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RESONANT QUADRUPOLE AMPLITUDES IN THE $N \to \Delta$ TRANSITION

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A series of high precision, exclusive measurements on the $N \to \Delta$ transition involving polarized beams, out-of-plane detection and focal plane polarimetry in the $\mathrm{H}(\vec{e}, e'p)\pi^0$ and $\mathrm{H}(\vec{e}, e'\pi^+)n$ channels have been pursued at Bates during the last three years. They are geared towards the precise determination of the quadrupole amplitudes in the $N \to \Delta$ transition and the isolation of the coherent, competing processes (*e.g.* born terms, tails of higher resonances). The issue is of fundamental interest to hadronic physics as it pertains to interquark forces and the structure of hadrons. It is pursed intensively both theoretically and experimentally at many laboratories. The recent precise Bates electroproduction data do not support earlier claims for a strong Coulomb quadrupole amplitude. The available theoretical calculations fail to reproduce the isolated responses.

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1. Introduction

The study of the $N \to \Delta(1232)$ transition in the nucleon provides access to an amplitude of key interest in hadronic physics namely the one involving the resonant quadrupole excitation of the $\Delta(1232)$. It contains the physical information pertaining higher order processes in the hadronic wave function in leading order, which can be understood in different ways depending on the particular theoretical model describing the nucleon (ranging from bag models to the Skyrmion). Each theoretical model — independent of its interpretative scheme — provides a set of observables which could be used to test it. The most common ones are the CMR and EMR ratios, the C2/M1 and E2/M1 amplitude ratios in the $N \to \Delta$ transition, and their momentum dependence. Spin-parity selection rules allow only three (M1, E2 and C2) multipoles to contribute. All models predict low values for these ratios at low momentum transfers, typically in the range of 0.005 to 0.05. In the constituent quark model [1, 2] the quadrupole resonant excitation comes on the account of color tensor interaction among quarks; the resulting d-state admixture in the wave function gives rise to quadrupole excitation which in leading order (the naive quark model or the spherical MIT bag) is absent. The d-state admixture resulting from the presence of tensor forces is reminiscent of the deformation of the deuteron and it leads to the popularization of this issue under the name of "the issue of nucleon deformation". Unfortunately the isolation of the quadrupole amplitude is particularly difficult to accomplish for a number of reasons. The amplitude itself is very much smaller than the leading dipole amplitude; however, far more difficult is the isolation of the resonant (spin flip) from non resonant amplitudes which derive from coherent processes, such as tails of higher resonances or born terms — the so called "background" processes.

In our approach the separation of the background from the resonant terms is attempted through measurements of the isolated responses over a wide range of the invariant hadron mass, where interference effects between different reaction mechanisms are varying.

Recently, photoproduction experiments with polarized photon beams yielded high precision results which constrain the transverse electric amplitude [3, 4]. Polarization measurements at finite Q^2 are still entirely missing, although Focal Plane Polarimetry (FPP) experiments have begun or are planned at Mainz and Jefferson Laboratory. The available older extractions of the CMR have significant systematic errors due to background contamination in addition to the significant instrumental error [5]. A recent measurement at Bonn [16] at $Q^2 = 0.127$ (GeV/c)^2 claims an unexpectedly high CMR of about 13%.

2. Structure function extraction

The Out-Of-Plane Spectrometer (OOPS) collaboration[1] over the last several years has developed a coherent program of investigations requiring out-of-plane detection capability and has designed and built the necessary equipment to pursue it. The program is geared towards addressing specific issues of current interest though the isolation of all five responses which manifest themselves in $A(\vec{e}, e'x)B$ reactions [6–8]. Primary among them is the study of the $N \rightarrow \Delta(1232)$ transition.

[1] Arizona State University; Bates Linear Accelerator Center; California State University at Los Angeles; Florida State University; National and Capodistrian University of Athens; Massachusetts Institute of Technology; Old Dominion University; Tohoku University; University of Illinois at Urbana-Champaign; University of Mainz; University of Massachusetts at Amherst.

The $A(\vec{e}, e'x)B$ cross section corresponding to the reaction depicted in Fig. 1 can be written [6,7] in the one-photon exchange approximation as:

$$\begin{aligned} d\sigma &= d\sigma_{\mathrm{Mott}}(v_{\mathrm{L}}R_{\mathrm{L}} + v_{\mathrm{T}}R_{\mathrm{T}} + v_{\mathrm{LT}}R_{\mathrm{LT}}\cos\phi_{xq} + v_{\mathrm{TT}}R_{\mathrm{TT}}\cos 2\phi_{xq} \\ &\quad + hv'_{\mathrm{LT}}R'_{\mathrm{LT}}\sin\phi_{xq})\,, \end{aligned} \tag{1}$$

where ϕ_{xq} is the azimuthal reaction angle for the emitted particle "X" as defined in Fig. 1, $v_{\alpha\beta}$ is the lepton tensor and h denotes the electron helicity. The maximum structure information that can be obtained in a given electro-coincidence experiment involves the isolation and mapping of the individual responses $R_{\alpha\beta}$ [7,8].

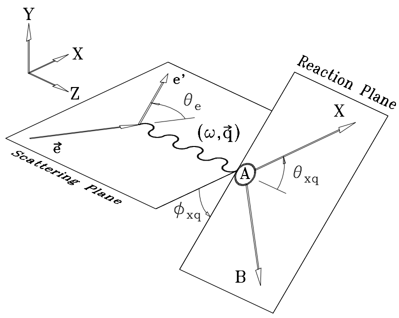

Fig. 1. Kinematic definitions for the A($\vec{e}, e'x$)B reaction.

It is well understood [8] that isolation of all five responses of Eq. (1) is possible if out-of-plane capability is implemented where ϕ_{xq} is used as a lever arm. If, in addition, these measurements are performed simultaneously with multiple detectors, systematic errors are substantially reduced, as the structure functions can be derived (up to a normalization factor) from counting rate asymmetries between detectors placed at these optimally chosen positions. This extraction method is known as the Separation Through Asymmetries Method (STAM) [8]. For instance, the response function R_{LT} can be related to the counting rate asymmetry A_{LT} which in STAM requires simultaneous measurements of the coincidence cross section at $\phi_{xq} = 0$ and $\phi_{xq} = \pi$ (left and right of the momentum transfer direction):

$$A_{\mathrm{LT}} = \frac{N(\phi_{xq}=0) - N(\phi_{xq}=\pi)}{N(\phi_{xq}=0) + N(\phi_{xq}=\pi)} = \frac{v_{\mathrm{LT}}R_{\mathrm{LT}}}{v_{\mathrm{L}}R_{\mathrm{L}} + v_{\mathrm{T}}R_{\mathrm{T}} + v_{\mathrm{TT}}R_{\mathrm{TT}}}\,. \tag{2}$$

R'_{LT}, the so called *fifth response* can be observed if the incident electrons are longitudinally polarized and out-of-plane detection is implemented. The helicity dependent term in the cross section is proportional to R'_{LT}, can be isolated with small systematic error through an asymmetry measurement:

$$A_h = A'_{\mathrm{LT}} = \frac{d\sigma_{+h} - d\sigma_{-h}}{d\sigma_{+h} + d\sigma_{-h}}\,. \tag{3}$$

The function $R'_{\rm LT}$ (or $A'_{\rm LT}$) arises from the interference between two or more complex reaction amplitudes with different phases. The sensitivity of the fifth structure function to interfering amplitudes can be used to help the isolation of resonant from competing channels in the study of nucleon [9] resonances.

The first realization of STAM is occurring at Bates, through the construction of the OOPS (Out-Of-Plane Spectrometer) system, shown in Fig. 2. It consists of four 850 MeV/c magnetic spectrometers [10] [11] of good resolution ($\Delta E/E = 10^{-3}$), fully instrumented, which can be positioned with high accuracy (within a volume of 1 mm^3 and pointing accuracy of 1 mrad) symmetrically around the momentum transfer axis at any of the $n \cdot \pi/4$ out-of-plane angles.

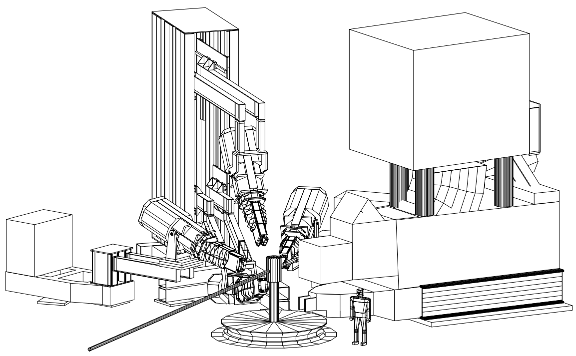

Fig. 2. One of the planned arrangements of a cluster of OOPS spectrometers with the OHIPS spectrometer in a typical out-of-plane $(\vec{e}, e'p)$ geometry.

3. The OOPS $N \to \Delta$ program

The OOPS program for the study of the $N \to \Delta$ transition as originally envisioned [9] and as it has been enriched since then [12–14], is based on the following approach:

High precision measurements are needed, which should be able to measure a CMR of 0.01 or smaller. With an eye upon the suppression of systematic error, we opted for simultaneous coincident measurements which necessitated the use of multipole spectrometers.

While the most sensitive electromagnetic response is the transverse–longitudinal response, $R_{\rm LT}$, we argued that in order to test for small amplitudes, it will be necessary to test the ability of the various theoretical models to correctly predict all accessible responses which should be measured accurately and if possible simultaneously with the $R_{\rm LT}$.

Particular care should be devoted in measuring the imaginary part of the transverse–longitudinal response $R'_{\rm LT}$ which provides an observable of key interest in helping isolate resonant from background amplitudes. Its detection requires out-of plane detection and polarized beams.

Detailed and accurate mapping of the W and Q^2 dependence of the cross section and the isolated responses provides equally important dynamical variables for testing the various theoretical models and for isolating resonant and background contributions.

Finally the study of all possible decay channels (π^0, π^+ and γ) provides essential information on the isospin dependence of the process and another way of testing our understanding of background contributions as they have different manifestation in each of these channels.

The response functions in Eq. (1) can be expanded in terms of pion partial-wave multipole amplitudes which are functions of W and Q^2. Truncated to S and P waves only, these expansions can be written as follows [15]:

$$R_{\rm L}(\theta_{pq}=0^\circ) = |S_{0+} - 4S_{1+} - S_{1-}|^2 \,, \tag{4}$$

$$R_{\rm T}(\theta_{pq}=0^\circ) = |E_{0+} - 3E_{1+} - M_{1+} + M_{1-}|^2 \,, \tag{5}$$

$$R_{\rm LT} = \Re e\, [(S_{0+} + 6S_{1+}\cos\theta_{pq})^* M_{1+}]\, \sin\theta_{pq} \,, \tag{6}$$

$$R_{\rm TT} = -\tfrac{3}{2}\Big(|M_{1+}|^2 + 2\Re e\, [(E_{1+} + M_{1-})^* M_{1+}]\Big) \sin^2\theta_{pq} \,, \tag{7}$$

$$R^n_{\rm LT} = \Im m\, [(S_{0+} - 4S_{1+} - S_{1-})^* M_{1+}] \,, \tag{8}$$

where, in the interference response functions $R_{\rm LT}$, $R_{\rm TT}$, $R^n_{\rm LT}$, terms not containing M_{1+} are also neglected. The response function $R^n_{\rm LT}$ is similar to $R'_{\rm LT}$ as it also is an imaginary part of a longitudinal–transverse interference and it can be determined by measuring the polarization of the recoiling proton. With an unpolarized electron beam and target, the final state proton polarization in parallel kinematics has only a component normal to the scattering plane which is proportional to $R^n_{\rm LT}$:

$$P_n = \frac{v_{\rm LT} R^n_{\rm LT}}{\sigma_{||}} \,, \tag{9}$$

where $\sigma_{||} = \sigma(\theta_{pq}) = 0$ is the parallel kinematics cross section.

In the absence of background, M_{1+}, E_{1+} and S_{1+} correspond to the magnetic dipole, electric quadrupole and Coulomb quadrupole (photon) amplitudes, respectively, and all other multipoles vanish. The parallel kinematics cross section $\sigma_{||}$ is dominated by $R_{\rm T}$, which contains $|M_{1+}|^2$. The most sensitive response functions to the multipole S_{1+} are $R_{\rm L}$ and $R_{\rm LT}$, $R_{\rm LT}$ can be extracted more easily from the asymmetry $A_{\rm LT}$ (see Eq. (2)). Finally, $R'_{\rm LT}$ and $R^n_{\rm LT}$ are sensitive to the background contributions, since without such contributions S_{1+} and M_{1+} would have the same phase and this response function would vanish. In our experiments these four, P_n, $\sigma_{||}$, $A_{\rm LT}$ and $A'_{\rm LT}$ were measured.

3.1. First results from $H(e,e'p)\pi^0$ and $H(e,e'\vec{p})\pi^0$

In the fall of 1996 and as a precursor to both the FPP and OOPS programs [9, 12] the two collaborations teamed up to measure, using the One Hundred Inch Proton Spectrometer (OHIPS) and the Medium Energy Pion Spectrometer (MEPS), the $R_{\rm LT}$ and the $R^n_{\rm LT}$ response. This precursor experiment which to some degree was a response to results from Bonn [16] claiming unexpectedly high CMR ratio and partly a warm-up for both the OOPS and FPP programs, proved exceedingly valuable for refining the subsequent in addition to providing material for three PhD thesis [17–19].

The experiment was conducted with an unpolarized electron beam of a 0.85% duty factor. A cryogenic liquid H_2 target was used in a cylindrical cell of 3 cm diameter with a 10 μm thick Havar wall. The scattered electrons were detected in MEPS, which has a QQSP configuration, and the coincident protons in OHIPS, which has a QQD configuration. The focal plane instrumentation of each spectrometer consisted of one crossed vertical drift chamber for track reconstruction and scintillators for triggering.

For the polarization measurement, OHIPS was additionally equipped with a Focal Plane Polarimeter (FPP) [17]. The overall efficiency of the system was calibrated using elastic electron scattering data on the liquid H_2 target. The polarimeter was calibrated by the H_2 elastic measurement, in which P_n vanishes in OPEA. Finally, a detailed Monte Carlo model was developed which provides the phase space normalization of the cross section and various corrections applied to the data.

In Fig. 3 the measured P_n is compared with π^0 electroproduction calculations of Sato and Lee [20]. Two calculations are shown, one for which the $\gamma N \to \Delta$ dressed-vertex form factors $G_{\rm E}$ (electric) and $G_{\rm C}$ (Coulomb) are set equal to zero and one for which their ratios to the magnetic form factor $G_{\rm M}$ are $G_{\rm E}/G_{\rm M} = 1.8\%$ and $G_{\rm C}/G_{\rm M} = -9.3\%$ at $Q^2 = 0$. The measured P_n is almost a factor of two larger than the predictions indicative of the importance of the background contributions.

The cross section at parallel kinematics $\sigma_{||}$ is shown in Fig. 4. There are two sets of points in the range of W between 1.21 and 1.27 GeV which were measured with two different beam energies of 719 MeV and 799 MeV. In these measurements, the spectrometer central angles and the scattered electron central momentum were different in order to keep W and Q^2 the same. The two data sets agree within statistical errors, which shows that systematic effects in the experiment are well under control. The data are

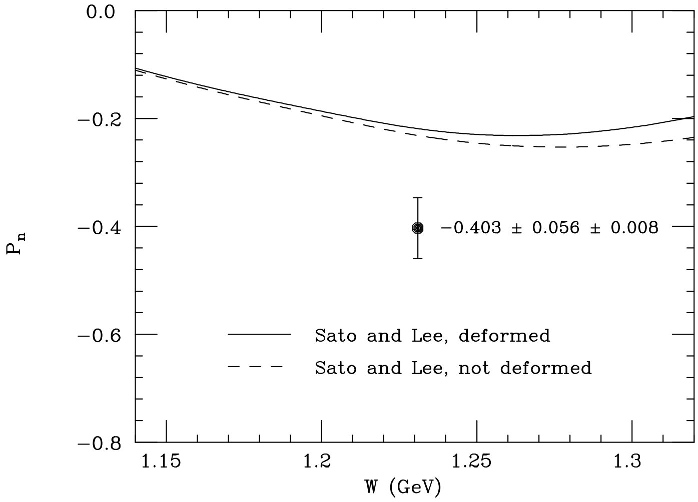

Fig. 3. Induced proton polarization P_n. The long dashes represent the calculation of Mehrotra and Wright; the solid and dotted curves those of Sato and Lee.

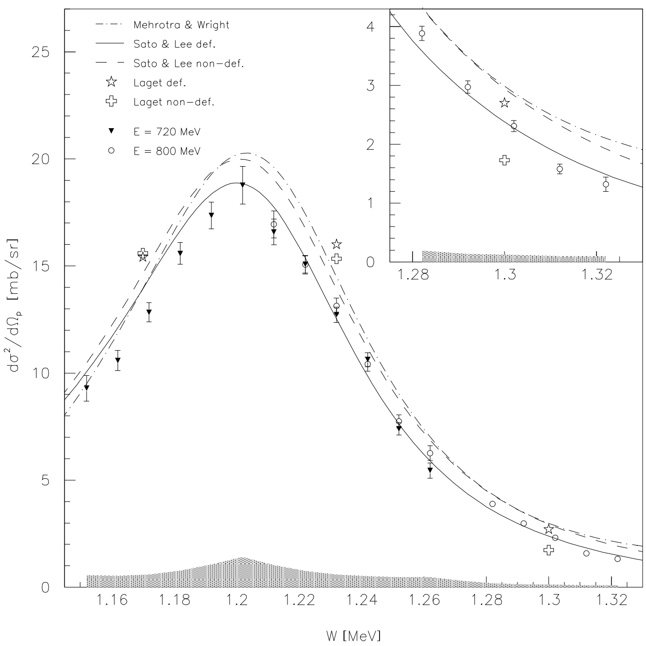

Fig. 4. CM cross section in parallel kinematics. The data points are compared with various model calculations. The shaded areas indicate the systematic error (standard deviation). The graph in the upper corner is an enlargement of the high W region.

compared with predictions of Sato and Lee [20], of Mehrotra and Wright [21] and of Laget [22]. The "deformed" model of Sato and Lee predicts the data very well. Only for the low W range they overestimate the cross section slightly. The curve from the model of Mehrotra and Wright is about 7% to large for most of the W range, only for high W is this disagreement considerably larger. The predictions of Laget's model are the furthest from the data points. The ability of the models to describe the cross section, which is dominated by $R_{\rm T}$, indicates that the transverse response is fairly well understood.

Fig. 5 shows the preliminary asymmetry ($A_{\rm LT}$) data below, above and at the resonance, compared with model predictions of Sato and Lee, of Mehrotra and Wright [21] and of Laget [22]. For each model, the lower curve corresponds to finite quadrupole $\gamma N \to \Delta$ form factors ($G_E/G_M = G_C/G_M = -4\%$ in the model of Laget) and the upper curve corresponds to zero quadrupole form factors, except for the model of Mehrotra and Wright which does not consider such form factors. It should be noted that $A_{\rm LT}$ is much smaller above and below the resonance than $A_{\rm LT}$ at the resonance, while it is expected to be equal in the absence of background. This result and the strong P_n result show important background contributions to both the real and the imaginary part of the response tensor.

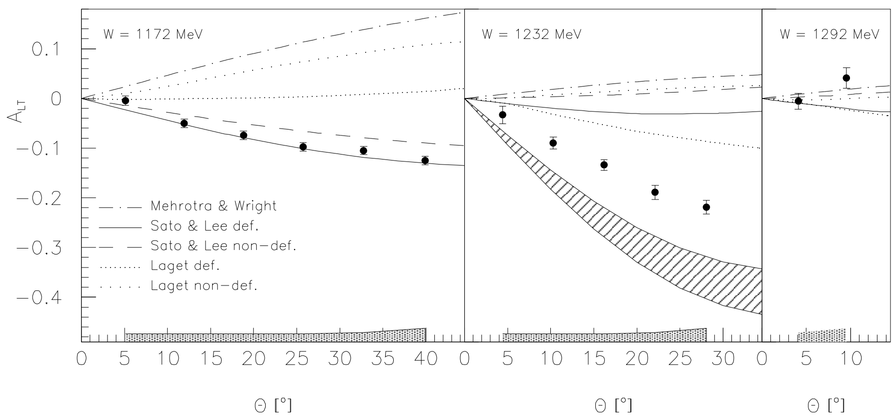

Fig. 5. The asymmetry $A_{\rm LT}$ for three different central values of W. The shaded areas at the bottom of the graph indicate the systematic errors (standard deviation). The band in the middle graph is the prediction with error using the assumption and results of Ref. [16].

The failure of the models examined here to predict the measured observables related to the LT-interference presumably is not primarily due to the transverse part, since the parallel kinematics cross section is fairly well predicted by all these models. Therefore, it is the longitudinal electromagnetic couplings, either the resonant quadrupole coupling or other background-related ones or both, which are not well understood. Also, the phases of non-resonant terms arising from the final state interaction of the $\pi - N$ system can play an important role, particularly in the induced proton polarization. The data presented here have sufficient sensitivity to constrain considerably the longitudinal couplings, since the measured observables are much stronger than the statistical errors. A final analysis for the determination of the magnetic dipole and Coulomb quadrupole $\gamma N \to \Delta$ amplitudes is currently underway.

3.2. Most recent measurements

The 1996 experiment was followed by measurements in the first half of 1998 in the same kinematic region (see Tab. I) but using a different detection system. During the run detection of π^+s was successfully attempted with the OOPS spectrometers; subsequent with the same geometry as in the $\mathrm{H}(\vec{e}, e'p)\pi^0$ an $\mathrm{H}(\vec{e}, e'\pi^+)n$ measurement was performed. Electrons were detected in the modified and upgraded OHIPS and the protons were detected in two OOPS modules (see Fig. 2) placed at the same θ but different ϕ, namely 45° and 135°. This arrangement and the fact that for those measurements polarized beam was employed allows for the measurement of A_{LT} and A'_{LT} (see Eq. (2) and (3)) simultaneously. The expected statistical errors are plotted in Fig. 6. A first analysis already shows, that A'_{LT} is larger in magnitude for the π^+ channel than for the π^0 channel. This is expected as the background (Born) terms are larger in the π^+ channel.

TABLE I

Kinematics and reactions of the OOPS measurements

Q^2 $[(\mathrm{GeV}/c)^2]$	W [MeV]	reaction
0.071	1155	$\mathrm{H}(e, e'p)\pi^0$
0.127	1170	$\mathrm{H}(\vec{e}, e'p)\pi^0$
0.127	1232	$\mathrm{H}(\vec{e}, e'p)\pi^0, \mathrm{H}(\vec{e}, e'\pi^+)n$

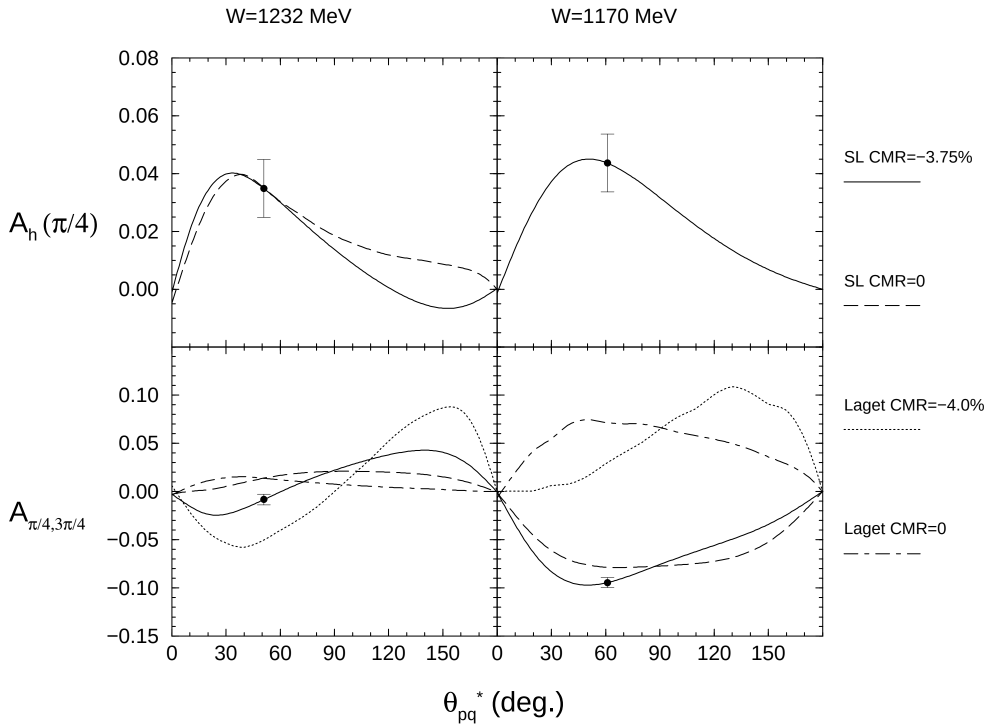

Fig. 6. Expected statistical errors of $\mathrm{A_{LT}}$ and A_h for the $\mathrm{H}(\vec{e}, e'p)\pi^0$ reaction at $Q^2 = 0.127$ (GeV/c)^2.

4. Conclusion and future prospects

The first data from Bates concerning high precision electroproduction studies of the $\Delta^+(1232)$ resonance show that we lack the detailed understanding of the nucleon as no available theoretical model can describe them. Our data are in apparent disagreement with recent claims for a large CMR at $Q^2 = 0.127$ (GeV/c)^2. Our data indicate that CMR is small — in line with the recent EMR measurements with polarized tagged photons. They also highlight the fact that background processes are important and the main obstacle in elucidating the issue of "nucleon deformation".

The data shown here are only the first to be released from the recent experiments on $N \to \Delta$ from Bates. More experiments are planned on this issue with the fully developed OOPS equipment in the near future.

Most of the data presented here derive from the PhD theses of OOPS students G. Warren (MIT), C. Mertz (ASU) and C. Vellidis (Athens). I am indebted to my colleagues at the University of Athens — IASA, C. Mertz, S. Stiliaris and C. Vellidis for significant contributions to this presentation.

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