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NON-PERTURBATIVE VEVS FROM A LOCAL EXPANSION *

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We propose a method for the calculation of vacuum expectation values (VEVs) given a non-trivial, long-distance vacuum wave functional (VWF) of the kind that arises, for example, in variational calculations. The VEV is written in terms of a Schrödinger-picture path integral, then a local expansion for (the logarithm of the) VWF is used. The integral is regulated with an explicit momentum cut-off, Λ. The resulting series is not expected to converge for Λ larger than the mass-gap but studying the domain of analyticity of the VEVs allows us to use analytic continuation to estimate the large-Λ limit. Scalar theory in $1+1$ dimensions is analyzed, where (as in the case of Yang-Mills) we do not expect boundary divergences.

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1. Introduction

The use of the Schrödinger picture in field theory provides a natural framework for non-perturbative calculations based on ansätze for the vacuum wave functional (VWF). Although the Yang-Mills VWF has been analyzed numerically by lattice simulations [1, 2] and analytical approximations [3–5], it is an open problem to compute VEVs with such a VWF because we have to address the calculation of non-Gaussian path integrals. One way to systematically compute the path integral

$$\int \mathcal{D}\phi \; A[\phi]\, |\Psi_0[\phi]|^2 \tag{1}$$

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would be to expand $\Psi_0[\phi]$ about a Gaussian, so we can use Wick's theorem to obtain the VEV. In principle, this would involve a perturbation expansion with (dressed) propagators and an infinite number of non-local vertices with increasing dependence on high momenta. Within perturbation theory, it can be explicitly shown that these diagrams will not generate new divergences other than the ones which are substracted by renormalization [6]. Outside perturbation theory we may encounter divergences (which will have to cancel when we resum the expansion) due to the fact that the Gaussian term (around which we expand) does not damp the high momentum modes strongly enough. We will show how to compute such a path integral by performing a further expansion in terms of local expressions. Therefore, we will give a method to compute finite VEVs, from a given $\Psi_0[\phi]$, by expanding the logarithm of $\Psi_0[\phi]$ in power of ϕ and its derivatives (and considering the quadratic term in ϕ as the unperturbed part). Notice that now (if we compute an equal-time VEV) the $|\Psi_0[\phi]|^2$ can be interpreted as the exponential of (minus) a euclidean local action (which lives in the quantization surface $t = 0$) with an infinite number of (non-renormalizable) terms which apparently generate ultra-violet divergences. These arise because the path integral includes configurations with momentum far beyond the convergence radius of our local expansion. In [6] we have argued that that (provided the theory has a mass-gap) the convergence radius of the local expansion is non-zero and finite. Therefore, we introduce an explicit cut-off, Λ, on the Fourier components of the field configurations of Eq. (1). Now the equal-time VEV of an operator can be computed from the Ψ_0-local expansion, provided Λ is smaller than the convergence radius. Of course, we want to send Λ to infinity. In [6] we have studied the domain of analyticity of the (equal-time) VEVs as functions of this cut-off and used it to compute the $\Lambda \to \infty$ value by analytic continuation. The particular method chosen for the continuation is not essential, although the procedure should be suitable for numerical analysis. We found to be most convenient a method related to Borel resummation (which itself is commonly used to re-sum asymptotic series). We will discuss the method in the context of scalar theory in $1+1$ dimensions and we will give an example based on perturbation theory (where we can check it).

2. Analyticity in the Schrödinger cut-off

In [7] it was shown that for Yang-Mills theory $\Psi_0[A(x/\sqrt{s})/\sqrt{s}]$, is analytic in the cut s-plane with the cut in the negative real axis. The same holds for scalar theory [6]. This analyticity of the scaled VWF provides a method to recover the full $\Psi_0[\phi]$ from its functional Taylor expansion around slowly varying configurations. This analyticity can be extended to scaled equal-time VEVs [6], so they can be reconstructed from the resummation of a perturbation expansion (the expansion parameter will be Λ/m, with m the mass scale which appears in the chosen VWF and which will be related to the energy spectrum)generated by locally expanding the VWF.

For scalar theory in $1+1$ dimensions, we have shown [6] that

$$K(s) = s^{n/2}\langle\Psi_0|A[\tilde{\phi}(\frac{p}{\sqrt{s}})]|\Psi_0\rangle_{\frac{\Lambda}{\sqrt{s}}} \tag{2}$$

is analytic in the cut s-plane, with the cut going from $s=-\Lambda^2$ to $s=0$ (and with n being a integer number). We will recover $K(1)$ (which gives an approximation to the VEV if Λ is large enough) by analytic continuation from $s=\infty$ (where a local expansion can be used) to $s=1$. The method for the analytic continuation is based on constructing a function, $I(\lambda)$, from $K(s)$:

$$I(\lambda) = \frac{1}{2\pi i}\int_C ds\,\frac{e^{\lambda(s-1)}}{s-1}\,\tilde{K}(\frac{p_i}{\sqrt{s}},\frac{\Lambda}{\sqrt{s}}). \tag{3}$$

The integration contour C is shown in Fig. 1 together with the $[-\Lambda^2,0]$ cut.

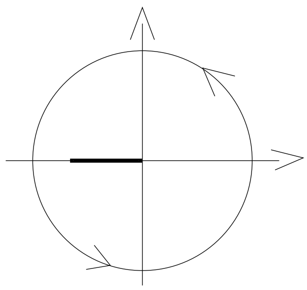

Fig. 1. Large radius contour for the integral $I(\lambda)$.

A local expansion of the VWF will give a series in p^k

$$\tilde{K} = \sum_k a_k\frac{1}{\sqrt{s}^k} = \sum_k b_k\frac{1}{(s-1)^k}, \tag{4}$$

which has been rearranged in a series in $1/(s-1)^n$, so the Eq. (3) will give

$$I(\lambda) = \sum_n b_n\frac{\lambda^n}{n!}. \tag{5}$$

It is shown in [6] (by collapsing the contour to an infinitesimal circle around $s=1$ and a contribution from the cut, which will be damped by the exponential term) that the $\lambda\to\infty$ limit of Eq. (3) gives the VEV:

$$\tilde{K}(p_i,\Lambda) = \lim_{\lambda\to\infty} I(\lambda) = \lim_{\lambda\to\infty}\sum_n b_n\frac{\lambda^n}{n!}. \tag{6}$$

Because we expect that the convergence radius of Eq. (5) will be infinite [6], we can obtain an approximation for $I(\infty)$ by truncating the (alternating) series at N terms, and taking λ as large as is consistent with this. In practice this means that we take $b_N\lambda^N/N!$ to be a small fraction of the value of the truncated sum. We have to assume that the function $I(\lambda)$ does not have a plateau at finite λ, which could incorrectly indicate that the $I(\infty)$ limit has been reached (and therefore the truncated approximant would give the value of $I(\lambda)$ at the plateau, instead of $I(\infty)$). We may instead estimate $I(\infty)$ by looking at the point where the approximant (a polynomial in λ) is stationary, then we have to assume that $I(\lambda)$ has no stationary point other than the one at $\lambda \to \infty$.

3. Example

In this section we will illustrate how to compute the, equal-time, two-point function in a $1+1$-dimensional scalar theory model where the VWF is given, for slowly varying configurations, by

$$\begin{aligned}|\Psi_0|^2 \;=\; & N\exp\left(-\frac{1}{2}\int\tilde\phi\tilde\phi(\alpha_0+\right. \\ & \left.\alpha_2p^2+\cdots)-\frac{1}{4!}\int\tilde\phi\tilde\phi\tilde\phi\tilde\phi(\beta_0+\beta_2\sum_i p_p^2+\cdots)\right). \qquad (7)\end{aligned}$$

And we assume that this VWF has a sensible UV limit (high frequency modes for the configurations), so the analyticity of the VEV (which was proven in [6] for the true vacuum) also would hold here. We write the (connected) two-point function for small momenta as

$$\langle\Psi_0|\tilde\phi(p)\tilde\phi(-p)|\Psi_0\rangle = c_0 + c_2p^2 + \cdots \qquad (8)$$

the coefficients c_0 and c_2 can be computed by performing a perturbation expansion with a small cut-off Λ. They will be given by a series in Λ. In Fig. 2 we give the result of the computation of c_0 and c_2 until order $O(\Lambda^5)$ and $O(\Lambda^2p^2)$ respectively.

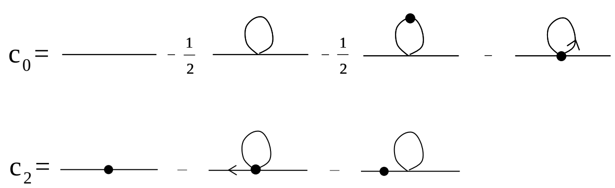

Fig. 2. Diagrams for the lowest Λ^n terms of c_0 and c_2.

Where the propagators are $1/\alpha_0$, the momenta insertion (denoted by a dot) are $-\alpha_2 q^2$ and the dotted vertex with arrow is $\beta_2 q^2$ (the arrow shows where the momentum is sitting). In ϕ^4 theory at first order in perturbation theory, the VWF has a good UV limit [6] and the VEV analyticity holds. The values for the α_0, α_2, β_0 and β_2 are given by

$$\begin{aligned}
\alpha_0 &= 2m\,, \\
\alpha_2 &= (m - \frac{m}{12\pi}\frac{g}{m^2})\frac{1}{m^2}\,, \\
\beta_0 &= \frac{m}{2}\frac{g}{m^2}\,, \\
\beta_2 &= -\frac{m}{16}\frac{g}{m^2}\frac{1}{m^2}\,,
\end{aligned} \tag{9}$$

and the Fig. 2 gives for c_0

$$\frac{1}{2m} - \frac{1}{32\pi m}\frac{g}{m^2}\frac{\Lambda}{m} + \frac{1}{128\pi m}\frac{g}{m^2}\frac{\Lambda^3}{m^3} + \frac{1}{m}O(\frac{g^2}{m^4}\frac{\Lambda^5}{m^5})\,, \tag{10}$$

and after resummation

$$c_0 = \frac{1}{2m} - \frac{1.88}{32\pi m}\frac{g}{m^2} \tag{11}$$

to be compared with

$$c_0 = \frac{1}{2m} - \frac{2}{32\pi m}\frac{g}{m^2}\,. \tag{12}$$

We need to go to the next order in Λ in order to be able to give a resummed value for c_2. Finally, in [6] we have shown that $\alpha_0 \neq 0$ provided $\dot{\phi}(0)|0\rangle \neq 0$.

4. Conclusion

We have shown how to compute VEVs by using a local expansion of the true VWF within the context of scalar theory in $1+1$ dimensions, where no extra counterterms are needed (this feature is also shared by Yang-Mills theory in $3+1$ dimensions). In [6] we have shown how the analyticity properties of the true vacuum in the cut-off, Λ, can be used to recover the VEV from a series in positive powers of Λ (which is the usual output of a local expansion of the VWF). If we want to use a variational ansatz for the VWF, we will have to asssume (or prove) that its short distance behaviour is such that the $\Lambda \to \infty$ is finite and, furthermore, that the analyticity behaviour of VEVs in Λ is preserved (so we can use the analytic continuation). We have given a diagrammatic approach to compute the first terms of a series in Λ for a VEV in the scalar theory, which (after our resummation) will give the

$\Lambda \to \infty$ value of the VEV. We expect that this method can be generalized to Yang-Mills theories in $3+1$ dimensions where, in the strong coupling limit, we expect to have a local VWF and therefore use the resummation to compute the VEV of a large Wilson loop.

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