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HADRONS IN NUCLEAR MEDIUM AND COLOR TRANSPARENCY*

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Coherent and incoherent propagations of hadrons in nuclear medium are discussed in connection with the "mass shift", the nuclear transparency and the color transparency.

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1. Introduction

The study of hadron properties in nuclear medium is one of the current topics in nuclear and hadron physics. For example, there have been many speculations on the "mass shift" [1–8]. I have attached the quotation mark here since the word is often misleading due to the lack of covariance in the medium. To see the in-medium properties of a hadron, one has to measure its energy, E, and momentum, p, separately and should not combine them to form "invariant mass", m^*, with $E^2 - p^2 = m^{*2}$.

The main issue here is how hadrons propagate in nuclear medium. I would like to stress that it is important to distinguish between the coherent propagation, in which the nuclear final state coincides with the initial state as is often the case in exclusive experiments, and the incoherent propagation, in which the nuclear final states are summed over as usually done in inclusive experiments. I will also show that the distinction is important in discussing the nuclear transparency and modeling the color transparency.

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2. Coherent and incoherent propagations

The coherent propagation is described by the usual Green's function defined by

$$G^{(A)}_{nn}(x',t';x,t) = \langle A,n|T(\psi(x',t')\psi^*(x,t))|A,n\rangle\,, \tag{1}$$

where $\psi(x,t)$ is the hadron field operator and $|A,n\rangle$ denotes a nuclear state. It is a diagonal element with respect to nuclear states and satisfies the Lippmann–Schwinger equation

$$G^{(A)}_{nn} = G^{(0)} + G^{(0)}U^{(A)}_{nn}G^{(A)}_{nn}\,, \tag{2}$$

where $U^{(A)}_{nn}$ is the optical potential. Most analyses have so far been restricted to the nuclear ground state ($n = 0$). For infinite nuclear medium, $G^{(\infty)}$ and $U^{(\infty)}$ are diagonal in momentum, p, and Eq. (2) can be solved in the form

$$G^{(\infty)}(p,E)^{-1} = G^{(0)}(p,E)^{-1} - U^{(\infty)}(p,E)\,. \tag{3}$$

The pole of $G^{(\infty)}$ for a fixed p as a function of E is given by

$$G^{(\infty)}(p,E) = 0, \tag{4}$$

and gives the dispersion relation,$E(p)$, for the hadron in nuclear medium. $E(p)$ is generally complex and $|G^{(\infty)}(p,E)|^2$ as a function of real E has a peak at $\mathrm{Re}\,E(p)$ with the width $-2\mathrm{Im}\,E(p)$. In an exclusive experiment, however, one cannot observe it in this manner since E is determined by kinematics and cannot be changed freely.

In most of high energy experiments, nuclear final states are not observed but summed over and the propagation should be considered as incoherent. One now needs non-diagonal elements of $G^{(A)}$ as well as the diagonal ones *i.e.*

$$G^{(A)}_{n'n}(x',t';x,t) = \langle A,n'|T(\psi(x',t')\psi^*(x,t))|A,n\rangle\,, \tag{5}$$

where a non-diagonal element can always be expressed as

$$G^{(A)}_{n'n} = G^{(A)}_{n'n'}\tilde{T}^{(A)}_{n'n}\tilde{G}^{(A)}_{nn}\,, \tag{6}$$

with the reduced T-matrix and Green's function, $\tilde{T}^{(A)}_{n'n}$ and $\tilde{G}^{(A)}_{nn}$, which involve no intermediate transition to n'. For infinite medium, the diagonal elements are functions of E and p and an experimentally observable quantity has the form

$$I(p,E) = \sum_{n'} |G^{(\infty)}_{n'n'}(p,E)|^2 W_{n'}\delta(E+\varepsilon_{n'}-E_i)\,, \tag{7}$$

where E_i is the initial energy and we have used Eq. (6) to factorize the diagonal elements $G^{(\infty)}_{n'n'}$. One sees that a different E picks up a different n' and thus, if $G^{(\infty)}_{nn}$ is only weakly n-dependent, the previously considered quantity, $|G^{(\infty)}(p,E)|^2$, becomes observable. Inclusive experiments may therefore have better chance of offering direct information on the medium modification of hadrons.

3. Nuclear transparency

The nuclear transparency is a measure of initial and/or final state interactions of a hadron involved in a hard process (large Q^2) and defined generally by

$$T_A(Q^2) = \sigma_A(Q^2)/A\sigma_N(Q^2)\,. \tag{8}$$

In the case of $(e,e'N)$ on nuclei, where the incident electron is scattered by a nucleus, A, changing its momentum from k to k', emitting a nucleon with a momentum, p, and leaving the residual nucleus, $A-1$, in a state n, the differential cross section for the exclusive process is given by

$$\frac{d\sigma_A(n)}{dk'dp} = |\langle k';p;A-1,n|T|k;A\rangle|^2(2\pi)^{-5}\delta(E_f - E_i)\,. \tag{9}$$

The scattered electron energy is usually integrated over and the measured differential cross section is

$$\frac{d\sigma_A(n)}{d\Omega dp} = \int dk'k'^2\frac{d\sigma_a(n)}{dk'dp}. \tag{10}$$

The nuclear transparency is defined by

$$T_A(Q^2;p,n) = \left(\frac{d\sigma_A(n)}{d\Omega dp}\right)_{\rm exp} \Bigg/ \left(\frac{d\sigma_A(n)}{d\Omega dp}\right)_{\rm PWIA}\,. \tag{11}$$

The nucleon propagates coherently in this case and the final state interaction is described by the optical potential. For a nucleon with a large enough momentum, the Glauber approximation works very well and the nuclear transparency is given by

$$T_A(Q^2;p,n) = \int dr\rho_A(r)P_c^{(-)}(r,p)\,, \tag{12}$$

where the survival probability of the scattered nucleon as it travels from the struck point, r, to the nuclear surface is

$$P_c^{(-)}(r,p) = \exp(-\sigma_{NN}^{\rm tot}\int_z^\infty dz'\rho_{A-1}(z';b))\,, \tag{13}$$

with z axis taken in the direction of p and b is the transverse component of r.

In an inclusive experiment, where the residual nuclear state, n, and the emitted nucleon momentum, p, are summed over, the differential cross section becomes

$$\frac{d\sigma_A}{d\Omega} = \int dp \sum_n \frac{d\sigma_A(n)}{d\Omega dp}\,, \tag{14}$$

and the nuclear transparency is defined by

$$T_A(Q^2) = \frac{d\sigma_A}{d\Omega} \bigg/ A\frac{d\sigma_N}{d\Omega}\,. \tag{15}$$

The Glauber expression of the transparency is the same as the previous one except the nucleon-nucleon total cross section is replaced by the reaction cross section [9, 10], *i.e.*

$$T_A(Q^2) = \int dr \rho_A(r) P_{ic}^{(-)}(r,q)\,, \tag{16}$$

with the incoherent survival probability

$$P_{ic}^{(-)}(r,q) = \exp\left(-\sigma_{NN}^r \int_z^\infty dz' \rho_{A-1}(z';b)\right)\,. \tag{17}$$

We have used the transferred momentum,q, for the average of the summed nucleon momenta and, as usual, $q^2 = -Q^2$.

We have compared the calculated transparencies with those extracted from the SLAC experiment [11, 12]. The experimental condition is neither exclusive nor fully inclusive and the experimental ones are actually sandwitched between the calculated lines for the coherent and incoherent propagations in the region of Q^2 between 2 and 8 $(GeV/c)^2$. There is thus no clear indication of deviation from the conventional Glauber calculations, though the large A, large Q^2 data tend to become larger than the incoherent curves, which show the maximum values in the conventional approach.

4. Modeling color transparency

Let us now examine how the internal dynamics of the hadron traveling in nuclear medium affect the transparency. We again consider the case of $(e, e'N)$ and study the time evolution of the nucleon internal state after it is struck by the electron at $t = 0$ [13].

The time evolution in the free space is described by the hamiltonian, H_0, and the nucleon with momentum, q, is its eigen-state, *i.e.*

$$H_0|0,q\rangle = E_q|0,q\rangle\,. \tag{18}$$

The in-medium evolution is then described by

$$H(t) = H_0 + V(t)\,, \tag{19}$$

where $V(t)$ is the interaction with the medium. Denoting the hard interaction operator with the electron by $J(q)$, we have for the relevant amplitude

$$M(q,t) = \left\langle 0,q|T\exp\left(-i\int\limits_0^t dt' H(t')\right) J(q)|0,0\right\rangle\,, \tag{20}$$

which gives the probability amplitude of the nucleon internal state to be in the ground state with momentum q, $|0,q\rangle$, at time t after the interaction with the electron at time 0. The amplitude reduces to the form factor times a phase in free space ($V=0$),

$$M_0(q,t) = \langle 0,q|e^{-iH_0t}J(q)|0,0\rangle = e^{-iE_qt}F(q)\,. \tag{21}$$

We further note that, in the case of inert nucleon for which no internal excitation is allowed, the time evolution is given by replacing H by its ground state expectation value, $\langle 0,q|H|0,q\rangle$, and we get the expression which is equivalent to that given by the Glauber approximation with the replacement, $v_qt \to z$.

With the same replacement and introducing the nucleon survival amplitude by the ratio

$$R(q,t) = M(q,t)/M_0(q,t)\,, \tag{22}$$

we obtain the nucleon survival probability with internal dynamics included as

$$P^{(-)}(q,r) = |R(q,t(r))|^2\,, \tag{23}$$

where $t(r)$ is the travelling time of the nucleon from the position, r, to the nuclear surface. The nuclear transparency is then given by the same expression as Eq. (12) and Eq. (16), *i.e.*

$$T_A(Q^2) = \int dr\rho_A(r)P^{(-)}(q,r)\,. \tag{24}$$

We have so far neglected the dynamics of nuclear medium. Even though the nuclear excitation energies are small compared with the internal excitation energies of the nucleon, the possibility of transition to various states other than the ground state due to the interaction with the travelling nucleon should not be neglected. Actually, the interaction, V, is a function of the coordinates of nucleons in the medium and its time dependence is due to that of the relative coordinates between the travelling nucleon and those in the medium. We can thus write the interaction as

$$V(t,\xi) = V(r_1 - v_q t, r_2 - v_q t, \cdots)\,, \tag{25}$$

where ξ stands for the coordinates $(r_1, r_2, \cdots)$. If the nuclear excitation energies are small, we can use the fixed scatterer approximation and the nucleon survival amplitude becomes a function of ξ. The nucleon survival probability, $P^{(-)}$, which gives the nuclear transparency through Eq. (24), is then obtained for the coherent case as

$$P_c^{(-)}(q,r) = |\int d\xi |\Psi_A(\xi)|^2 R(q,t(r),\xi)|^2\,, \tag{26}$$

and for the incoherent case as

$$P_{ic}^{(-)}(q,r) = \int d\xi |\Psi_A(\xi)|^2 |R(q,t(r),\xi)|^2\,. \tag{27}$$

It is clear from these expressions that the incoherent transparency is always larger than the coherent one. Actual evaluations of the multi-dimensional integrals in these expressions are impossible and all the models of color transparency so far proposed use the approximation corresponding to taking the average neither of R, nor of $|R|^2$ but of V. The interaction parameters are then adjusted so that, for the inert nucleon, the result coincides with the Glauber expression, either Eq. (12) for the coherent case or Eq. (16) for the incoherent case. This prescription may take care of major effects of nuclear dynamics but the approximation should be further examined since the averages such as those appearing in Eq. (26) and Eq. (27) can give rise to interesting phenomena.

In the following, we will use the same approximation to model the color transparency [14]. The model we use for the nucleon internal dynamics is a relativistic harmonic oscillator quark model [15,16] and the Hamiltonian, H_0, for a large momentum, q, is given by

$$H_0 = |q| + \hat{M}^2/2|q|\,, \tag{28}$$

where the mass operator, $\hat{M}$, is

$$\hat{M}^2 = \eta \sum_{i=1}^{3} (p_i^2 + \alpha^2 x_i^2) + C\,, \tag{29}$$

with the center-of-mass part subtracted. The parameters, η, α and C, are determined by the nucleon mass, its charge radius and the mass of the Roper resonance, which is identified as a first positive parity excited state in this model. The interaction with the medium, V, is taken to be

$$V = -ic_0 \sum_{i=1}^{3} x_{i\perp}^2 \,, \tag{30}$$

reflecting the idea of the color transparency that the interaction becomes weak as the transverse size of the system decreases [17–20]. We assume a purely absorptive interaction and the strength is adjusted so as to reproduce the Glauber result for the inert nucleon, as mentioned previously. The hard interaction operator, $J(q)$, is chosen as

$$J(q) = \exp(iqx_1)\exp(\nu q^2 \sum_{i=1}^{3} x_{i\perp}^2) \,, \tag{31}$$

where the second factor is introduced to simulate the longitudinal-transverse correlation in the internal dynamics, which is important in the problem of color transparency but is absent in the harmonic oscillator model. In this model, all the calculations can be done analytically and we get a closed expression for the survival amplitude, R.

$$R(q,t) = \frac{(1-(\frac{\alpha_\perp-\alpha+2\nu q^2}{\alpha_\perp+\alpha-2\nu q^2})(\frac{\alpha_\perp-\alpha}{\alpha_\perp+\alpha}))^2}{(1-(\frac{\alpha_\perp-\alpha+2\nu q^2}{\alpha_\perp+\alpha-2\nu q^2})(\frac{\alpha_\perp-\alpha}{\alpha_\perp+\alpha})e^{-i\frac{\eta}{|q|}2\alpha_\perp t})^2} e^{-i\frac{\eta}{|q|}2\alpha_\perp t} \,, \tag{32}$$

where $\alpha_\perp$ is given by

$$\alpha_\perp^2 = \alpha^2 - i\frac{2|q|c_0}{\eta} \,. \tag{33}$$

We have calculated the survival amplitude and the nuclear transparency with the interaction parameter, c_0, corresponding to the incoherent case, for several choices of the correlation strength, ν. I refer to our paper [14] for the detailed description of the results and give here only its brief summary. Compared with the Glauber calculation which coincides with the case of the inert nucleon, the internal dynamics always increases the survival probability and thus the nuclear transparency, and the effect becomes more conspicuous as Q^2 increases and also as the correlation strength, ν, increases. If we compare our results with those of the SLAC experiments, we can probably exclude the possibility of very strong correlation. However, the comparison is based on the assumption that the experimental condition corresponds to the incoherent case. If the condition is closer to the coherent case, the data for heavy nuclei seem to indicate the color transparency and require the strongest correlation in our model. It is therefore essential to clarify how the nuclear final states are treated in the experiments. For theorists, the calculations are clearer for the extreme cases, *i.e.* purely exclusive or fully inclusive.

5. Concluding remarks

I conclude my talk with a few remarks.

I wish to stress again that the coherent and the incoherent propagations should be clearly distinguished in constructing models and in comparing with experiments. The coherent propagation is described by the usual optical potential but the direct observation of its effect in the form of the strength function in an exclusive experiment is difficult due to kinematical constraints. Such an observation is feasible in an inclusive experiment, if the optical potential depends only weakly on nuclear states.

Distinguishing coherent and incoherent propagations is crucial in observing the on-set of the color transparency. In the case of $(e, e'N)$ experiments, the on-set seems sensitive to the longitudinal-transverse correlation in the internal dynamics of the nucleon. Experiments with better statistics and clearer kinematics would be most desirable.

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