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THE INTERACTION BETWEEN CONSTITUENT QUARKS*

D.O. Riska

Department of Physics, 00014 University of Helsinki
Helsinki, Finland

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The symmetry structure and the dynamical mechanisms that underlie the effective interaction between constituent quarks are discussed. The need for a flavor dependent hyperfine interaction is emphasized. Such a hyperfine interaction arises from exchange of the octet of light pseudoscalar mesons, which are the Goldstone bosons of the approximate chiral symmetry of QCD, between the quarks. The role of gluon exchange between quarks is small, but gluonic "dressing" of the meson exchange interaction is important for eliminating the tensor component of the meson exchange interaction.

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1. The interaction between quarks and antiquarks

The spectra of charmonium and bottomonium are fairly well described by a simple Hamiltonian model operator, with an interaction potential formed of a linear confining interaction with a string tension of about $c \simeq 1$ GeV/fm and an attractive Coulombic gluon exchange interaction. The (squared) quark-gluon coupling strength for the latter has been determined by numerical lattice methods to be $\alpha_{\rm S} = 0.37$ for charm quarks ($m_s \simeq 1.3$ GeV) and 0.22 for beauty quarks ($m_b \simeq 4.1$ GeV) respectively [1]. Full lattice calculations of the Hamiltonian for heavy quarkonia confirm this picture, and leave only minor gaps in the understanding of the spectrum [2]. These gaps concern the resilient underprediction of the absolute value of the splitting between the lowest positive and negative parity states, and the relative sizes of the hyperfine splitting of the latter (the χ_{cJ} and χ_{bJ} states, respectively).

There is in any case no doubt that heavy quarkonia admit a Schrödinger equation based description with a potential. In this interaction potential

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there is a need for a delicate cancellation between the spin-orbit interactions associated with the linear confining interaction, and that due to one-gluon exchange [3]:

$$V_{\rm LS} = -\frac{c}{2m_q^2 r}\vec{S}\cdot\vec{L} + \frac{2\alpha_{\rm S}}{m_q^2 r^3}\vec{S}\cdot\vec{L}\,. \qquad (1)$$

The former term arises because of the required relativistic scalar nature of confinement [4]. The cancellation between these two terms works well for both charmonium and bottomonium for which $\langle r\rangle \simeq 0.35$ fm and $\langle r\rangle \simeq 0.20$ fm, respectively.

The validity of the description of the confining interaction between quarks and antiquarks as a linear potential is nicely illustrated by the rate of the basic M1 transition $J/\psi \to \eta_{\rm C}$, which contains a crucially important exchange current term to which the confining interaction, but not the gluon exchange interaction contributes. This rate may (in the well justified long wavelength approximation) be expressed as

$$\Gamma = \frac{16}{27}\alpha\frac{q^3}{m_c^2}\frac{M_{\eta_c}}{M_{J/\psi}}\left(1-\frac{2}{3}\left\langle\frac{p^2}{m_c^2}\right\rangle - \left\langle\frac{cr}{m_c}\right\rangle\right)^2. \qquad (2)$$

Here the first two terms on the rhs represent the contributions from the single quark currents with a (lowest order) relativistic correction, whereas the last term represents the contribution from the confinement exchange current. Without the last two terms on the rhs the calculated decay rate would be 3.7 KeV, whereas the empirical decay rate is only 1.14 ± 0.35 KeV. The contribution of the last term has to be of the order $c\langle r\rangle/m_c$, which with $c \simeq 1$GeV/fm, $m_c = 1.3$ GeV and $\langle r\rangle \simeq 0.35$ fm comes to about 0.27, which is a sizeable correction. The relativistic correction is also substantial: with $\langle p^2\rangle \simeq 1/\langle r^2\rangle$ it is $\simeq 0.12$. With these estimates for the correction terms the calculated decay rate drops to 1.4 KeV, which is much closer to the empirical value. Although this decay rate — in particular the relativistic correction — is sensitive to the wave function model [5], the large pair term correction also appears in explicitly relativistic approaches [6].

2. The interaction between constituent quarks

While there is thus at least a qualitative and phenomenological understanding of the interaction between heavy quarks and antiquarks both the form and the dynamical origin of the effective interaction between the constituent quarks, which form the baryons, remain largely open issues. The small spin-orbit splittings between the lowest negative parity states in the baryon spectrum suggests that both the spin-orbit and tensor components of the effective quark-quark interaction should be weak. This rules out a strong quark–gluon coupling, although there may still be a need for a partial cancellation between the spin-orbit components associated with the scalar confining interaction and the gluon exchange interaction, both of which have the same form as in Eq. (1), but with only half the strength in the case of the interaction between quarks [7].

Another issue is to what extent the nonperturbative vacuum of QCD supports gluon exchange. Cooled lattice calculations suggests that the quark-gluon coupling should be very weak [8]. The valence-QCD approximation suggests the presence of a residual weak but nonzero gluon exchange interaction between quarks [9]. I is therefore reasonable to assume that the gluons either decouple from the constituent quarks below the confinement scale $\Lambda_{\rm QCD}$ or the chiral restoration scale $\Lambda_\chi \simeq 1$ GeV or that their coupling freezes at some small value below these momentum scales. This agrees with recent phenomenological studies of the behavior of the running QCD coupling strength in the infrared limit [10, 11].

Another reason for believing that gluon exchange cannot be the main cause of the hyperfine interaction between constituent quarks is the presence of low lying positive parity states below the lowest negative parity states in all sectors of the baryon spectrum without flavor singlet states. The latter feature is most readily explained if the main component of the hyperfine interaction between the quarks is an attractive flavor dependent spin-spin interaction [12].

Consider *e.g.* the spectrum of the nucleon, the lowest excitations of which are the $N(1440)$ $1/2^+$ and the $N(1535)$ $1/2^-$ states. Any monotonic confining interaction would organize the spectrum in "normal order" so that the lowest excited state would be the negative parity state. The "hyperfine interaction" between quarks has therefore to be strong enough to reverse this normal ordering of the baryon spectrum. Because these three nucleon states all have the same mixed [21] color-spin symmetry, the color magnetic hyperfine component of the single gluon exchange interaction, the color-spin structure of which is $\vec{\lambda}^1_{\rm C} \cdot \vec{\lambda}^2_{\rm C} \vec{\sigma}^1 \cdot \vec{\sigma}^2$, cannot reverse this ordering. This is because this color-spin operator is a Casimir operator in the decomposition of $\mathrm{SU}(6)_{\rm CS}$ into $\mathrm{SU}(3)_{\rm C} \times \mathrm{SU}(2)_{\rm S}$. Reversal of this spectral ordering does however become possible if the interaction has the flavor-spin structure $\vec{\lambda}^1_{\rm F} \cdot \vec{\lambda}^2_{\rm F} \vec{\sigma}^1 \cdot \vec{\sigma}^2$, which is characteristic of the Goldstone boson (*i.e.* π,K,η) exchange interaction between quarks. This is because while both the nucleon and the $N(1440)$ resonance have the same completely symmetric flavor-spin state $[3]_{\rm FS}$, the flavor spin state of the $N(1535)$ has mixed $[21]_{\rm FS}$ symmetry, and thus the flavor-spin operator shifts this state relative to the other ones.

This argument carries directly over to the spectrum of the Λ^0 hyperon, where the lowest excited states (with exception of the flavor singlet state

$\Lambda(1405)$, are the $\Lambda(1600)$ $1/2^+$ and the $\Lambda(1670)$ $1/2^-$ resonances. But the argument against a strong color magnetic hyperfine interaction is even stronger in the case of the Δ resonance spectrum, where the lowest excited states (with spin 3/2) are the $\Delta(1600)$ $3/2^+$ and the $\Delta(1700)$ $3/2^-$ resonances. The color-spin structure of the $\Delta(1232)$ and the $\Delta(1600)$ is completely antisymmetric [111], whereas that of the $\Delta(1700)$ is mixed [21]. For these states the color-magnetic hyperfine interaction is therefore an active operator, which however shifts the negative parity state downwards in energy relative to the positive parity states - and thus worsens the disagreement with the empirical ordering. In this case again the flavor-spin dependent Goldstone boson exchange interaction brings about the desired reversal of the normal ordering.

It is in fact possible to provide a dynamical underpinning to this symmetry argument. Provided that the tensor component of the Goldstone boson exchange interaction is dropped, it proves possible to explain the spectrum of the nucleon and the strange hyperons up to the small spin-orbit splittings with that interaction in combination with a linear confining interaction with conventional strength along with a relativistic kinetic energy operator for the constituent quarks [13]. This however leaves the tensor component of the Goldstone boson exchange interaction to be explained away, somewhat as the large spin-orbit interaction of the gluon exchange interaction has to be explained away in attempts to describe the baryon spectrum in terms of gluon exchange alone [14].

3. The spin-orbit splittings of in the *P*-shell

The empirical spin-orbit splittings in the P shell of the baryon spectrum are mostly small and consistent with zero, with the one exception of the flavor-singlet doublet $\Lambda(1405) - \Lambda(1520)$, which is split by 115 MeV. Because the $\Lambda(1405)$ is situated at threshold for $\bar{K}N$ decay, it has been successfully described as a $\bar{K}N$ molecular state [15] rather than a 3 quark state. This view is consistent with the Skyrme model description of the $\Lambda(1405) - \Lambda(1520)$ as bound states of a topological soliton and a $\bar{K}$, which moreover predicts the size of the spin-orbit splitting correctly [16].

The tensor component of the Goldstone boson exchange interaction does not contribute at all to the splitting of this multiplet, so that even in an attempt to describe it as a three-quark system, the dynamical origin of this splitting cannot be due to pseudoscalar meson exchange [12], which may be due to the strong coupling to the $\bar{K}N$ channel.

But in addition the tensor component of the Goldstone boson exchange interaction would imply small, but even so contraindicated spin orbit splittings among the other low lying spin-flavor multiplets with negative parity.

Therefore another dynamical mechanism is required to cancel out this tensor interaction. One possibility to achieve such a cancelling effect is to consider vector meson exchange interactions between the constituent quarks along with the pseudoscalar meson exchange interaction [12]. The tensor components of the pseudoscalar and vector meson exchange interactions have opposite signs, and largely cancel at short range, whereas their spin-spin components add, and therefore such a combination has the phenomenologically required features.

An alternative to balancing the pion exchange tensor interaction at short range by a corresponding vector meson exchange interaction of opposite sign is to consider a weak gluon exchange interaction along with the Goldstone boson exchange interaction, and in addition the irreducible π-gluon exchange interaction as done in Ref. [17]. This is because the tensor component of the $\pi-$gluon exchange interaction is of the same order of magnitude as the pion exchange interaction, but has the opposite sign. As a result the net tensor interaction is very weak.

4. The irreducible π-gluon exchange interaction

The $\pi-$gluon exchange loop mechanisms allow combination of short range gluon exchange with long range pion exchange, and thus combines long and short range physics, even with the conventional assumption that the Goldstone bosons decouple from constituent quarks above the chiral restoration scale $\Lambda_\chi \simeq 4\pi f_\pi$.

The calculation of the $\pi-$gluon exchange interaction may be performed within the framework of the Blankenbecler–Sugar quasipotential framework, which allows a covariant extraction of the iterated single pion and gluon exchange interactions from the Bethe–Salpeter equation kernel [18]. The resulting interaction is then real and almost energy independent.

If the Goldstone boson exchange interaction between quarks is complemented with a fairly weak gluon exchange interaction along with the associated irreducible $\pi-$gluon exchange interaction the problem of the tensor component of the former is eliminated because the tensor component of the $\pi-$gluon exchange interaction is of the same order of magnitude as the pion exchange interaction, but with the opposite sign. The former dominates at short and the latter at large distances. As a result the net tensor interaction is very weak [17].

The $\pi-$gluon exchange interaction has an attractive spin-spin component, which adds to, but is somewhat weaker than that of single pion exchange at short range. Its detailed behavior at very short range is very sensitive to the high momentum behavior of the pion and gluon exchange interactions. Finally the spin-orbit and central components of the $\pi-$gluon exchange interaction turn out to be very weak. The $\pi-$gluon exchange interaction thus appears to provide part of the explanation for why the effective interaction between constituent quarks should have the form of an attractive flavor dependent spin–spin interaction and an at most very weak tensor interaction.

5. Discussion

This analysis suggests that the hyperfine interaction between light constituent quarks should be formed of a (weak) single gluon exchange component, a pion exchange component with conventional strength and an irreducible $\pi-$gluon exchange interaction. The single gluon exchange interaction is weak because the effective quark-gluon coupling is weak in the infrared limit [10,11], and therefore the problem of a large gluonic spin-orbit interaction [14] is avoided. The pion exchange tensor interaction is in effect cancelled by the large tensor component of the $\pi-$gluon exchange interaction, and thus the incorrect (small) spin-orbit splitting of the low lying negative parity resonances is avoided. The $\pi-$gluon exchange and single pion exchange interactions combine to a strong attractive flavor dependent spin-spin interaction, which brings the low lying positive parity resonances below the lowest negative parity resonances in agreement with experiment.

There are a large number of other exchange mechanisms that may contribute significantly to the hyperfine interaction between quarks. Among these are vector meson exchange [12] and two-pion exchange. The two-pion exchange interaction mainly leads to a flavor independent attractive interaction, which contributes to the strength of the effective confining interaction at short range. The associated (negative) spin-orbit interaction would then add to the effective spin-orbit interaction associated with the linear confining interaction (*cf.* Eq. (1)), which suggests a need for some additional interaction mechanism that can provide balancing flavor independent spin-orbit interaction.

The presence of an irreducible $\pi-$gluon exchange interaction is an immediate consequence if there exists a pion exchange and (however screened) gluon exchange interaction between quarks. It is also independent of the structure of the pion itself — whether it arises as a succession of instanton induced quark–antiquark interactions, according to a common view, or whether is has a simpler quark-antiquark structure.

The substantial size of the $\pi-$gluon tensor and spin-spin interaction components suggests that the interaction between constituent quarks in the end may prove to be as complex as the nucleon-nucleon interaction proved to be, and that it — at least partly — has to be constructed phenomenologically as is the case with the latter [19]. Given this situation a purely phenomenological approach to the interaction may be well motivated, and then the main requirements that the effective mass operator model satisfy the fundamental symmetries, as Poincaré invariance, in addition to the dynamical symmetries that are implied by the spectrum. An example of a simple mass operator model, which satisfies these requirements based on instant form kinematics, and which with a few adjustable parameters in the flavor spin dependent hyperfine interaction term is able to describe the presently known part of the baryon spectrum is given in Ref. [20].

REFERENCES

[1] C. T. Davies *et al.*, it Phys. Rev. **D56**, 2755 (1997).
[2] G. S. Bali, K. Schilling, A. Wachter, *Phys. Rev.* **D56**, 2566 (1997).
[3] S. Godfrey, N. Isgur, *Phys. Rev.* **D320**, 189 (1985).
[4] D. Gromes, *Phys. Lett.* **B202**, 262 (1988).
[5] X. Zhang, K. J. Sebastian, H. Grotch, *Phys. Rev.* **D44**, 3163 (1991).
[6] J. Linde, H. Snellman, *Nucl. Phys.* **A619**, 346 (1997).
[7] S. Capstick, N. Isgur, *Phys. Rev.* **D34**, 2809 (1986).
[8] M. C. Chu, *Phys. Rev.* **D49**, 6039 (1994).
[9] K.-F. Liu, private communication.
[10] A. C. Mattingly, P. M. Stevenson, *Phys. Rev.* **D49**, 437 (1994).
[11] S. J. Brodsky *et al.*, *Phys. Rev.* **D57**, 245 (1997).
[12] L. Ya. Glozman, D. O. Riska, *Phys. Rep.* **268**, 263 (1996).
[13] L. Ya. Glozman *et al.*, LANL hep-ph/9706507, unigraz-utp-25-06-97.
[14] J. Carlson, J. B. Kogut, V. J. Pandharipande, *Phys. Rev.* **D28**, 2807 (1983).
[15] R. H. Dalitz, A. Deloff, *J. Phys. G* **17**, 289 (1991).
[16] D. O. Riska, N. Scoccola, *Phys. Lett.* **B265**, 188 (1991).
[17] C. Helminen, D. O. Riska, hep-ph/9805280.
[18] R. Blankenbecler, R. Sugar, *Phys. Rev.* **142**, 1051 (1966).
[19] M. Taketani, S. Nakamura, M. Sasaki, *Prog. Theor. Phys.* **6**, 581 (1951).
[20] F. Coester, K. Dannbom, D. O. Riska, LANL hep-ph/9711458.