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DEEP INELASTIC SCATTERING AND THE PION LIGHT-CONE WAVE FUNCTION *

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Light-cone QCD sum rules are used to calculate the quark distribution function for the pion and place a new constraint on the pion twist-2 wave function. When combined with information available from analysis of other experimental data, we conclude that the wave function nearly achieves its asymptotic form.

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1. Introduction

This talk reports on an investigation of the pion wave function within the framework of light-cone QCD sum rules, with special emphasis on the twist-2 component of this wave function and its relationship to the quark distribution function of the pion [1].

In the light-cone QCD sum rule approach, the pion is characterized in terms of transition matrix elements of nonlocal quark and gluon field operators sandwiched between the vacuum and the state $|\pi(k)\rangle$ in which a pion has momentum k near the light cone $k^2 = 0$. The twist-2 pion wave function $\varphi_\pi(v)$ appears in the twist expansion of the matrix element

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$$\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|\pi(k)\rangle = ik_\mu f_\pi \int_0^1 dv \mathrm{e}^{-i(kx)v}\left(\varphi_\pi(v) + x^2 g_1(v) + O(x^4)\right)$$
$$+ f_\pi\left(x_\mu - \frac{x^2}{kx}k_\mu\right)\int_0^1 du \mathrm{e}^{-i(kx)v}(g_2(v) + O(x^2))\,, \tag{1}$$

where the quantities $g_1(v)$ and $g_2(v)$ are twist-4 pion wave functions. The three two-particle wave functions appearing in Eq. (1) are only a few of those needed to completely specify the pion, with the remainder characterizing other quark and gluon field configurations. The twist-2 wave function makes the leading contribution to the left-hand side of the QCD sum rule in many applications and has therefore been the object of much theoretical interest.

The first few moments of the function $\varphi_\pi(v)$ have been calculated by Chernyak and Zhitnitsky [2] using the conventional QCD sum rule expansion of Shifman, Vainshtein, and Zakharov [3], and from these it was suggested that $\varphi_\pi(v)$ have a quadratic "double hump" dependence on the variable u. Subsequent work [4] based on nonlocal condensates challenged this conclusion, suggesting rather that the shape of $\varphi_\pi(v)$ is closer to asymptotic, *i.e.*, $6u(1-u)$. In the meantime, there have been several attempts to determine $\varphi_\pi(u)$ from experiment using light-cone QCD sum rules: the calculation of the pion-nucleon coupling constant [5] and the analysis of $\gamma\gamma^* \to \pi$ [6]. We will use these empirical constraints, along with our new constraint $\varphi_\pi(0.3) = 1.0 \pm 0.2$ [1] as discussed in Sect. 2, to determine an empirical picture of $\varphi_\pi(v)$. The result of our constrained analysis, described in Sect. 3, is that $\varphi_\pi(v)$ is asymptotic to within experimental errors.

2. Light-cone sum rule for the quark distribution function of the pion

As in [1], we consider the correlator

$$T_{\mu\rho\lambda}(p,q,k) = -i\int d^4x d^4z \mathrm{e}^{ipx+iqz}\langle 0|T\{j^5_\mu(x), j^d_\rho(z), j^d_\lambda(0)\}|\pi^-(k)\rangle \tag{2}$$

for our calculation of the pion structure function. Here k is the pion momentum,

$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 d, \quad j^d_{\rho,\lambda} = \bar{d}\gamma_{\rho,\lambda}d, \tag{3}$$

and the following kinematics is used:

$$\begin{aligned} k^2 &= 0; \quad q^2 = (p+q-k)^2; \quad t = (p-k)^2 = 0; \quad s = (p+q)^2\,; \\ Q^2 &= -q^2; \quad (2k, p+q) = s + Q^2; \quad (2pk) = p^2\,. \end{aligned} \tag{4}$$

The discontinuity in s at fixed p^2 and Q^2 of the correlator of Eq. (2) is calculated from

$$\mathrm{Im}T_{\mu\rho\lambda} = \frac{1}{2i}\left[T_{\mu\rho\lambda}(p^2, q^2, s+i\varepsilon) - T_{\mu\rho\lambda}(p^2, q^2, s-i\varepsilon)\right], \tag{5}$$

where p^2 and q^2 are space-like vectors, $p^2 < 0$, $q^2 < 0$, such that $|p^2|, |q^2| \gg \Lambda^2_{\mathrm{QCD}}$. In the scaling limit, we assume that $|p^2| \ll |q^2|$ and keep only the first nonvanishing terms in an expansion in powers of p^2/q^2.

The connection to the deep inelastic structure function is possible because the optical theorem relates the hadronic tensor $W_{\mu\nu}$ to the virtual Compton amplitude that appears when the pion contribution to $\mathrm{Im}T_{\mu\rho\lambda}$ is calculated in the physical region of the s-channel,

$$\mathrm{Im}T_{\mu\rho\lambda} = p_\mu \frac{f_\pi}{p^2}\mathrm{Im}\left\{ i\int d^4z e^{iqz}\langle\pi(p)|T\{j^d_\rho(z), j^d_\lambda(0)\}|\pi(k)\rangle\right\}. \tag{6}$$

In this fashion we find

$$\lim_{t\to 0}\mathrm{Im}T_{\mu\rho\lambda} = -4\pi\frac{x_{\mathrm{B}}^2}{Q^2}\left(p - \frac{pq}{q^2}q\right)_\rho \left(p - \frac{pq}{q^2}q\right)_\lambda t(p^2, x_{\mathrm{B}}) + \ldots\,, \tag{7}$$

where

$$t(p^2, x_{\mathrm{B}}) = \left(\frac{q^d(x_{\mathrm{B}})}{p^2} + \int \frac{\rho(s, x_{\mathrm{B}})}{s - p^2}ds\right). \tag{8}$$

Here $q^d(x_{\mathrm{B}})$ is d-quark distribution function of a pion. This constitutes the right-hand side of the QCD sum rule. We will compare terms on the right- and left-hand sides having the tensor structure $p_\mu p_\rho p_\lambda$.

For the calculation of the left-hand side of the QCD sum rule, it is important to avoid the boundaries at $x_{\mathrm{B}} = 0$ and $x_{\mathrm{B}} = 1$ [7] in order that the amplitude is determined by small distances in the t channel. It is also important to avoid expansion in p/k, which is naturally accomplished by making use of the light-cone QCD sum rule.

Making the operator-product expansion of the correlator in Eq. (2), the result of a very simple calculation with free d-quark propagators in Eq. (1) gives

$$\begin{aligned} T_{\mu\rho\lambda}(p,q,k) &= i\int d^4x d^4z e^{ipx+iqz}\frac{(x-z)_\alpha z_\beta}{4\pi^4(x-z)^4 z^4} \\ &\quad\times\langle 0|\bar{u}(x)\gamma_\mu\gamma_5\gamma_\alpha\gamma_\rho\gamma_\beta\gamma_\lambda d(0)|\pi(k)\rangle\,. \end{aligned} \tag{9}$$

From this, we find the relevant tensor for the twist-2 contribution ($p^2 \to \infty$) to be

$$T_{\mu\rho\lambda} = -4 f_\pi p_\mu p_\rho p_\lambda \int_0^1 \frac{(1-v)\varphi_\pi(v)}{p^2(s-(s+Q^2)v)} dv, \tag{10}$$

which gives the twist-2 contributions to the left-hand side of $t_2(p^2, x_\mathrm{B}) = \varphi_\pi(v)/p^2$.

The twist-4 contributions from the two-particle sector (see Eq. (1)) arise in a similar fashion and give the twist-4 quark contribution to the left-hand side to be

$$\begin{aligned} t_4^q(p^2, x_\mathrm{B}) &= \frac{4}{p^4}\left(\frac{g_1(x_\mathrm{B}) + G_2(x_\mathrm{B})}{x_\mathrm{B}} + \frac{1}{2} g_2(x_\mathrm{B}) - \frac{d g_1(x_\mathrm{B})}{d x_\mathrm{B}}\right) \\ &= \frac{1}{p^4} f_4(x_\mathrm{B})\,. \end{aligned} \tag{11}$$

There is one other twist-4 contribution, and it comes from a three-particle operator involving a gluon and two quarks, namely from the matrix element $\langle 0|\bar{u}(x) g_s G_{\alpha\beta}\gamma_\gamma\gamma_5(z) d(0)|\pi(k)\rangle$. The calculation of the gluon contribution is a lengthy calculation, which is explained in Ref. [1]. We distinguish two contributions of the gluons, giving $t_4^g(p^2, x_\mathrm{B}) = (f_g(x_\mathrm{B}) + f_{g1}(x_\mathrm{B}))/p^4$.

The light-cone QCD sum rule is now obtained by taking the Borel transform of $t(p^2, x_\mathrm{B})$ on the right-hand and left-hand sides to find

$$q^d(x_\mathrm{B}) = \varphi_\pi(x_\mathrm{B}) - (f_4(x_\mathrm{B}) + f_g(x_\mathrm{B}) + f_{g1}(x_\mathrm{B}))\left(\frac{1}{M^2} - \frac{\mathrm{e}^{-m_{A_1}^2/M^2}}{m_{A1}^2}\right). \tag{12}$$

where we have approximated the continuum as a pole at the mass of the A_1 meson to stabilize the sum rule for large values of the Borel mass M.

3. Numerical results for the quark distribution function

We have carried out numerical calculations using the results of Ref. [8], in which a series expansion of light-cone wave functions was suggested to separate the longitudinal and transverse degrees of freedom, with the higher-order terms corresponding to operators with increasing conformal spin. In the case of the twist-2 pion wave function, this expansion is

$$\begin{aligned} \varphi_\pi(u) &= 6u(1-u)\Big\{1 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1) \\ &\quad + a_6 C_6^{3/2}(2u-1) + \dots\Big\}\,. \end{aligned} \tag{13}$$

Here $C_n^{3/2}$ are the Gegenbauer polynomials;

$$C_2^{3/2}(x) = \frac{3}{2}(5x^2 - 1)\,, \quad C_4^{3/2}(x) = \frac{15}{8}(21x^4 - 14x^2 + 1)\,.$$

We made numerical calculations using the asymptotic parameterization of the quark and gluon wave functions. The asymptotic parameterization for $\varphi_\pi(v)$ corresponds to the expression in Eq. (13) when $a_2 = a_4 = a_6 = \ldots = 0$. This is suggested as a reasonable starting point by the results of Ref. [9], in which an evaluation of the twist-2 and twist-4 wave functions of the matrix element $\langle 0|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|\pi(k)\rangle$ in Eq. (1) is made using the light-front quark model.

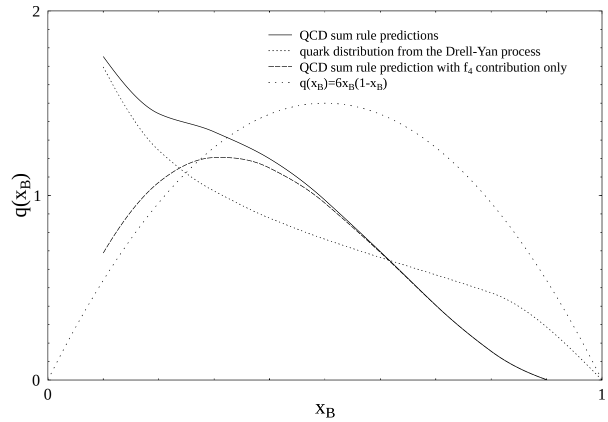

Fig. 1. Theoretical results for the d-quark distribution function of the pion compared to experimental results. The solid curve is the complete result appearing in Eq. (13), evaluated with the asymptotic twist-4 and twist-4 wave functions.

Using the asymptotic parameterization of the pion wave function, we show In Fig. 1 a comparison of our QCD sum rule in Eq. (12) to the Drell–Yan measurements evolved to $\mu^2 = 1$ GeV^2 [10]. Note that the calculation has the same trend as the data, although it does not agree perfectly. The level of agreement could be improved by adjusting the parameters of the pion wave function. It is difficult to justify a detailed fit to the experiment, since there the QCD sum rule approach has errors that are not completely under control, particularly at the end points. An alternative idea suggests itself from the fact that in the vicinity of $x_{\rm B} = 0.3$ the two-particle and three-particle twist-4 contributions are small and of opposite sign. Thus, in the region of $0.25 < x_{\rm B} < 0.4$ the quark distribution function is given essentially by $\varphi_\pi(v)$, which makes it possible to deduce the empirical constraint

$$\varphi_\pi(u) = 1 \pm 0.2 \quad (u = 0.3)\,. \tag{14}$$

4. Constrained analysis of the twist-2 pion wave function

If we assume that the pion wave function is not very different from its asymptotic form, then we can expect that the higher terms in Eq. (13) are small. This assumption means that there should be the following relations:

$$1 \gg a_2 \gg a_4 \gg a_6 \gg \ldots \,. \tag{15}$$

In the present analysis, which is based on Ref. [11], we take into consideration only the three leading terms in the expansion Eq. (13): $1, a_2, a_4$. At the end of our analysis, we find the relations in Eq. (15) to be approximately satisfied.

First, consider the constraint on the second moment based on the calculation of Chernyak and Zhitnitsky [2], where we have adjusted the value and added an error to take account of the considerations of [12]. In the parameterization (13), this takes the following form,

$$m_2 = \int_0^1 u^2 \varphi_\pi(u) du = \frac{3}{70}(7 + 2a_2) = 0.35 \pm 0.05\,. \tag{16}$$

Note that the higher terms of expansion (13) do not contribute to the relation (16). The constraint of Braun and Filyanov [5], $\varphi_\pi(0.5) = 1.25 \pm 0.25$, leads to the following result,

$$\varphi_\pi(0.5) = \frac{3}{2}\left(1 - \frac{3}{2}a_2 + \frac{15}{8}a_4\right) = 1.25 \pm 0.25\,. \tag{17}$$

The constraint of Radyushkin and Ruskov [6] is an integral constraint, and in the parameterization (13) it gives

$$I = \int_0^1 \frac{\varphi_\pi(u)}{u} du = 3(1 + a_2 + a_4) = 2.4\,. \tag{18}$$

And, finally, the constraint of Eq. (14) gives us the following formula:

$$\varphi_\pi(0.3) = 1.26(1 - 0.3a_2 - 1.317a_4) = 1 \pm 0.2\,. \tag{19}$$

It is convenient to present all existing constraints on a plot with axis a_2, a_4 (see Fig. 2). Note that due to the relatively small coefficient of a_2 in Eq. (16), a small uncertainty in the value of m_2 leads to a big uncertainty for a_2. Assuming that $m_2 = 0.35 \pm 0.05$ we obtain

$$0 < a_2 < 1.2\,. \tag{20}$$

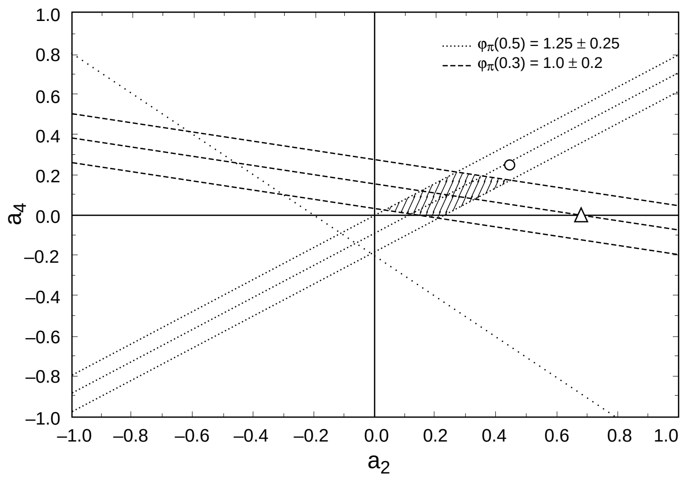

Fig. 2. Constraints on the first two coefficients of the twist-2 pion wave function of Eq. (15). The circle corresponds to the values of the coefficients in the Braun–Filyanov wave function, and the triangle to the coefficients in the Chernyak–Zhitnitsky wave function; the asymptotic wave function sits at the origin. The dotted line is the result with Eq. (18).

The relation (20) does not determine the value of a_2 very accurately, but it is useful, showing that $a_2 > 0$.

The constraints for $\varphi_\pi(0.5)$ and $\varphi_\pi(0.3)$ are more sensitive to the parameters a_2 and a_4. From the relations of Eqs (17), (19) it follows that

$$a_2 = 0.25 \pm 0.25\,; \quad a_4 = 0.1 \pm 0.12\,, \tag{21}$$

and we can not exclude the possibility that the pion wave function attains its asymptotic form. From relation (21) we obtain the following prediction:

$$I = 4 \pm 1\,. \tag{22}$$

5. Summary and conclusions

Using the light-cone QCD sum rule method, a new constraint on the twist-2 pion wave function has been shown to arise using phenomenological Drell–Yan results. The result of applying the sum rule leads to the result that $\varphi_\pi(0.3) = 1 \pm 0.2$. A combined analysis of all light-cone QCD sum rule constraints is indicating a twist-2 amplitude $\varphi_\pi(v)$ nearly asymptotic in form for $\mu^2 = 1$ GeV^2 in which the coefficients a_2 and a_4 in the expansion of Eq. (13) have the values given in Eq. (21).

We furthermore quoted results that indicate that the light-front quark model and the light-cone QCD sum rule method are complementary sources of information about the pion at scales μ^2 =1 GeV^2, and that the two approaches are consistent with the asymptotic characterization of the twist-2 wave function.

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