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THE BALMER-LIKE FORMULA FOR MASS DISTRIBUTION OF ELEMENTARY PARTICLE RESONANCES*

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Elementary particle resonances have been systematically analyzed using all available experimental data. We have come to the conclusion that resonance decay product momenta and masses of resonances are to be quantized. The Balmer-like formula for mass distribution of elementary particle resonances has been obtained. These observations allow us to formulate a strategy of experimental searches for new resonances and systematize the already known one.

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One of the most remarkable corner-stones for foundation of quantum theory was the Balmer formula for the spectrum of a hydrogen atom. This formula was obtained phenomenologically from experimental observations. It allowed Bohr to reproduce the spectrum of hydrogen atom and his results coincide with those following from quantum mechanics. We note the well-known fact that Bohr solved the problem of quantization of a hydrogen atom in 1913 long before the creation of quantum theory. Sommerfeld generalized Bohr's results to the relativistic case. Bohr and Sommerfeld formulas are precise. Here, we want to demonstrate that the Balmer-like formula for the mass distribution of elementary particle resonances and excited states of nucleons can be obtained from a systematic analysis of all available experimental data and is based on the fundamental conservation law of energy-momentum and the Bohr–Sommerfeld quantization rule. We will consider the foregoing statements in detail [1].

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Relativistic elementary particles are classified according to the continuous unitary irreducible projective representations of the Poincaré group $\mathcal{P}$. The eigenvalues m^2 and $j(j+1)$ of invariant operators of $\mathcal{P}$ uniquely characterize these representation spaces of the Poincaré group. Thus, elementary particles are characterized by two parameters m^2 and $j(j+1)$ interpreted as mass and spin, corresponding to the generators $P_{\mathcal{M}}$ and $J_{\mathcal{M}\nu}$ of the Poincaré group. Those generators accord to momentum and angular momentum operators. However, distinctions between the notions of the momentum and angular momentum are conventional. In the space with constant curvature, for example, these notions are indistinguishable. Indeed, the translation on the sphere is also rotation. The equivalence between two types of motion is established via the constant of dimensionality of length — radius of the sphere.

Analysis of the procedure of quantizing some classical systems shows that the SU(2) group plays a fundamental role in the process of quantization. Remembering the known isomorphism between the SO(4) and SU(2) groups, which within vector–parametrization of the SO(4) group is given by:

$$T[\mathrm{SO}(4)] = \frac{(1+\hat{a}_+)(1+\hat{b}_-)}{\sqrt{(1+\vec{a}^2)(1+\vec{b}^2)}} = T_+(\vec{a})T_-(\vec{b})\,, \tag{1}$$

we can also obtain the geometrical interpretation of this process. For that purpose let us define the generators of the SO(4) group $\vec{M} = [\vec{r} \times \vec{p}]$, $\vec{N} = r_4\vec{p} - \vec{r}p_4$. Linear combinations of these orthonormal operators $\vec{\mathcal{M}}_\pm = (\vec{M} \pm \vec{N})$ form two sets of generators of the SU(2) group. Thus, the SU(2) group generates the action on the three-dimensional sphere S^3. This action consists of the translation with whirling around the direction of translation. Now as an example, let us recall the Schrödinger–Coulomb problem where the SO(4) symmetry is used to obtain the spectrum of a hydrogen atom. In that case, the operator N accepts the form of the normalized Runge–Lenz vector

$$\vec{A} = (-2mH)^{-1/2}\left(\frac{\vec{r}}{r} + (2m\alpha)^{-1}(\vec{M}\times\vec{p} - \vec{p}\times\vec{M})\right). \tag{2}$$

The eigenvalues of the Schrödinger–Coulomb problem follow directly from the invariant operator (Casimir operator) of SU(2) by writing the Hamiltonian as

$$H = \frac{1}{2(((\vec{M}\pm\vec{A}),\vec{\sigma}) + 2\hbar)^2} \to -mc^2\frac{\alpha^2}{2(n+1)^2},\ n = 0,1,2,\ldots; \tag{3}$$

these eigenvalues (including degeneracy) are given by standard group theoretical arguments.

The operator $\vec{N}$ on S^3 can be written as $\vec{N} = R\vec{p} + \vec{r}(\vec{r}\vec{p})/R$, where R is the radius of the sphere. Comparing this vector with the normalized Runge–Lenz vector we find for hydrogen atom

$$R_n = \frac{n^2}{\alpha^2 m e^2} = n^2 R_0\,. \tag{4}$$

The mass spectrum formula, which we are using here, is based on top model of elementary particles. According to this concept quantum top equations have to be formulated on S^3 [2]. So, formulation of the Hamiltonian purely in terms of the generators of the SU(2) group is achieved. We get

$$H = \frac{1}{2mR^2}\{2\hbar + (\vec{\mathcal{M}}_\pm, \vec{\sigma})\}\{2\hbar + (\vec{\mathcal{M}}_\pm, \vec{\sigma})\}\,, \tag{5}$$

where $\vec{\mathcal{M}}_\pm = (\vec{M} \pm \vec{N})$.

The spectrum of H may easily be found

$$H\Psi_n = \frac{\hbar^2}{2mR^2}(n+1)^2\;\Psi_n, \quad n = 0,1,2,\dots\,. \tag{6}$$

The discreteness of the energy spectrum is a consequence of the compactness of the group SU(2), the space of which is that space of solutions. When $R \to \infty$, the Hamiltonian tends to the Hamiltonian of the Pauli equation. In this case $\vec{\mathcal{M}}_\pm/R = (\vec{M} \pm \vec{N})/R \to \pm\vec{p}$, and $H = \!\!\to \frac{1}{2m}(\vec{p}, \vec{\sigma})^2$. This Hamiltonian at the classical level corresponds to spherical symmetrical classical top Hamiltonian on S^3: $H = J^2/2I$, where I is the moment of inertia.

Now let us generalize this concept to the relativistic case by generalizing the Pauli equation to the Dirac equation. This procedure can be displayed by the following scheme:

$$\begin{aligned}
2mH \;=\; & \frac{(\vec{\sigma}\vec{\mathcal{M}} + 2\hbar)}{R}\frac{(\vec{\sigma}\vec{\mathcal{M}} + 2\hbar)}{R} \\
& \to \left(\frac{H_D}{c} - mc\right)\left(\frac{H_D}{c} + mc\right) = (\vec{\sigma}\vec{\mathcal{M}} + 2\hbar)^2 \\
& \to \mathrm{Det}\begin{pmatrix} \frac{H_D}{c} - mc & \vec{\sigma}\vec{\mathcal{M}}_\pm + 2\hbar \\ \vec{\sigma}\vec{\mathcal{M}}_\pm + 2\hbar & \frac{H_D}{c} + mc \end{pmatrix} = 0 \\
& \to \frac{H_D}{c}\Psi_\pm = \left(\vec{\alpha}\frac{\vec{\mathcal{M}}_\pm}{R} + \beta mc + \gamma_5\,\frac{2\hbar}{R}\right)\Psi_\pm, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}
\end{aligned} \tag{7}$$

The spectrum of this Hamiltonian may be found by taking into account (6). We find the following formula for the spectrum of (7)

$$\mathcal{E} = c\sqrt{m^2c^2 + \frac{\hbar^2(n+1)^2}{R^2}}, \; n = 0,1,2,\dots\,. \tag{8}$$

From a physical point of view, it is clear that formula (8) will correspond to the properties of a classical top. Indeed, at the classical limit we obtain

$$\mathcal{E} = \frac{c}{R}\sqrt{m^2c^2R^2 + J^2}\,. \tag{9}$$

We will consider decay channels of resonances consisting of two decay products. Taking into account (7)–(8) we suggest the following operator for the mass spectrum formula for resonances

$$M(R) = \sqrt{m_a^2 + \frac{1}{R^2}((\vec{\sigma}\vec{\mathcal{M}}) + 2\hbar)^2} + \sqrt{m_b^2 + \frac{1}{R^2}((\vec{\sigma}\vec{\mathcal{M}}) + 2\hbar)^2}\,, \tag{10}$$

where m_a and m_b are masses of decay products of the resonance, R is the characteristic length of the group of resonances. The spectrum of this operator is obviously given by

$$M_n = \sqrt{m_a^2 + P_n^2} + \sqrt{m_b^2 + P_n^2}\,, \tag{11}$$

where $P_n = nP_1,\ P_1 = \hbar/R$.

Some of the resonances have a dominant decay channel, and we suppose that the momentum of this channel must manifest itself in decays properties through other channels. Let us consider a few dominant decay channels of the resonances: $\Xi^- \to \Lambda\pi^-$ with fraction 99.887 $\pm$0.035%, $\Xi(1530)^0 \to \Xi^-\pi^+$ with fraction 100%, $\Sigma^0 \to \Lambda\gamma$ with fraction 100%, $\Sigma^- \to n\pi^-$ with fraction 99.848 ± 0.005%.

The masses of heavier resonances were calculated by formula (11) where m_1, m_2 and P_1 are the masses and momenta of decay products from one of the dominant channels cited above. The results of our calculations and the corresponding experimental data [3] are illustrated in Fig. 1. Here we presented only the fragment of our calculations. More complete analysis you can find in the paper [4]. The X-axis characterizes the families of resonances (baryonic or mesonic) and the Y-axis represents their masses (in MeV). The figure shows that momenta P_1 to be proposed generate the families of resonances with different quantum numbers. We think that the results given in the figure convincingly demonstrate the empirical fact that resonance decay product momenta and their masses are quantized independently on the type of interaction between resonance decay products, quantum numbers of resonances, and the type of particles. There arises an excellent possibility of predicting new resonances and verifying masses of the existing ones.

It seems [1] that a system of resonators corresponds to stable systems (like proton), when all the frequencies of a system of resonators are ideally coordinated and equal to each other or are commensurable. In other words, all

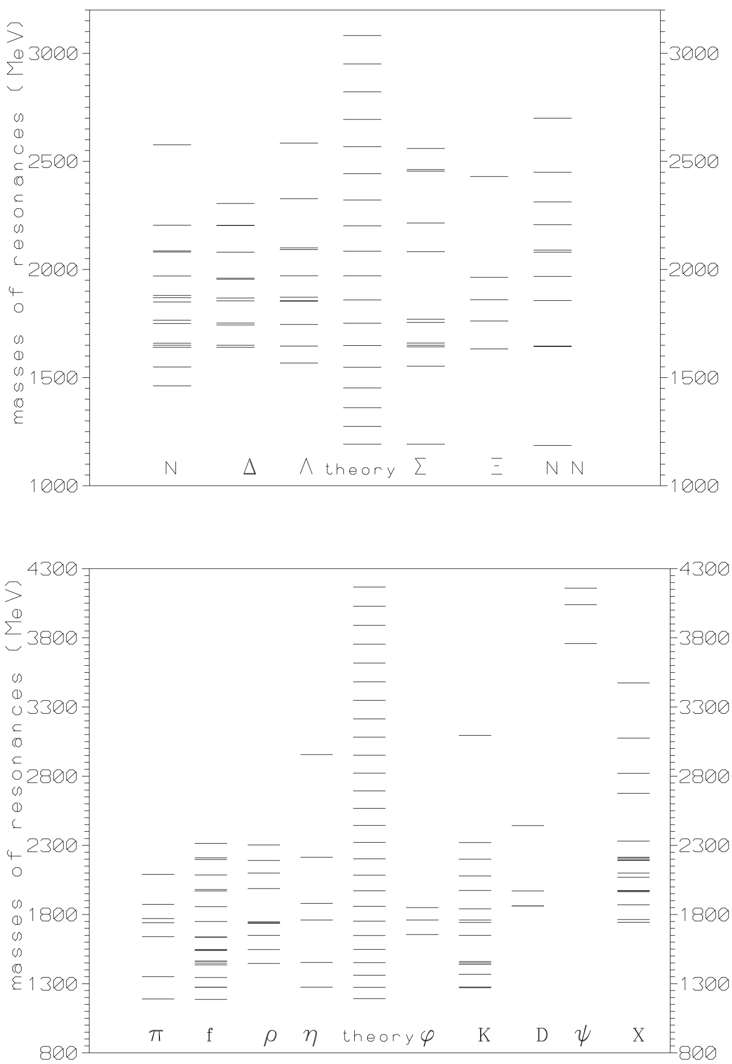


Fig. 1. The mass distribution of baryonic and mesonic resonances with momenta multiples of 74.39 MeV/c. The basic momentum was taken from the channel $\Sigma^0 \rightarrow \Lambda\gamma$.

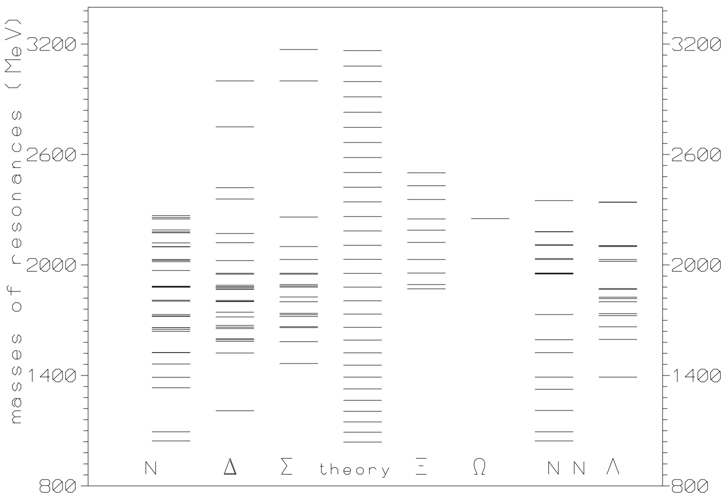
channel motions in stable systems are exactly synchronous. The same conclusion was obtained by Gryzinski [5] that the atom can remain in stationary state if motion in the whole atom is perfectly synchronic. Let us consider how our approach allows one to investigate internal structure of nucleons. The excited states of nucleons have been investigated in a large number of experiments. The main information about the masses, widths and elasticities for N and Δ resonances comes from the partial-wave analysis of $N\pi$, $N\eta$, ΛK and ΣK data sets (for details see [3]). We would like to propose that the properties of the ground state of a nucleon can be studied from experiments aimed at extracting information about excited states of nucleons. The proton is practically stable (mean lifetime $\tau > 1.6 * 10^{25}$ years — independently of the decay mode [3]). It can decay via different channels. For example, $p \to e^+\pi^0$, $p \to \mu^+\gamma$, $p \to \nu K^*(892)^+$, $p \to e^+K^*(892)^0$... But the proton does not decay despite the possibility from the energy-momentum conservation law. We decided to investigate some of these channels, for example, $p \to \nu K^*(892)^+$ and $p \to e^+K^*(892)^0$. The masses of p, ν, $K^*(892)^+$, e^+ and $K^*(892)^0$ are known. So we are able to calculate the decay momentum P_1 and then to evaluate the masses of excited states of a proton using formula (11) where m_1, m_2 are the masses of hypothetical decay particles (ν, $K^*(892)^+$, e^+ and $K^*(892)^0$) and P_1 is the momentum of their relative motion. The results of our calculations and the corresponding experimental data [3] are illustrated in Fig. 2. The X-axis characterizes the families of resonances and the Y-axis represents their masses (in MeV). The figure shows that momenta P_1 to be proposed also generate the families of resonances with different quantum numbers and confirms our conclusion that resonance decay product momenta and their masses are quantized.

The well-known results in high-energy physics indicate that there is a profound connection between spins and masses of strongly interacting elementary particles, hadrons. The spin J of some baryons and mesons appears to be nearly proportional to the square of their mass M: $M^2 \propto J$. The correlation between spin and mass of experimentally known low mass hadrons is represented by a straightline Regge trajectory. The general formula, which connects the maximal spin J and mass M of heavy hadrons, was obtained in [6] by using simple arguments of dimensional analysis and similarity principle

$$J = \hbar (M/m_p)^{1+1/\zeta}\,, \tag{12}$$

where m_p is the proton mass, the number ζ takes values ζ=1, 2, 3 and characterizes the spatial dimensionality of hadrons.

The plot [6] $\lg J$ versus $\lg M$ for astrophysical bodies shows a remarkable regularity, and the theoretical lines describe not only the shape, but also absolute values in tremendous mass and spin intervals (the mass interval is about 34 orders of magnitude, the corresponding interval for angular momenta covers about 60 orders of magnitude) without invoking arbitrary parameters. Therefore, Muradian's approach incorporates in a natural way fundamental quantum mechanical constants $\hbar$ and m_p. It is worthwhile to note that the Regge-like trajectories can be obtained from (11) by using the Bohr–Sommerfeld quantization conditions $Pr = n\hbar \equiv J$ and the assumption that the masses of a ζ-dimensional rotational object can be written in



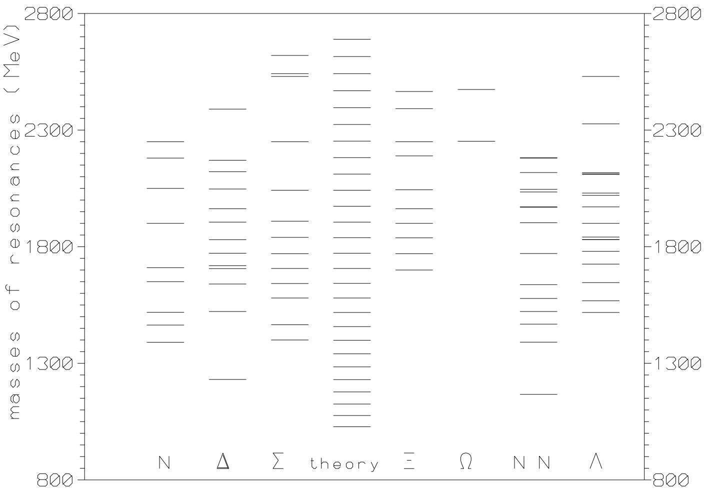


Fig. 2. The mass distribution of baryonic resonances with momenta multiples of 45.52 MeV/c (top Fig.) and 41.22 MeV/c (bottom Fig.). The basic momentum was taken from the hypothetical channel of a proton $p \to \nu K^*(892)^+$ (top Fig.) and $p \to e^+ K^*(892)^0$ (bottom Fig.)

the form $m = \rho r^\zeta$, where ρ is the constant density of the object. Therefore, equation (11) acquires the form

$$M = \sqrt{m^2 + P^2/c^2} = \sqrt{(\rho r^\zeta)^2 + J^2/c^2 r^2}\,. \tag{13}$$

The minimization of this expression over r provides Regge-like trajectories. These results are remarkable because the observed and well-established

Regge-like trajectories in micro- and macrosystems were obtained from first principles.

In conclusion, we would like to say that the above presented method is able to describe the existing experimental data with high accuracy. It was clearly demonstrated that information about the inner structure of a nucleon can be extracted using experimental data for excited nucleon states. An excellent possibility of predicting new resonances and verifying masses of existing ones arises in any case. So, we have established the Balmer-like parameter-free formula for masses of elementary particle resonances in accordance with the systematic analysis of experimental data. Interest in our results is not only in their closeness to the experimental data, but also in the derivation of formula (11) from the two invariants: the conservation law of energy-momentum and the Ehrenfest adiabatic invariant (Bohr–Sommerfeld quantization rule).

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