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PROTON SPIN CONTENT: LINKS TO QCD TOPOLOGICAL SUSCEPTIBILITY* **

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The part of the proton spin Σ carried by u, d, s quarks is calculated in the framework of the QCD sum rules in the external fields. The operators up to dimension 9 are accounted. An important contribution comes from the operator of dimension 3, which in the limit of massless u, d, s quarks is equal to the derivative of QCD topological susceptibility $\chi'(0)$. The comparison with the experimental data on Σ gives $\chi'(0) = (2.3 \pm 0.6) \times 10^{-3}$ GeV^2. The limits on Σ and $\chi'(0)$ are found from selfconsistency of the sum rule, $\Sigma \gtrsim 0.05$, $\chi'(0) \gtrsim 1.6 \times 10^{-3}$ GeV^2. The values of $g_A = 1.37 \pm 0.10$ and $g_A^8 = 0.65 \pm 0.15$ are also determined.

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In the last years, the problem of nucleon spin content and particularly the question which part of the nucleon spin is carried by quarks, attracts a strong interest. The valuable information comes from the measurements of the spin-dependent nucleon structure functions $g_1(x, Q^2)$ in deep inelastic $e(\mu)N$ scattering (for the recent data see [2,3], for a review [4]). The parts of the nucleon spin carried by u, d and s-quarks are determined from the measurements of the first moment of $g_1(x, Q^2)$

$$\Gamma_{p,n}(Q^2) = \int\limits_0^1 dx g_{1;p,n}(x, Q^2)\,. \tag{1}$$

The data allows one to find the value of Σ — the part of nucleon spin carried by three flavours of light quarks $\Sigma = \Delta u + \Delta d + \Delta s$, where $\Delta u, \Delta d, \Delta s$

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are the parts of nucleon spin carried by u, d, s quarks. On the basis of the operator product expansion (OPE) Σ is related to the proton matrix element of the flavour singlet axial current $j^0_{\mu 5}$

$$2ms_\mu \Sigma = \langle p, s| j^0_{\mu 5} |p, s\rangle \,, \tag{2}$$

where s_μ is the proton spin 4-vector, m is the proton mass. The renormalization scheme in the calculation of perturbative QCD corrections to $\Gamma_{p,n}$ can be arranged in such a way that Σ is scale independent.

An attempt to calculate Σ using QCD sum rules in external fields was done in Ref. [5]. Let us shortly recall the idea. The polarization operator

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0| T\{\eta(x), \bar{\eta}(0)\}|0\rangle \tag{3}$$

was considered, where

$$\eta(x) = \varepsilon^{abc} \left(u^a(x) C\gamma_\mu u^b(x) \right) \gamma_\mu \gamma_5 d^c(x) \tag{4}$$

is the current with proton quantum numbers [6], u^a, d^b are quark fields, a, b, c are colour indeces. It is assumed that the term

$$\Delta L = j^0_{\mu 5} A_\mu \,, \tag{5}$$

where A_μ is a constant singlet axial field, is added to QCD Lagrangian. In the weak axial field approximation $\Pi(p)$ has the form

$$\Pi(p) = \Pi^{(0)}(p) + \Pi^{(1)}_\mu(p) A_\mu. \tag{6}$$

$\Pi^{(1)}_\mu(p)$ is calculated in QCD by OPE at $p^2 < 0, |p^2| \gg R_{\rm c}^{-2}$, where $R_{\rm c}$ is the confinement radius. On the other hand, using dispersion relation, $\Pi^{(1)}_\mu(p)$ is represented by the contribution of the physical states, the lowest of which is the proton state. The contribution of excited states is approximated as a continuum and suppressed by the Borel transformation. The desired answer is obtained by equalling of these two representations. This procedure can be applied to any Lorenz structure of $\Pi^{(1)}_\mu(p)$, but as was argued in [7,8], the best accuracy can be obtained by considering the chirality conserving structure $2p_\mu \hat{p}\gamma_5$.

An essential ingredient of the method is the appearance of induced by the external field vacuum expectation values (v.e.v). The most important of them in the problem at hand is

$$\langle 0| j^0_{\mu 5} |0\rangle_A \equiv 3f_0^2 A_\mu \tag{7}$$

of dimension 3. The constant f_0^2 is related to QCD topological susceptibility. Using (5), we can write

$$\begin{aligned}\langle 0|j^0_{\mu 5}|0\rangle_A &= \lim_{q\to 0}\, i\int d^4x \mathrm{e}^{iqx}\langle 0|T\{j^0_{\nu 5}(x), j^0_{\mu 5}(0)\}|0\rangle A_\nu \\ &\equiv \lim_{q\to 0} P_{\mu\nu}(q)A_\nu\,. \end{aligned} \tag{8}$$

The general structure of $P_{\mu\nu}(q)$ is

$$P_{\mu\nu}(q) = -P_L(q^2)\delta_{\mu\nu} + P_T(q^2)(-\delta_{\mu\nu}q^2 + q_\mu q_\nu)\,. \tag{9}$$

Because of anomaly there are no massless states in the spectrum of the singlet polarization operator $P_{\mu\nu}$ even for massless quarks. $P_{T,L}(q^2)$ also have no kinematical singularities at $q^2 = 0$. Therefore, the nonvanishing value $P_{\mu\nu}(0)$ comes entirely from $P_L(q^2)$. Multiplying $P_{\mu\nu}(q)$ by $q_\mu q_\nu$, in the limit of massless u, d, s quarks we get

$$\begin{aligned} q_\mu q_\nu P_{\mu\nu}(q) &= -P_L(q^2)q^2 = N_f^2(\alpha_s/4\pi)^2 i\int d^4x \mathrm{e}^{iqx} \\ &\qquad \times\langle 0|TG^n_{\mu\nu}(x)\tilde{G}^n_{\mu\nu}(x), G^m_{\lambda\sigma}(0)\tilde{G}^m_{\lambda\sigma}(0)|0\rangle\,, \end{aligned} \tag{10}$$

where $G^n_{\mu\nu}$ is the gluonic field strength, $\tilde{G}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\lambda\sigma}G_{\lambda\sigma}$. (The anomaly condition was used, $N_f = 3$.) Going to the limit $q^2 \to 0$, we have

$$f_0^2 = -\frac{1}{3}P_L(0) = \frac{4}{3}N_f^2\chi'(0), \tag{11}$$

where $\chi(q^2)$ is the topological susceptibility

$$\chi(q^2) = i\int d^4x \mathrm{e}^{iqx}\langle 0|TQ_5(x), Q_5(0)|0\rangle\,, \tag{12}$$

$$Q_5(x) = (\alpha_s/8\pi)\; G^n_{\mu\nu}(x)\tilde{G}^n_{\mu\nu}(0)\,. \tag{13}$$

As is well known (see, *e.g.*, the review [9]), $\chi(0) = 0$ if there is at least one massless quark. The attempt to find $\chi'(0)$ itself by QCD sum rules failed: it was found [5] that OPE does not converge in the domain of characteristic scales for this problem. However, it was possible to derive the sum rule, expressing Σ in terms of f_0^2 (7) or $\chi'(0)$. The OPE up to dimension $d = 7$ was performed in Ref. [5]. Among the induced by the external field v.e.v.'s besides (7), the v.e.v. of the dimension 5 operator

$$g\langle 0|\sum_q \bar{q}\gamma_\alpha(1/2)\lambda^n\tilde{G}^n_{\alpha\beta}q|0\rangle_A \equiv 3h_0A_\beta, \quad q = u, d, s \tag{14}$$

was accounted and the constant h_0 was estimated using a special sum rule, $h_0 \approx 3\times 10^{-4}\mathrm{GeV}^4$. There were also accounted the gluonic condensate $d=4$ and the square of quark condensate $d=6$ (both times the external A_μ field operator, $d=1$). However, the accuracy of the calculation was not good enough for reliable calculation of Σ in terms of f_0^2: the necessary requirement of the method — the weak dependence of the result on the Borel parameter was not well satisfied.

In this paper we improve the accuracy of the calculation by going to higher order terms in OPE up to dimension 9 operators. Under the assumption of factorization — the saturation of the product of four-quark operators by the contribution of an intermediate vacuum state — the dimension 8 v.e.v.'s are accounted (times A_μ):

$$-g\langle 0|\bar{q}\sigma_{\alpha\beta}\tfrac{1}{2}\lambda^n G^n_{\alpha\beta}q\cdot\bar{q}q|0\rangle = m_0^2\langle 0|\bar{q}q|0\rangle^2, \tag{15}$$

where $m_0^2 = 0.8\pm 0.2$ GeV^2 was determined in [10]. In the framework of the same factorization hypothesis the induced by the external field v.e.v. of dimension 9

$$\alpha_s\langle 0|j^{(0)}_{\mu 5}|0\rangle_A\langle 0|\bar{q}q|0\rangle^2 \tag{16}$$

is also accounted. In the calculation we used the following expression for the quark Green function in the constant external axial field [8]:

$$\begin{aligned}
&\langle 0|T\{q^a_\alpha(x),\ \bar{q}^b_\beta(0)\}|0\rangle_A = i\delta^{ab}\hat{x}_{\alpha\beta}/2\pi^2x^4\\
&+\left(\frac{1}{2}\pi^2\right)\delta^{ab}(Ax)(\gamma_5\hat{x})_{\alpha\beta}/x^4 - \frac{1}{12}\delta^{ab}\delta_{\alpha\beta}\langle 0|\bar{q}q|0\rangle\\
&+\frac{1}{72}i\delta^{ab}\langle 0|\bar{q}q|0\rangle(\hat{x}\hat{A}\gamma_5 - \hat{A}\hat{x}\gamma_5)_{\alpha\beta}\\
&+\frac{1}{12}f_0^2\delta^{ab}(\hat{A}\gamma_5)_{\alpha\beta} + \frac{1}{216}\delta^{ab}h_0\left[\frac{5}{2}x^2\hat{A}\gamma_5 - (Ax)\hat{x}\gamma_5\right]_{\alpha\beta}.
\end{aligned} \tag{17}$$

The terms of the third power in x-expansion of quark propagator proportional to A_μ are omitted in (17), because they do not contribute to the tensor structure of Π_μ of interest. Quarks are considered to be in the constant external gluonic field and quark and gluon QCD equations of motion are exploited (the related formulae are given in [11]). There is also an another source of v.e.v. h_0 to appear besides the x-expansion of quark propagator given in Eq. (17): the quarks in the condensate absorb the soft gluonic field emitted by other quark. A similar situation takes place also in the calculation of the v.e.v. (16) contribution. The accounted diagrams with dimension 9 operators have no loop integrations. There are others v.e.v. of dimensions $d\le 9$ particularly containing gluonic fields. All of them, however, correspond to at least one loop integration and are suppressed by the numerical factor $(2\pi)^{-2}$. For this reason they are disregarded.

The sum rule for Σ is given by

$$\Sigma + C_0 M^2 = -1 + \frac{8}{9\tilde{\lambda}_N^2} e^{m^2/M^2} \Big\{ a^2 L^{4/9} + 6\pi^2 f_0^2 M^4 E_1 \left(\frac{W^2}{M^2}\right) L^{-4/9}$$
$$+ 14\pi^2 h_0 M^2 E_0 \left(\frac{W^2}{M^2}\right) L^{-8/9} - \frac{1}{4}\,\frac{a^2 m_0^2}{M^2} - \frac{1}{9}\pi\alpha_s f_0^2\,\frac{a^2}{M^2} \Big\} \,. \qquad (18)$$

Here M^2 is the Borel parameter, $\tilde{\lambda}_N$ is defined as $\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2 = 2.1$ GeV^6, $\langle 0|\eta|p\rangle = \lambda_N v_p$, where v_p is proton spinor, W^2 is the continuum threshold, $W^2 = 2.5$ GeV^2,

$$a = -(2\pi)^2 \langle 0|\bar{q}q|0\rangle = 0.55 \text{ GeV}^3 \,, \qquad (19)$$

$$E_0(x) = 1 - e^{-x}, \quad E_1(x) = 1 - (1+x)e^{-x} \,,$$

$L = \ln(M/\Lambda)/\ln(\mu/\Lambda)$, $\Lambda = \Lambda_{\mathrm{QCD}} = 200$ MeV and the normalization point μ was chosen $\mu = 1$ GeV. When deriving (18) the sum rule for the nucleon mass was exploited what results in appearance of the first term, -1, in the right hand side (rhs) of (18). This term absorbs the contributions of the bare loop, gluonic condensate as well as α_s corrections to them and essential part of terms, proportional to a^2 and $m_0^2 a^2$. The values of the parameters, $a, \tilde{\lambda}_N^2, W^2$ taken above were chosen by the best fit of the sum rules for the nucleon mass (see [12], Appendix B) performed at $\Lambda = 200$ MeV. It can be shown, using the value of the ratio $2m_s/(m_u + m_d) = 24.4 \pm 1.5$ [13] that $a(1 \text{ GeV}) = 0.55$ GeV^3 corresponds to $m_s(1 \text{ GeV}) = 153$ MeV. α_s corrections are accounted in the leading order (LO) what results in appearance of anomalous dimensions. Therefore Λ has the meaning of effective Λ in LO. The unknown constant C_0 in the left-hand side (lhs) of (18) corresponds to the contribution of inelastic transitions $p \to N^* \to interaction\, with A_\mu \to p$ (and in inverse order). It cannot be determined theoretically and may be found from M^2 dependence of the rhs of (18) (for details see [12,14]). The necessary condition of the validity of the sum rule is $|\Sigma| \gg |C_0 M^2| \exp[(-W^2 + m^2)/M^2]$ at characteristic values of M^2 [14]. The contribution of the last term in the rhs of (18) is negligible. The sum rule (18) as well as the sum rule for the nucleon mass is reliable in the interval of the Borel parameter M^2 where the last term of OPE is small less than 10–15% of the total and the contribution of continuum does not exceed 40–50% . This fixes the interval $0.85 < M^2 < 1.4$ GeV^2. The M^2-dependence of the rhs of (18) at $f_0^2 = 3 \times 10^{-2}$ GeV^2 is plotted in Fig. 1. The complicated expression in rhs of (18) is indeed an almost linear function of M^2 in the given interval! This fact strongly supports the reliability of the approach. The best values of $\Sigma = \Sigma^{\rm fit}$ and $C_0 = C_0^{\rm fit}$ are found from the χ^2

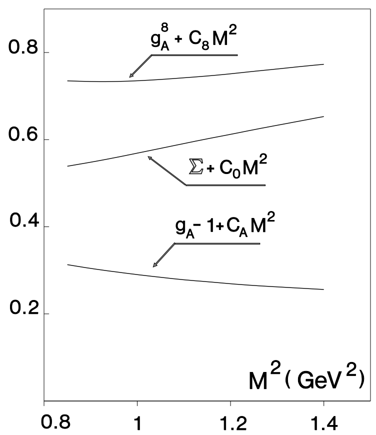

Fig. 1. The M^2-dependence of $\Sigma + C_0 M^2$ at $f_0^2 = 3 \times 10^{-2}$ GeV^2, Eq. (18), $g_A^8 + C_8 M^2$, and $g_A - 1 + C_A M^2$, Eq. (22).

fitting procedure

$$\chi^2 = \frac{1}{n} \sum_{i=1}^{n} \left[\Sigma^{\rm fit} + C_0^{\rm fit} M_i^2 - R(M_i^2) \right]^2 = \min, \tag{20}$$

where $R(M^2)$ is the rhs of (18).

The values of Σ as a function of f_0^2 are plotted in Fig. 2 together with $\sqrt{\chi^2}$. In our approach the gluonic contribution cannot be separated and

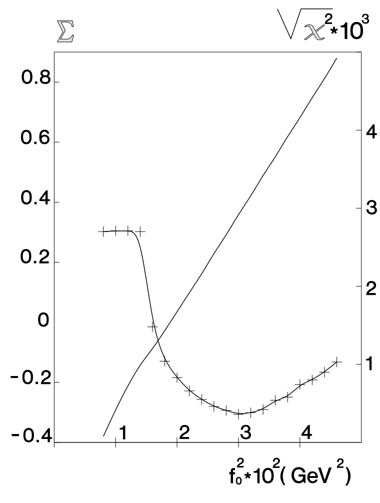

Fig. 2. Σ (solid line, left ordinate axis) and $\sqrt{\chi^2}$, Eq. (20), (crossed line, right ordinate axis). as a functions of f_0^2.

is included in Σ. The experimental value of Σ can be estimated [2,3] (for discussion see [15]) as $\Sigma = 0.3 \pm 0.1$. Then from Fig. 2 we have $f_0^2 = (2.8 \pm 0.7) \times 10^{-2}$ GeV^2 and $\chi'(0) = (2.3 \pm 0.6) \times 10^{-3}$ GeV^2 . The error in f_0^2 and χ' besides the experimental error includes the uncertainty in the sum rule estimated as equal to the contribution of the last term in OPE (two last terms in Eq.18) and a possible role of NLO α_s corrections. At $f_0^2 < 0.02$ GeV^2 χ^2 is much worse and the fit becomes unstable. This allows us to claim (with some care, however,) that $\chi'(0) \geq 1.6 \times 10^{-3}$GeV^2 and $\Sigma \geq 0.05$ from the requirement of self consistency of the sum rule. The χ^2 curve also favours an upper limit for $\Sigma \lesssim 0.6$. At $f_0^2 = 2.8 \times 10^{-2}$ GeV^2 the value of the constant C_0 found from the fit is $C_0 = 0.19$ GeV^{-2}. Therefore, the mentioned above necessary condition of the sum rule validity is well satisfied. Recently, the first attempt to calculate $\chi'(0)$ on the lattice was performed [16]. The result is $\chi'(0) = (0.4 \pm 0.2) \times 10^{-3}$ GeV^2, much below our value. However, as mentioned by the authors, the calculation has some drawbacks and the result is preliminary.

Let us discuss the role of various terms of OPE in the sum rules (18) To analyze it we have considered sum rules (18) for 4 different cases, *i.e.* when we take into consideration: (a) only contribution of the operators up to $d = 3$ (the term-1 and the term, proportional to f_0^2 in (18)); (b) contribution of the operators up to $d = 5$ (the term $\sim h_0$ is added); (c) contribution of the operators up to $d = 7$ (three first terms in (18)), (d) our result (18), *i.e.* all operators up to $d = 9$. We choose for this analysis the most reasonable value of $f_0^2 = 0.03$ GeV^2, but the conclusion we will come appears to be the same for all more or less choice of f_0^2. Results of the fit of the sum rules are shown in the Table I for all four cases. Fit is done in the region of Borel masses $0.9 < M_{\rm B}^2 < 1.3$ GeV^2. In the first column the values of Σ are shown , in the second — values of the parameter C, and in the third — the ratio $\gamma = |\sqrt{\chi^2}/\Sigma|$, which is the real parameter, describing reliability of the fit. From the Table one can see, that reliability of the fit monotonically improves with increasing of the number of accounted terms of OPE and is quite satisfactory in the case (d).

TABLE I

case	Σ	C(GeV^{-2})	γ
(a)	-0.019	0.31	10^{-1}
(b)	0.031	0.3	5.10^{-2}
(c)	0.54	0.094	9.10^{-3}
(d)	0.36	0.21	$1.3 \cdot 10^{-3}$

From the same sum rule (18) it is possible to find g_A^8 — the proton coupling constant with the octet axial current, which enters the QCD formula for $\Gamma_{p,n}$ [4]. There are two differences in comparison with (18):

1. Instead of f_0^2 it appears the square f_8^2 of the pseudoscalar meson coupling constant with the octet axial current. In the limit of strict SU(3) flavour symmetry it is equal to f_π^2, $f_\pi = 133$ MeV. However, it is known, that SU(3) symmetry is violated and the kaon decay constant, $f_K \approx 1.25 f_\pi$. In the linear in s-quark mass m_s approximation $f_\eta = 1.31 f_\pi$. We put for f_8^2 the value $f_8^2 = 2.6 \times 10^{-2}$ GeV^2, intermediate between f_π^2 and f_η^2 .

2. h_0 should be substituted by $m_1^2 f_\pi^2$. The constant m_1^2 is determined by the sum rules suggested in [17]. A new fit corresponding to the values of the parameters used above, was performed and it was found; $m_1^2 = 0.16$ GeV^2.

The M^2-dependence of $g_A^8 + C_8 M^2$ is presented in Fig. 1 and the best fit according to the fitting procedure (20) at $1.0 \leq M^2 \leq 1.3$ GeV^2 gives

$$g_A^8 = 0.65 \pm 0.15\,, \quad C_8 = 0.10 \text{ GeV}^{-2}\,, \quad \sqrt{\chi^2} = 1.2 \times 10^{-3}\,. \tag{21}$$

(The error includes the uncertainties in the sum rule as well as in the value of f_8^2.) The obtained value of g_A^8 within the errors coincides with $g_A^8 = 0.59 \pm 0.02$ [18] found from the data on baryon octet β-decays under assumption of strict SU(3) flavour symmetry and contradicts the hypothesis of bad violation of SU(3) symmetry in baryon axial octet coupling constants [19].

A similar sum rule with the account of dimension 9 operators can be derived also for g_A — the nucleon axial β-decay coupling constant. It is an extension of the sum rule found in [7] and has the form

$$\begin{aligned} g_A + C_A M^2 &= 1 + \frac{8}{9\tilde{\lambda}_N^2} \mathrm{e}^{m^2/M^2} \\ &\times \left[a^2 L^{4/9} + 2\pi^2 m_1^2 f_\pi^2 M^2 - \frac{1}{4} a^2 \frac{m_0^2}{M^2} + \frac{5}{3}\pi \alpha_s f_\pi^2 \frac{a^2}{M^2} \right]. \end{aligned} \tag{22}$$

The main term in OPE of dimension 3 proportional to f_π^2 occasionally was cancelled. For this reason the higher order terms of OPE may be more important in the sum rule for g_A than in the previous ones. The M^2 dependence of $g_A - 1 + C_A M^2$ is plotted in Fig. 1, lower curve; the curve is almost the straight line, as it should be. The best fit gives

$$g_A = 1.37 \pm 0.10, \quad C_A = -0.088 \text{ GeV}^{-2}, \quad \sqrt{\chi^2} = 1.0 \times 10^{-3} \tag{23}$$

in comparison with the world average $g_A = 1.260 \pm 0.002$ [20]. The inclusion of dimension 9 operator contribution essentially improves the result: without it g_A would be about 1.5 and χ^2 would be much worse.

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