NONRELATIVISTIC DESCRIPTION OF HEAVY QUARKONIA *

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(Received June 15, 1998)

It is argued that a very good description of heavy quarkonia can be given in the framework of a nonrelativistic model, in spite of the fact that the relativistic corrections are expected to be significant. This suggests that most of the relativistic corrections can be absorbed into the phenomenological nonrelativistic potentials. How exactly this happens, is a difficult, open problem.

PACS numbers: 13.20.Gd, 14.40.Gx

Let us characterize as "ordinary" each meson, which in the valence approximation consists of two partons – a quark and an antiquark. This leaves out glueballs, mesons composed (in the valence approximation!) of more than one quark-antiquark pair and various hybrids. Heavy quarkonia are ordinary mesons, where both the partons, the quark Q and the antiquark $\overline{Q}$, are heavy, but not too heavy. Heavy, means that the mass is much larger than the parameter $\Lambda_{\rm QCD}$, *i.e.* than about 200 MeV. This is a simplifying feature, because the virtual gluons, which mediate the $Q\overline{Q}$ interactions, transfer typically momenta of the order of $\Lambda_{\rm QCD}$. Therefore, for $m_Q \gg \Lambda_{\rm QCD}$ the recoil velocities at absorption or emission of virtual gluons by the valence partons are negligible. Consequently, the valence partons propagate smoothly, and not by jumps and jerks, when virtual gluons are being absorbed or emitted. The assumption of large mass eliminates the u and d quarks, which have masses of a few MeV, and also the s quark with its mass of about 150 MeV. The other condition is that Q and $\overline{Q}$ should have masses smaller than about 85 GeV. Above this limit decays into a physical

* Presented at the NATO Advanced Research Workshop, Cracow, Poland, May 26–30, 1998.

† Also at the Institute of Nuclear Physics, Kraków, Poland

W boson and a lighter quark become possible and the heavy quark becomes too unstable to form heavy quarkonia. This is the case for the t quark with its mass of about 175 GeV. The inability of the t quark to form quarkonia can be seen from simple arguments. Visualizing the heavy quarkonium in the spirit of the Bohr (1912) model as a kind of microscopic double star, one finds that the lifetime of the t quark is only a small fraction of the time necessary to perform a complete revolution. Therefore, a well-defined quarkonium cannot be formed. Alternatively, one could notice that the level spacing for heavy quarkonia depends little on the masses of the quarks and is of the order of few hundreds MeV. The width of the t quark is of the order of some GeV, thus the observation of the level structure in the $t\overline{Q}$ or $Q\bar{t}$ system is impossible. Finally, we are left with three families of quarkonia – the $c\bar{c}$, the $b\bar{c}$ or $c\bar{b}$ and the $b\bar{b}$.

Somewhat surprisingly, a very good description of many features of the quarkonia can be obtained from simple potential models. It is known from QCD that the $Q-\overline{Q}$ interaction at small distances is approximately coulombic and can be described by the potential

$$V_s(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \text{const.} \tag{1}$$

Strictly speaking the coupling α_s depends on r, this dependence, however, is weak (logarithmic) and has been often neglected. At large distances between Q and $\overline{Q}$, the gluon field is all squeezed into a string connecting the two valence partons. The energy of the string is approximately proportional to its length. Thus the potential energy is

$$V_L(r) = \sigma r, \tag{2}$$

where the constant σ is known as the string tension. This formula must be interpreted with care, because it corresponds to the "quenched" version of QCD, where the production of light quark pairs is forbidden. In real life, as soon as the $Q\overline{Q}$ string gets an energy large compared to Λ_{QCD}, it breaks into a $q\overline{Q}$ string and a $Q\bar{q}$ string, where $q\bar{q}$ denotes a light quark-antiquark pair. Thus, the increase of energy with increasing $Q\overline{Q}$ distance is, at large distance, much slower than linear in r. For intermediate distances, which are relevant for the quarkonia, no simple formula for the potential is known. Let us describe three guesses.

The Cornell potential [1] is just a sum of the potentials (1) and (2):

$$V_{\text{Cornell}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + \text{const.} \tag{3}$$

This is consistent, because both the potential $V_L(r)$ for small values of r and the potential $V_s(r)$ for large values of r are negligible. The logarithmic potential [2]

$$V_{QR}(r) = a \ln\frac{r}{r_0} \tag{4}$$

is motivated by the observation that this is the only potential, which gives for the quarkonia an excitation spectrum independent of the quark masses. This independence had been observed experimentally. There are dozens of rather successful potential models of heavy quarkonia [3]. Some of them include relativistic corrections, but this does not improve the fits. Motyka and myself [4] analysed the predictions for a generic potential of the form

$$V(r) = -\frac{a}{r^\alpha} + br^\beta + \text{const} \tag{5}$$

with a, b, *const*, α, β being constants. When applied to masses (averaged over fine and hyperfine splittings), leptonic decay widths and dipole electric transition probabilities in the $b\overline{b}$ quarkonia, one gets predictions depending on the parameters. We found that the best fits are obtained for potentials not too different from

$$V(r) = 0.706380 \left(\sqrt{r} - \frac{0.46042}{r} \right) + 8.81715, \tag{6}$$

where all the quantities are in suitable powers of GeV, and choosing the quark mass

$$m_b = 4.80303. \tag{7}$$

Actually, only the narrow quarkonia below the threshold for strong decays have been considered, because it is believed that the broad quarkonia above this threshold require complicated and highly model-dependent coupled channel calculations. For potential (6), using all the experimental data available, we found $\chi^2 = 6.5$ for seven degrees of freedom, which is a spectacular agreement, when one realizes that the masses of the quarkonia are measured with precision of a few tenth of an MeV, *i.e.* with a relative error of order 10^{-5}. What makes it even more surprising is that this success is quite undeserved. From the wave function of the $q\overline{q}$ pair it is easy to calculate the kinetic energies of the quark and antiquark and from that to estimate the expected relativistic corrections. One finds about 60 MeV – two orders of magnitude above the errors of our fits. Thus potential (6) must be an effective potential, which somehow includes almost all the relativistic corrections. How and why this happens is an open problem. Let us also note that the high precision estimate of the quark mass m_b is less interesting than it might seem. The mass of, say, an electron is a well-defined quantity. One can determine it by isolating an electron, so that it may be considered free, by then measuring its energy and momentum, and finally using the formula $m^2 = E^2 - p^2$. Attempts to obtain a free quark, in order to apply this procedure, are as hopeless as attempts to obtain a piece of string with one end only. Thus, there is no simple operational definition of the quark mass. This mass can be deduced from various theoretical formulae, but the result depends on the formula chosen. Result (7) is the mass, which should be used in the nonrelativistic Schrödinger equation with potential (6), in order to reproduce the mass spectrum of the low laying $b\bar{b}$ quarkonia. Its relation to masses defined from other formulae, *e.g.* to the masses chosen by the Particle Data Group [5], which corresponds to the position of the singularity in the quark propagator, is unknown.

It is possible to generalize the analysis sketched here [6]. Introducing two more parameters (the mass of the c quark m_c and the QCD coupling $\alpha_s(M_z)$), and Fermi–Breit type terms into the Hamiltonian, one can partly reproduce and partly predict the mass spectra and the leptonic decay widths of all the $b\bar{b}$, $b\bar{c}$ or $c\bar{b}$ and $c\bar{c}$ quarkonia, below their respective strong decay thresholds. The agreement with experiment, where possible to check, is somewhat worse than for the $b\bar{b}$ quarkonia alone, but the model remains one of the best on the market. The predictions for the $b\bar{c}$ and $c\bar{b}$ quarkonia are particularly interesting, because they should soon become possible to check against experiment. Let us repeat, however, that the reason for the success of the nonrelativistic potential model is not understood. Therefore, more fundamental methods – QCD sum rules [7] or lattice calculations [8] – are of great interest. From the phenomenological point of view, somehow, the potential models give the best results.

To summarize: an elementary, nonrelativistic potential model, supplied with rather standard spin dependent corrections, can describe with amazing precision many features of the known heavy quarkonia and give plausible predictions for the yet undiscovered ones. The relativistic corrections, which must be there, are somehow absorbed into the nonrelativistic potential. How this happens is an interesting, though probably very difficult, problem.

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