Vol. 29 (1998) ACTA PHYSICA POLONICA B No 9

RELATIVITY DAMPS OPEP IN NUCLEAR MATTER* **

Manoj K. Banerjee

Department of Physics, University of Maryland
College Park, MD, USA, 20742-4111
e-mail: banerjee@quark.umd.edu

(Received June 15, 1998)

Using a relativistic Dirac–Brueckner analysis the OPEP contribution to the ground state energy of nuclear matter is studied. In the study the pion is derivative-coupled. We find that the role of the tensor force in the saturation mechanism is substantially reduced compared to its dominant role in a usual nonrelativistic treatment. We show that the damping of derivative-coupled OPEP is actually due to the decrease of M^*/M with increasing density. We point out that if derivative-coupled OPEP is the preferred form of nuclear effective Lagrangian nonrelativistic treatment of nuclear matter is in trouble. Lacking the notion of M^* it cannot replicate the damping. We suggest an examination of the feasibility of using pseudoscalar coupled πN interaction before reaching a final conclusion about nonrelativistic treatment of nuclear matter.

PACS numbers: 12.39.Pn, 11.10.Ef

The one pion exchange potential, OPEP, in momentum space is given by the expression:

$$\begin{aligned}&\left\langle \tfrac{1}{2}\vec{P}+\vec{p}\,', \tfrac{1}{2}\vec{P}-\vec{p}\,' \mid v \mid \tfrac{1}{2}\vec{P}+\vec{p}, \tfrac{1}{2}\vec{P}-\vec{p}\right\rangle \\ &= -\left(\frac{g_{\pi NN} m_\pi}{2M}\right)^2 \frac{\vec{\sigma}_1\cdot(\vec{p}\,'-\vec{p})\vec{\sigma}_2\cdot(\vec{p}\,'-\vec{p})}{(\vec{p}\,'-\vec{p})^2+m_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2\,.\end{aligned}$$

* Presented at the NATO Advanced Research Workshop, Cracow, Poland, May 26–30, 1998. The talk is based on a recent paper written by Tjon and myself [1].
** In honor of Josef Speth's 60th birthday.

With $\frac{1}{4\pi}\left(\frac{g_{\pi NN} m_\pi}{2M}\right)^2 \simeq 11\,\mathrm{MeV}$ the OPEP in coordinate space is given by the expression:

$$\begin{aligned} v_\pi(\vec{r}) &= 11\mathrm{Mev}\left\{-4\pi\frac{\vec{\tau}_1\cdot\vec{\tau}_2}{3}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\frac{\delta(\vec{r})}{m_\pi^2}\right. \\ &+ \frac{\vec{\tau}_1\cdot\vec{\tau}_2}{3}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\frac{\mathrm{e}^{-m_\pi r}}{r} \\ &+ \left.\frac{\vec{\tau}_1\cdot\vec{\tau}_2}{3}\,S_{12}\,\frac{\mathrm{e}^{-m_\pi r}}{r}\left(1+\frac{3}{m_\pi r}+\frac{3}{m_\pi^2 r^2}\right)\right\}, \end{aligned}$$

where the last term is the tensor force and

$$S_{12} = 3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

is the tensor operator. In a Hartree–Fock calculation in nuclear matter the $\delta(\vec{r})$ potential is capable of contributing at saturation density, $\rho_0 \simeq \frac{1}{2}m_\pi^3$, as much as $\sim$ 20MeV. But we must ignore it as it will be wiped out by the short-range correlation. The Yukawa potential and the tensor force contribute a mere $\simeq -2$MeV. Really important contributions of the tensor force come from second and higher orders. The large matrix element, $\langle {}^3D_1 \mid S_{12} \mid^3 S_1\rangle = \sqrt{8}$, shown in the matrix below,

$$S_{12}^2 = 8 - 2S_{12}.$$

	$\mid^3 S_1\rangle$	$\mid^3 D_1\rangle$
$\langle^3 S_1 \mid$	0	$\sqrt{8}$
$\langle^3 D_1 \mid$	$\sqrt{8}$	-2

is responsible for this feature.

The effect of the tensor force and its dominance in nonrelativistic nuclear physics are seen most dramatically from the following results for the deuteron [2]

$$\langle \mathrm{Deuteron} \mid V_{\mathrm{central}} \mid \mathrm{Deuteron}\rangle \sim 0\,,$$

$$2\langle^3 D_1 \mid V_{\mathrm{tensor}} \mid^3 S_1\rangle \sim -22\,\mathrm{MeV}\,,$$

$$\langle \mathrm{Deuteron} \mid \frac{\vec{p}^2}{2M} \mid \mathrm{Deuteron}\rangle \sim +20\,\mathrm{MeV}\,.$$

In a nonrelativistic Bethe–Brueckner calculation of nuclear matter one finds typically [3]

$$\langle \mathrm{N\,Matter} \mid V_\pi \mid \mathrm{N\,Matter}\rangle_{\mathrm{nonrelativistic}} \sim -34\,(\rho/\rho_0)^{0.45}\,\mathrm{MeV}\,. \tag{1}$$

The exponent of the density ρ is markedly less than the nominally expected value of 1 because of Pauli blocking.

Relativistic nuclear physics is heavily based on summing Bethe–Salpeter ladders using some form of Blankenbecler–Sugar–Logunov–Tavkhelidze [5] prescription to obtain quasipotentials. In general quasipotentials are not simply described by tree graphs with dressed vertices. The OPEP is an exception. Other terms have the characteristics of four-point functions. In practice, one uses OBEP forms with form factors for approximate representation of quasipotentials. The parameters are fixed by fitting NN data.

The relativistic results for the contribution of V_π to the deuteron [6] is

$$\langle \mathrm{Deuteron} \mid V_\pi \mid \mathrm{Deuteron} \rangle_{\mathrm{relativistic}} = -22\,\mathrm{MeV},$$

suggesting an equally important role of the pion. In sharp contrast, this seems not to be the case in a relativistic treatment of nuclear matter.

Strong scalar (S) and vector (V) fields of the order of a few hundred MeV are typical for relativistic theories [7–9] based on a meson theoretical description of the nuclear force. These values are consistent with expectations based on the studies of scattering of ~ 1 GeV protons by nuclei. The large scalar fields have far reaching consequences in nuclear matter through the strongly medium modified nucleon mass $M^* = M + S$. The saturation mechanism is believed to rest upon the decrease of magnitude of S with increasing density. Of course, in a Mean Field Theory (MFT) like the QHD [10] it is the only possible mechanism for saturation.

The contributions of a particular meson exchange potential, V_α, to the binding energy can be calculated using the Hellmann–Feynman theorem

$$\langle \mathrm{N\,Matter} \mid V_\alpha \mid \mathrm{N\,Matter} \rangle = g^2_{\alpha NN} \frac{\partial}{\partial\, g^2_{\alpha NN}} (E/A). \qquad (2)$$

We find the following for the pion field contribution to E/A:

$$\langle \mathrm{N\,Matter} \mid V_\pi \mid \mathrm{N\,Matter} \rangle_{\mathrm{relativistic}} \sim -20\,(\rho/\rho_0)^{0.16}\,\mathrm{MeV}. \qquad (3)$$

From this we see that the pion contribution is considerably suppressed compared to the value given by Eq. (1) for the nonrelativistic case. We will make clear that the suppression of OPEP is generic and not particular to the present calculation. Furthermore, OPEP has only a minor role in the saturation mechanism. This is exhibited in Fig. 1 where we plot our calculated results of E/A (curve a) and $E/A - \langle NM \mid v_\pi \mid NM \rangle - 17\,\mathrm{MeV}$ (curve b). The two curves have practically the same density dependence verifying that OPEP contributes little to the saturation mechanism. The subtraction of 17 MeV in curve b makes the scale more compact.

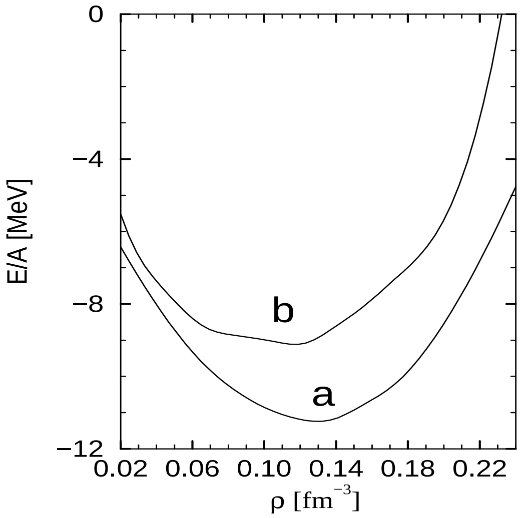

Fig. 1. Plots of the Dirac–Brueckner predictions of E/A (curve a) and $E/A - \langle NM \mid v_\pi \mid NM \rangle - 17\,\mathrm{MeV}$ (curve b).

The preceding numbers confirm the belief that in a relativistic treatment of nuclear matter the tensor force does not have the dominant role that it has in the usual nonrelativistic treatment. The radically different explanations of the saturation mechanism in nonrelativistic and relativistic studies of nuclear matter constitute a puzzling issue. A valid nonrelativistic treatment must reproduce the main physics of a valid relativistic treatment in leading order in v/c. Although the issue is a longstanding one, no resolution of it has been given to date. We have addressed this question. A Dirac–Brueckner (D–B) analysis [7, 8] is at present the best tool we have for a relativistic study of nuclear matter. We examine here the role of derivative-coupled OPEP in D–B and show that it is substantially reduced due to relativity. Since the contribution of OPEP to the deuteron binding energy remains large in a relativistic treatment the damping in nuclear matter must be due to many-body effects. We find that it can be attributed to the decrease of M^*/M with increasing density.

The above results can be understood qualitatively by examining the second order contributions to the G-matrix. Keeping only the positive energy M^* state contributions in the intermediate states we have,

$$\langle \vec{p}' \mid G(P) \mid \vec{p} \rangle = \langle \vec{p}' \mid V(P) \mid \vec{p} \rangle + \sum_{\lambda,i} \int \frac{d^3k}{(2\pi)^3}$$
$$\times \langle \vec{p}' \mid V(P) \mid \vec{k} \rangle \frac{1}{M^*} \frac{Q_{\mathrm{Pauli}}}{(\vec{p}/M^*)^2 - (\vec{k}/M^*)^2 + \Delta/M^*} \langle \vec{k} \mid V(P) \mid \vec{p} \rangle . \quad (4)$$

The tensor force contributes mainly to the second term of Eq. (4) which makes the structure of the two-nucleon propagator important. Normally the boson masses in OBEP provide the scales for momenta in nuclear physics. But here we notice that the two-nucleon propagator provides a new scale, *viz.*, M^*. To exploit this new scale let us introduce dimensionless momenta, $\vec{\ell} = \vec{p}/M^*$, $\vec{n} = \vec{k}/M^*$, *etc.*, and exhibit a few OBEP matrix elements in terms of these.

σ Exchange

$$\begin{aligned}\langle \vec{p} \mid v_\sigma \mid \vec{k} \rangle &= -\frac{g_\sigma^2}{(\vec{p}-\vec{k})^2 + m_\sigma^2} \\ &= -\frac{1}{M^{*\,2}} \frac{g_\sigma^2}{(\vec{\ell}-\vec{n})^2 + (m_\sigma/M^*)^2}. \end{aligned} \tag{5}$$

π Exchange (Derivative Coupling)

$$\begin{aligned}\langle \vec{p} \mid v_\pi^{dc} \mid \vec{k} \rangle &= (\frac{g_\pi}{2M})^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot (\vec{k}-\vec{p}) \vec{\sigma}_2 \cdot (\vec{k}-\vec{p})}{(\vec{p}-\vec{k})^2 + m_\pi^2} \\ &= (\frac{M^*}{M})^2 \frac{1}{M^{*\,2}} (\frac{g_\pi}{2})^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot (\vec{\ell}-\vec{n}) \vec{\sigma}_2 \cdot (\vec{\ell}-\vec{n})}{(\vec{\ell}-\vec{n})^2 + (m_\pi/M^*)^2}. \end{aligned} \tag{6}$$

π Exchange (Pseudoscalar Coupling)

$$\begin{aligned}\langle \vec{p} \mid v_\pi^{\gamma_5} \mid \vec{k} \rangle &= (\frac{g_\pi}{2M^*})^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot (\vec{k}-\vec{p}) \vec{\sigma}_2 \cdot (\vec{k}-\vec{p})}{(\vec{p}-\vec{k})^2 + m_\pi^2} \\ &= \frac{1}{M^{*\,2}} (\frac{g_\pi}{2})^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{\vec{\sigma}_1 \cdot (\vec{\ell}-\vec{n}) \vec{\sigma}_2 \cdot (\vec{\ell}-\vec{n})}{(\vec{\ell}-\vec{n})^2 + (m_\pi/M^*)^2}. \end{aligned} \tag{7}$$

Notice the differences in the M^* factors. The derivative-coupled pion-exchange potential has an extra damping factor of $(M^*/M)^2$ relative to the sigma-exchange potential. It is reasonable to expect that the M^*/M factor suppresses derivative-coupled OPEP, Pauli coupled ρ, *etc.*

No such damping factor is present for the pseudoscalar coupled pion-exchange potential. To our knowledge all published relativistic nuclear matter calculations have used derivative-coupling. Of course, the reason is well-known. The pair suppression problem is automatically taken care of with

TABLE I

Table of M^* factors for various OBEP potentials. The number of stars in the 2nd column indicates the importance of the OBEP in nuclear interaction.

Parity	Importance	Boson	Scaling Factor
E	* * *	σ	$(1/M^*)^2$
V	* * *	ω Dirac	$(1/M^*)^2$
E		ρ Dirac	$(1/M^*)^2$
N		δ	$(1/M^*)^2$
O	* *	π Der. Coupled	$(M^*/M)^2(1/M^*)^2$
D	*	ρ Pauli	$(M^*/M)^2(1/M^*)^2$
D		η Der. Coupled	$(M^*/M)^2(1/M^*)^2$
	-*	ω Pauli	$(M^*/M)^2(1/M^*)^2$

use of derivative-coupling. We will proceed as if derivative-coupling is correct. A discussion of pseudoscalar coupling *vs.* derivative-coupling follows at the end of the talk.

The M^*/M suppression is corroborated in more detail by the following calculation. Let us modify the S obtained from the self-consistent D–B calculation by multiplying it with the factor $\alpha \leq 1$ thus generating a $M^* = M + \alpha S$. By using the modified scalar self-energy in the nucleon propagators we recalculate first the G-matrices and then E/A and finally $\langle \mathrm{N\,Matter} \mid V_\pi \mid \mathrm{N\,Matter}\rangle$ using Eq. (2). Only the $\alpha = 1$ analysis is self consistent; others are not. But such a calculation is particularly suitable to exhibit the role of M^*/M on the OPEP contribution. Figure 2 exhibits clearly the damping due to decreasing M^*/M. We stress that the mechanism of damping is generic to any relativistic treatment using derivative-coupled pion and not particular to either Ref. [8] or the use of Ref. [5].

We want to be careful that the present work not be interpreted as providing support for MFT. Results of calculations of E/A using the same interaction, namely, that of Ref. [8], in both D–B and MFT treatments are shown in Fig. 3. We see that the results are distinctly different. Such differences are found in the results for scalar and vector fields in the two treatments. Results obtained upon using the intearction of Ref. [8] are listed below.

$$\begin{aligned}
S_{\mathrm{D-B}} &= -306\,(\rho/\rho_0)^{0.81}\,\mathrm{MeV},\\
V_{\mathrm{D-B}} &= 233\,(\rho/\rho_0)^{0.97}\,\mathrm{MeV},\\
S_{\mathrm{MFT}} &= -358\,(\rho/\rho_0)^{0.92}\,\mathrm{MeV},\\
V_{\mathrm{MFT}} &= 295\,(\rho/\rho_0)\,\mathrm{MeV}\,.
\end{aligned}$$

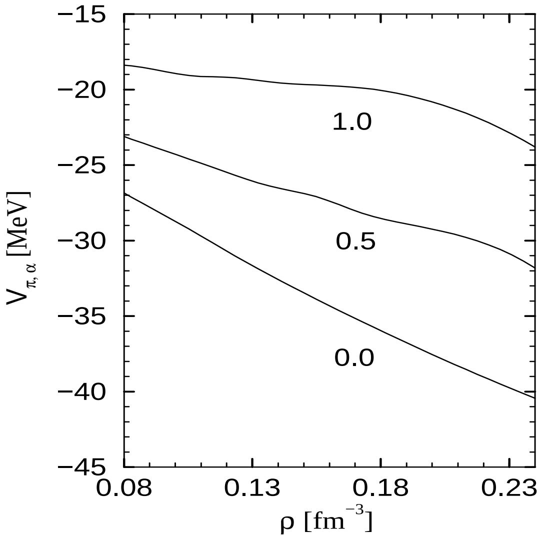

Fig. 2. Plots of $\langle NM \mid V_\pi \mid NM \rangle$ with the parameters of Ref. [8]. In the G-matrix calculations S is replaced with αS. The plots are for $\alpha = 0.$, 0.5 and 1.0. The last one is the result of a D–B self-consistent calculation. The other two are not self consistent.

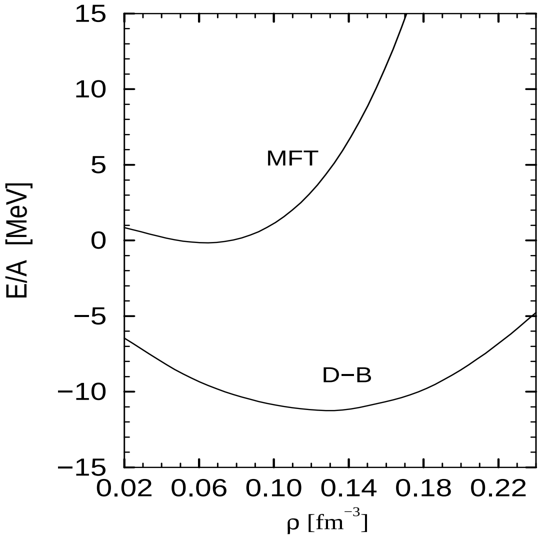

Fig. 3. Plots of E/A from a Dirac–Brueckner and a MFT calculation with the same quasipotential.

Undoubtedly, if one releases oneself from the constraint of fitting NN data and freely chooses the NN interaction one can obtain proper binding and saturation of nuclear matter with a MFT calculation.The results presented in this talk are the first explicit calculations showing that in the relativistic treatment the tensor force contributions generated by derivative-coupled OPEP[1] are reduced in size in nuclear matter. Because of this, in complete contrast to the nonrelativistic situation, they cease to play an essential role in the saturation mechanism. The reduction of the tensor force contributions is principally due to the relativistic M^*/M effect. But even the reduced role of OPEP is not negligible in the actual saturation properties of nuclear matter. As noted, it contributes -20MeV to E/A. The dominant mechanism of saturation of nuclear matter is basically very different in the two approaches. In the nonrelativistic approach it is the density-dependent reduction due to Pauli blocking of the attraction from tensor force, while in the relativistic approach it is the reduction of the rate of growth with increasing ρ of the attraction from the scalar field relative to the growth of repulsion from the vector field.

Finally, let us discuss the issue of derivative versus pseudoscalar πN coupling. If the former is the correct coupling for nuclear effective Lagrangian then nonrelativistic treatment of nuclear matter appears not to be valid. On the other hand, if a pseudoscalar coupling Lagrangian can be found which gives satisfactory results for nuclear matter the nonrelativistic treatment may be valid. Unfortunately there are no published results for nuclear matter with a pseudoscalar coupling Lagrangian. Needless to say, before doing any nuclear matter calculation the parameters must be fixed by fitting NN data.

It is useful to remind ourselves that we have been dealing with quasipotentials. These are still constrained to be chiral invariant. The interaction Lagrangians with which one could reproduce the quasipotentials via tree graphs with form factors are listed below.

$$\mathcal{L}_{\pi N}^{\text{derivative coupling}} = \frac{g_\pi}{2M}\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi\cdot\partial_\mu\vec{\pi} + g_\sigma\bar{\psi}\sigma\psi. \tag{8}$$

$$\mathcal{L}_{\pi N}^{\gamma_5\text{ coupling}} = g_\pi\bar{\psi}[\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}]\psi. \tag{9}$$

In the derivative-coupling Lagrangian the nucleon fields are unaffected by chiral transformation and the scalar field σ is a chiral singlet. In the pseudoscalar coupling Lagrangian the nucleon fields, ψ and $\bar{\psi}$, belong to chiral $(1/2,0),(0,1/2)$ representations, while σ and $\vec{\pi}$ form chiral $(1/2,1/2)$ representations. In Table II we list the parameters for two derivative-coupling Lagrangians, namely, Amorim–Tjon [8] and Bonn C [9]. The last line gives

[1] Pauli ρ tensor force is also damped.

TABLE II

Amorim–Tjon and Bonn C parameters for derivative-coupling and standard pseudoscalar coupling parameters. The value of m_σ is not specified.

Coupling	Source	Isospin	m_σ in MeV	g_σ	m_π in MeV	g_π
Derivative	Amorim-Tjon [8]		570	7.6	138	14.2
	Bonn C [9]	1	550	8.6	138	14.2
		0	720	17.6	138	14.2
Pseudoscalar			?	?	138	14.2

the standard pseudoscalar coupling Lagrangian parameters. Notice that both the mass and the coupling constant of the σ meson have been left unspecified. The reason is that, quite unlike OPEP, there will be considerable modification of the one-σ exchange potential as one goes from the form given by the original Lagrangian to the quasipotential. This happens through two distinct mechanisms. First, the σ couples to the pion clouds of each of the pair of interacting nucleons. Second, a pair of interacting pions are exchanged between two nucleons. The interaction must be isovector in the two-pion t-channnel[2]. The prospect of a pseudoscalar Lagrangian succeeding in the nuclear matter problem is not very good. The undamped tensor force will contribute an additional $\sim -15\,\mathrm{MeV}$. To compensate this the effective σ nucleon coupling in the quasipotential must decrease. It is difficult to see how such a reduction can come about. Still, the only recourse is to actually carry out the program of study with a pseudoscalar Lagrangian before we come to a definitive conclusion about the future nonrelativistic treatment of nuclear matter.

This research is supported by the U.S. Dept. of Energy under grant No. DE-FG02-93ER-40762.

REFERENCES

[1] M.K. Banerjee, J.A. Tjon, The Role of Tensor Force in Nuclear Matter Saturation, DOE/ER/40762-137, UMD PP#98-056, nucl-th/9711029.

[2] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, *Phys. Rev.* **C51**, 38 (1995).

[3] This result was generated with a Brueckner theory program of Gruemmer (Jülich) using the Bonn C potential described in Ref. [9].

[2] For example, by exchanging a ρ meson.

[4] E.E. Salpeter, H.A. Bethe, *Phys. Rev.* **84**, 1232 (1951).

[5] R. Blankenbecler, R. Sugar, *Phys. Rev.* **142**, 1051 (1966); A.A. Logunov, A.N. Tavkhelidze, *Nuovo Cim.* **29**, 380 (1963). *Annu. Rev. Nucl. Sci.* **21**, 93 (1971).

[6] E. Hummel, J.A. Tjon, *Phys. Rev.* **C49**, 21 (1994).

[7] B. ter Haar, R. Malfliet, *Phys. Rep.* **149**, 207 (1987).

[8] A. Amorim, J.A. Tjon, *Phys. Rev. Lett.*, **68**, 772 (1992).

[9] R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989).

[10] B.D. Serot, J.D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986).

[11] M.K. Banerjee, J.A. Tjon, *Phys. Rev.* **C56**, 497 (1997).

[12] C. Horowitz, B.D. Serot, *Nucl. Phys.* **A464** (1987) 613.

[13] B.C. Clark, R.L. Mercer, P. Schwandt, *Phys. Lett.* **122B** 211 (1983).

[14] J.A. McNeil, J.R. Shepard, S.J. Wallace, *Phys. Rev. Lett.*, **50**, 1439 (1983); **50**, 1443 (1983).

[15] N. Ottenstein, S.J. Wallace, J.A. Tjon, *Phys. Rev.* **C38**, 2272 (1988).