, Editorice Compositori, Bologna 1975.

[4] S. Mandelstam, *Phys. Rep.* **C23**, 245 (1976).

[5] G. 't Hooft, *Nucl. Phys.* **B190**, 455 (1981).
[6] A.S. Kronfeld et al., *Phys. Lett.* **B198**, 516 (1987).
[7] T. Suzuki, I. Yotsuyanagi, *Phys. Rev.* **D42**, 4257 (1990).
[8] S. Hioki et al., *Phys. Lett.* **B272**, 326 (1991).
[9] S. Kitahara, Y. Matsubara, T. Suzuki, *Prog. Theor. Phys.* **93**, 1 (1995).
[10] Z.F. Ezawa, A. Iwasaki, *Phys. Rev.* **D25**, 2681 (1982); *Phys. Rev.* **D26**, 631 (1982).
[11] H. Shiba, T. Suzuki, *Phys. Lett.* **B333**, 461 (1994).
[12] O. Miyamura, *Phys. Lett.* **B353**, 91 (1995).
[13] T. Suzuki, *Prog. Theor. Phys.* **80**, 929 (1988).
[14] S. Maedan, T. Suzuki, *Prog. Theor. Phys.* **81**, 229 (1989).
[15] H. Suganuma, S. Sasaki, H. Toki, *Nucl. Phys.* **B435**, 207 (1989).
[16] D. Zwanziger, *Phys. Rev.* **D3**, 880 (1971).
[17] H. Ichie, H. Suganuma, H. Toki, *Phys. Rev.* **D52**, 2944 (1995).
[18] S. Sasaki, H. Suganuma, H. Toki, *Prog. Theor. Phys.* **94**, 373 (1995).
[19] K. Amemiya, H. Suganuma, *INNOCOM Proceedings*, World Scientific 1998.
[20] R. Brower, K. Orginos, C. Tan, *Nucl. Phys.* **B53**, 488c (1997).
[21] M. Fukushima, A. Tanaka, S. Sasaki, H. Suganuma, H. Toki, D. Diakonov, *Phys. Lett.* **B399**, 141 (1997).