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THE DYNAMICS OF πN SCATTERING AND THE BARYON SPECTRUM*

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In the present analysis I will detail a procedure for calculating the baryon spectrum as a solution of an eigenvalue problem that generates both the mass and width of the state. This is illustrated for the case of the Δ and Roper resonances.

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1. Introduction

The baryon spectrum is one problem where QCD can confront the traditional approach to nuclear physics in terms of mesons and baryons. As a result, it could shed information on how to amalgamate the quark-gluon degrees of freedom of QCD with the classical meson-baryon degrees of freedom of nuclear physics. Since πN scattering has been the main source of information on the low lying baryon resonances, I will address the question of the baryon spectrum within the framework of πN scattering through the Δ and Roper (R) resonances.

A detailed examination of the baryon spectrum, up to 2 GeV, as given in the data tables [1], indicates that the baryon states or resonances decay to two- or three-body final states. In particular, the non-strange decay modes are πN, $\pi\pi N$, and ηN. This is with the understanding that the physical Δ is a πN resonance, and the ρ meson is a $\pi\pi$ resonance. Thus to understand this spectrum in terms of the observed degree of freedom, *i.e.* mesons and baryons, it is necessary to work in a minimal Hilbert space consisting of B, πB, and $\pi\pi B$, where B are those baryons with an underlying quark structure.

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Since the determination of these resonances is predominantly based on a phase shift analysis of the πN data, one needs to establish if rapid changes in the cross section or partial wave amplitude are due to the presence of states with quark-gluon sub-structure or are due to either threshold effect or a resonance generated by the dynamics of the coupling between the allowed channels. For this one needs to construct a model in which one can make a distinction between these three types of phenomena that can give rise to rapid variation in the πN amplitude.

In the present analysis I outline a model in which one can examine the rapid variation in the πN amplitude, and establish if such variations in the amplitude are due to states that belong to the baryon spectrum. In Sec. 2 I examine a simple model that includes the B and the πB Hilbert space for the Δ and R resonances. I then proceed in Sec. 3 to report how this model has been extended to include, in addition to the B and πB space, the $\pi\pi B$ Hilbert space, and finally I give some concluding remarks in Sec. 4.

2. A simple model

Here, I would like to consider a simple model that illustrates the relation between πN scattering and the determination of the baryon state that dominates the amplitude. For this I will assume that the only term in the interaction is a pion production and absorption vertex, *i.e.*, the interaction Lagrangian is of the form

$$\mathcal{L}_I = g_{\pi BB'}(q^2)\,\bar{\psi}_B\,\gamma_5\,\vec{\tau}\cdot\vec{\phi}\,\psi_{B'} \quad \text{for} \quad B, B' = N, \Delta, R\,, \tag{2.1}$$

where $g_{\pi BB'}(q^2)$ is a form factor which is taken, in this analysis from the Cloudy Bag Model (CBM) [2]. In taking the $\pi BB'$ form factor from the CBM, a connection between the quark model for the baryon and the meson-baryon degrees of freedom has been established. In the CBM, I can define 'bare' baryons in terms of three quark sub-structure. These baryons can then get mesonic dressing and as a result can decay by meson emission. Although the present analysis is based on the CBM, the formalism can be applied to any quark model in which the mesons can couple to the valence quarks to generate a $\pi BB'$ vertex.

I now can define my Hilbert space in terms of the number of pions present. Since I am interested in the baryon spectrum, the state with zero and $n = 1, 2, \ldots$ pions can be defined as:

$$P = |B\rangle\langle B| \quad \text{and} \quad Q_n = |n\pi, B\rangle\langle n\pi, B| \quad n = 1, 2, \ldots\,. \tag{2.2}$$

Since the Hamiltonian couples states of n and $n-1$ pions, my Hamiltonian in this basis generates an infinite set of coupled equations. To solve this coupled channel problem, these equations must be truncated. The minimum level of truncation will be determined by the degree of pionic dressing needed to describe the decay mode of the baryon. *E.g.*, for the $\Delta(1230)$, I will need to take the P and the Q_1 Hilbert spaces, if I am to determine the decay width of the Δ to a πN final state. In this case there are two coupled equations that couple the zero- and one-pion channels. I can formally eliminate the one pion channel with the resultant equation in the P-space describing the Δ, and include pionic dressing at the level of one pion only. This equation is of the form [3]

$$[E - \mathcal{H}_{PP}(E)]\; P\psi = 0\,, \tag{2.3}$$

where the effective energy dependent Hamiltonian $\mathcal{H}_{PP}$ is given by

$$\begin{aligned} \mathcal{H}_{PP}(E) &= H_{PP} + H_{PQ_1}\, [E - H_{Q_1Q_1}]^{-1}\, H_{Q_1P} \\ &\equiv H_{PP} + \Sigma(E)\,. \end{aligned} \tag{2.4}$$

Here $\Sigma(E)$ is the shift in the mass of the Δ due to pionic dressing. The solution of Eq. (2.3) results in the mass and form factor of the dressed Δ, and since the mass of the Δ is greater than the πN threshold, the Δ will have a width for decay to a πN final state. This procedure could be applied to any of the baryon states, but then the only decay mode would be the two-body πB decay mode.

In the above analysis, the determination of the baryon spectrum was reduced to an eigenvalue problem, with the eigenvalues being the mass and width of the baryon states. This was achieved by working in the P-space. On the other hand, I can formally eliminate the P-space and reduce the problem to a two-body scattering problem in the Q_1 space, *i.e.*,

$$[E - \mathcal{H}_{Q_1Q_1}]\; Q_1\psi = 0\,, \tag{2.5}$$

where the energy dependent effective Hamiltonian $\mathcal{H}_{Q_1Q_1}$ is

$$\mathcal{H}_{Q_1Q_1} = H_{Q_1Q_1} + H_{Q_1P}\, [E - H_{PP}]^{-1}\, H_{PQ_1}\,. \tag{2.6}$$

This Hamiltonian describes πB scattering by a potential which consists of an s-channel pole diagram only. In my simple model, $H_{Q_1Q_1}$ is just the kinetic energy of the πN system. The corresponding πB T-matrix can be written as

$$T(E) = H_{Q_1P}\, [E - H_{PP} - \Sigma(E)]\, H_{PQ_1}\,. \tag{2.7}$$

Here, I observe that a determination of the position of the pole of the T-matrix is identical to the determination of the energy E at which Eq. (2.3) has a solution, and the residue at this pole is the form factor as determined by solving Eq. (2.3).

Had I included a πN potential in my original Lagrangian,[1] $H_{Q_1Q_1}$ would include this potential, and both the πN amplitude, and $\Sigma(E)$, would get an additional contribution that depends on this potential. However, the solution of Eq. (2.3) with the modified $\Sigma(E)$ would still give the position of the baryon pole and its residue. On the other hand, the full T-matrix has, in addition to the baryon pole, information about threshold effects and possible resonances generated by dynamics of the additional πB potential introduced.

I now turn my attention to an illustration of the above simple model for the Δ and Roper resonances. I will assume that the N and Δ have a quark substructure that is given in the CBM by $(1S_{1/2})^3$ with the Δ in spin isospin 3/2, while the nucleon in spin isospin 1/2. The Roper is then considered as a radial excitation, *i.e.* $(1S_{1/2})^2(2S_{1/2})$, and in this case there are two possible spin isospin wave functions. These correspond to the [56] and [70] representation of SU(6). The parameters of this model are: the bare πNN coupling constant $f^0_{\pi NN}$, and the radius of the bag R which determines the range of the πNN form factor. All other coupling constants and form factors are related to the πNN parameters via the quark model, or SU(3).

The lowest energy baryon resonance is the $\Delta(1230)$ which decays to a final πN state. To describe this resonance the P and the Q_1 spaces are defined as:

$$P : \Delta \quad \text{and} \quad Q_1 : \pi N, \pi\Delta \,. \tag{2.8}$$

In this way the mesonic dressing of the Δ is due to $\Delta \to \pi B \to \Delta$ with $B = N$ or Δ. In Fig. 1, I present the phase shifts for three different radii R for the bag. Also included are the VPI [4] experimental phase shifts. Here, to get a reasonable set of phase shifts, the bare $\pi N\Delta$ coupling is taken to be twice the πNN coupling constant, with $f^2_{\pi NN} = 0.08$. The relation between the bare $\pi N\Delta$ and $\pi\Delta\Delta$ coupling was maintained as defined by the quark model. Note that the bare mass of the Δ, for the three case considered, has been adjusted for the phase to go through $\pi/2$ at the energy of 1230 MeV. The slope of the phase shifts in all cases is about the same, suggesting that the width predicted for the Δ might be independent of the bag radius.

Included also in the figure is the variation in the full width (*i.e.*, $\Gamma(E) = -2\Im[\Sigma(E)]$) of the resonance as a function of the imaginary part of the energy $E_i = \Im[E]$. The real part of the energy is fixed at $E_r = \Re[E] =$ 1230 MeV. Here, I find that the full width $\Gamma(E)$ is sensitive to both the radius of the bag R and the energy E_i at which the width is calculated. This suggests that any analysis of the data to extract the Δ mass and width requires that one analytically continue the πN amplitude to the Δ pole.

[1] The πB potential could be the sum of a u-channel baryon pole plus heavy meson exchange as is the case in meson exchange potentials.

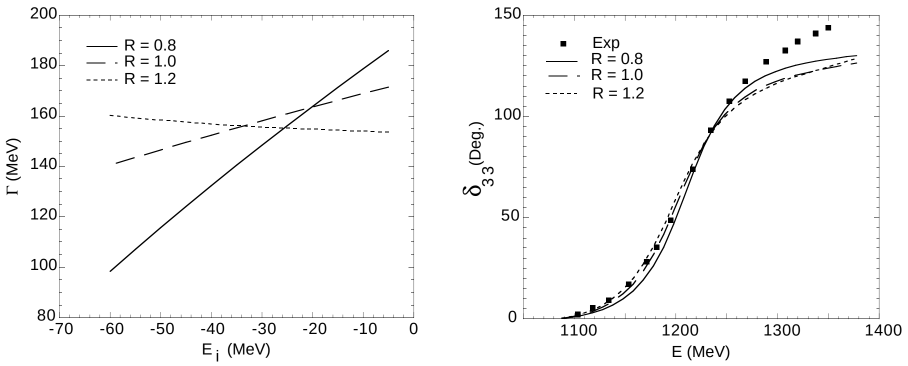

Fig. 1. The variation in the width of the Δ as function of the distance form the real energy axis for three different radii of the bag. Also included are the phase shifts for the three different values of R.

Although there is a similar variation in the mass-shift ($\Delta m = \Re[\Sigma(E)]$) for the Δ, this variation is not as pronounced.

I can extend this model to the Roper resonance provided I ignore the three-body decay mode of the Roper. In this case the P and Q_1 spaces are given by

$$P : N, [56], [70] \quad \text{and} \quad Q_1 : \pi N, \pi\Delta, \pi[56], \pi[70] \ , \tag{2.9}$$

respectively. Now Eq. (2.3) results in a set of three coupled equations with a bare mass for the N and the R as parameters. These bare masses are adjusted to give the physical mass of the nucleon of 940 MeV, and a physical Roper mass of 1440 MeV. Since the CBM predicts the same mass for the [56] and [70], I have kept the bare masses to be identical for the bare Roper states. The coupling constant in this case is taken to correspond to $f_\pi = 93$ while the radius of the bag is taken to be 1.0 fm.

TABLE I

The mass and width of the two P_{11} resonances as calculated from the position of the pole (i), and on the real energy axis (ii).

Baryon	Method (i)	Method (ii)
N	940–0.0 i	794.7–205.0 i
R_1	1440–2.8 i	1440.6–3.3 i
R_2	1470–51.3 i	1480.5–38.8 i

Here again I can examine the width of the Roper by calculating the width on the real energy axis (method (ii)) at $E = 1440$ MeV, or at the pole of the scattering amplitude, *i.e.* a solution of Eq. (2.3) (method (i)). In Table I, I give the mass and width of the nucleon, and the two P_{11} resonances, as determined by the two methods. It is clear that a determination of the width on the real energy axis is valid provided the width of the resonance is small. However, for the Roper resonance with a total width of $\approx$ 250 MeV [1], a calculation of the width on the real axis can be a poor approximation.

I would now like to turn my attention to the P_{11} phase shift in this model. Fig. 2 shows the phase shift as a function of center of mass energy. The fact that I have included both the nucleon and the two Roper states in the P-channel, results in the phase shifts changing sign. With two Roper resonances included, I expect the phase shifts to exhibit the effect of the two resonances. In fact that is exactly what is found. The wider of the two resonances takes the phase shifts through $\pi/2$, while the narrower of the two resonances is only a blip in the phase shift curve. Only when I magnify the scale of my plot do I see the effect of this narrow baryon resonance. This is an example of a baryon state that is very difficult to observe in πN scattering. Since I have not included the coupling to the $\pi\pi N$ channels, the width of the Roper resonances in this model are too small for a good agreement with the experimental phase shifts. I should remind the reader that the model considered here for the Roper is very simple and there have been no parameters to adjust.

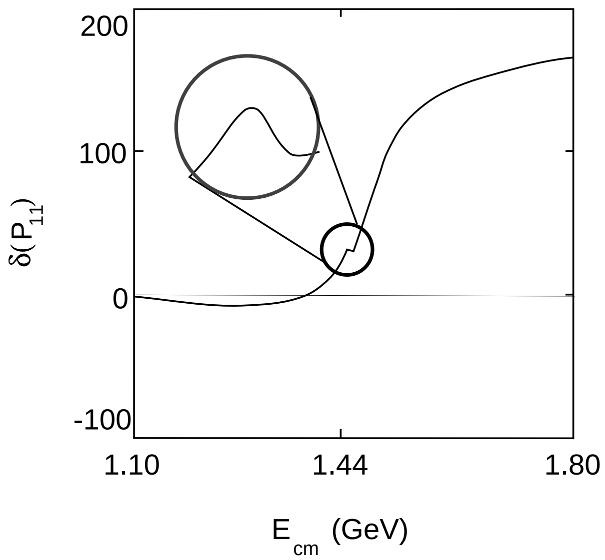

Fig. 2. The πN phase shifts in the P_{11} channel. Note that the phase shifts are a result of two Roper resonances, one very narrow, which causes a small bump in the phase shifts.

3. The $\pi B - \pi\pi B$ model

The fact that the Roper resonances came out too narrow, was a result of not including the $\pi\pi B$ channels. I can overcome this problem by the addition of the Q_2 Hilbert space. Now I have three coupled equations of the form

$$\begin{aligned}(E - H_{PP})\, P\psi &= H_{PQ_1}\, Q_1\psi\,, \\ (E - H_{Q_1Q_1})\, Q_1\psi &= H_{Q_1P}\, P\psi + H_{Q_1Q_2}\, Q_2\psi\,, \\ (E - H_{Q_2Q_2})\, Q_2\psi &= H_{Q_2Q_1}\, Q_1\psi\,. \end{aligned} \tag{3.1}$$

The formal elimination of the Q_2 Hilbert space gives an equation in the Q_1 part of the space that has effectively a potential as part of $H_{Q_1Q_1}$. This additional potential is nothing more than the crossed diagram or u-channel baryon pole diagram. If I take the next step of formally eliminating the Q_1 part of the space, then in the P space my equation is still of the form given by Eq. (2.3), but now $\Sigma(E)$ has an additional contribution from the $\pi\pi B$ part of the Hilbert space, and is of the form

$$\Sigma(E) = H_{PQ_1}\left[E - H_{Q_1Q_1} - H_{Q_1Q_2}\,(E - H_{Q_2Q_2})^{-1}\, H_{Q_2Q_1}\right]^{-1} H_{Q_1P}\,. \tag{3.2}$$

This is represented diagrammatically in Fig. 3, with the πN amplitude in the second diagram being due to the potential resulting from the elimination of the Q_2 Hilbert space.

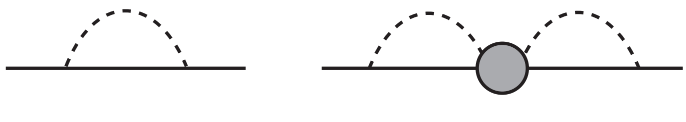

Fig. 3. The diagrammatic contribution to $\Sigma(E)$ when $H_{Q_1Q_1}$ includes a πB interaction either from the elimination of the Q_2 space, or the introduction of a πB potential.

Although this model includes the $\pi\pi B$ part of the Hilbert space, it still does not include the mechanism for the decay of baryons into $N\rho$ where the ρ is a $\pi\pi$ resonance. To include the $N\rho$ channel, I need to include the $\pi\pi$ interaction in the Q_2 Hilbert space. If in addition I include a πB interaction to represent heavy meson exchange, *e.g.* ρ-exchange, then this πB interaction needs to be included in both the Q_1 and the Q_2 spaces. This creates two problems: *(i)* The operator $H_{Q_1Q_2}\,[E - H_{Q_2Q_2}]\, H_{Q_2Q_1}$ is now the full three-body Green's function for the $\pi\pi B$ system with pair-wise interaction, and will require Faddeev technique to handle it. *(ii)* The πB interaction in the Q_1 Hilbert space is also present in the Q_2 Hilbert space.

This creates a bootstrap problem because this interaction is present in both $H_{Q_1Q_1}$ and $H_{Q_2Q_2}$, and the equation in the Q_1 Hilbert space that effectively includes the Q_2 Hilbert space is non-linear in the πB interaction.

The first of these problems is simple to overcome by employing standard Faddeev methods, and has been carried through using the above projection operators by Fuda [5], and using the classification of diagrams in time ordered perturbation theory by Afnan and Pearce [6]. The second of the problems can be resolved in time ordered perturbation theory by observing that the πB amplitude required in the Q_2 Hilbert space is at an energy m_π less than the energy at which the amplitude is required in the Q_1 Hilbert space. As a result, I can determine the πB amplitude below the pion production threshold within the P and Q_1 Hilbert spaces, and then use that amplitude to solve the equation above the production threshold. This has been carried out within the framework of the CBM with volume coupling for the P_{11} channel by Pearce and Afnan [7].

4. Conclusions

From the above analysis of a simple two channel model for the $\Delta(1230)$ and the Roper resonances, the following conclusions may be deduced.

(i) I can commence with a valence quark model for the baryon with meson coupling to the quarks and predict the baryon spectrum as resonances that decay to observed mesons and baryons. The width of these states is determined by the degree of meson dressing included.

(ii) By solving Eq. (2.3) as a complex eigenvalue problem I can determine the poles of the S-matrix that correspond to baryons with quark sub-structure. In this way I can identify if the rapid variation in the amplitude is due to a baryon state, a threshold effect, or a dynamically generated resonance with a corresponding S-matrix pole.

(iii) Finally, I note that the extraction of mass and widths of πN resonances will require the analytic continuation of the amplitude to the resonance pole, and this analytic continuation can be model dependent.

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