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MESON EXCHANGE MODEL OF MESON–MESON INTERACTIONS *

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We review the motivation for, and construction of, the Jülich meson exchange model of meson-meson interactions. The model employs both s- and t-channel exchanges of well-established mesons and gives a good quantitative fit to pseudoscalar–pseudoscalar meson scattering data from threshold to beyond 1.2 GeV. The model allows one to distinguish between "true" — *i.e.*, $q\bar{q}$ — meson states and dynamical effects. In the framework of the model, the f_0 and a_0 appear as instances of dynamical effects. Focussing on the $\pi\pi$ sector, limitationsof the model due to the lack of chiral symmetry are observed when the model is extended to pion interactions in nuclear matter. These are discussed, and one possible remedy is examined.

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1. Introduction

In the absence of a solution to the fundamental theory of hadronic interactions at low and intermediate energies, we are compelled to seek approximations or models which permit us to confront the data in this energy domain in a quantitative way. The nucleon-nucleon interaction was probably the first two-body hadronic interaction to be treated in this way. Early, purely phenomenological potentials [1] gave way to various forms of one-boson exchange potentials [2], which could be described as semi-phenomenological, in that the exchanged mesons are restricted to those observed experimentally. The expectation — or hope — is that a small number

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of parameters — coupling constants and cutoffs — will be sufficient to describe two-nucleon data up to the pion production threshold, and even a bit beyond.

The same philosophy has motivated the development of the meson exchange model of meson-meson interactions. In fact, from a logical, rather than an historic, point of view, the study of the meson-meson interaction should precede that of the nucleon-nucleon or pion-nucleon interaction since, in the meson exchange framework, a good model of the meson-meson interaction is needed for the former two. For example, the ubiquitous σ meson in the pure one-boson exchange NN models is generally acknowledged to be an approximation to the exchange of a virtual pair of pions in a relative s-state [3, 4]. Similar arguments apply to the πN interaction. In a sense, the observed meson-meson interaction justifies the one-boson exchange approach to the NN. The attempted reach of the approach has, however, been enlarged. The hope now is that a relatively small number of parameters can *consistently* describe baryon-baryon, meson-baryon, and meson-meson interactions in the low and intermediate energy domain. The hope is still far from realization, but considerable progress has been made. In what follows we will briefly describe the model, observe its successes, examine some of its problems, and discuss possible improvements.

2. The Jülich model: Elements

The basic ingredients of the Jülich model of meson-meson interactions [5,6] are single meson exchange, which provides the driving force (potential), and a scattering equation with relativistic kinematics that is used to ensure

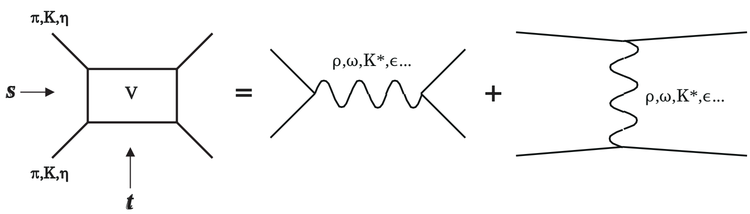

Fig. 1. The one-meson exchange potential for pseudoscalar meson-meson scattering.

unitarity. This is illustrated schematically in Fig. 1. The scattering equation chosen is usually the relativistic form of the Lippmann-Schwinger equation or the Blankenbecler-Sugar (BbS) equation:

$$M = V + VGM. \tag{1}$$

We focus our attention here on the scattering of pseudoscalar mesons, with emphasis on the $\pi\pi$ channel.

The original idea was that, in the absence of any well-defined low-mass scalar mesons, the potential should be due to the exchange of vector mesons. In addition, it is assumed that the coupling constants obey SU(3) flavor symmetry, so that the pseudoscalar-pseudoscalar-vector meson interaction is given by

$$\mathcal{L}_{\rm ppv} = -i\frac{g}{2}\mathrm{Tr}([\boldsymbol{P}, \partial_\mu \boldsymbol{P}]_- \boldsymbol{V}^\mu), \tag{2}$$

where $\boldsymbol{P}$ and $\boldsymbol{V}^\mu$ are, respectively, the 3×3 SU(3) matrix representations of the pseudoscalar meson and vector meson octets.

In addition to the coupling constant g, cutoffs must be applied to the interaction vertices in order to ensure convergence of the integrals in the scattering equation. To each 3-meson vertex is assigned a t-channel and an s-channel cutoff. For the vector mesons the t-channel cutoff is of the dipole form,

$$F_i^{(t)}(\boldsymbol{q}^2) = \frac{\Lambda^2 - m_i^2}{\Lambda^2 - \boldsymbol{q}^2}, \tag{3}$$

where $\boldsymbol{q}$ is the 3-momentum of the exchanged meson. For s-channel exchange exchange we adopt the form

$$F_i^{(s)}(\omega_p) = \frac{\Lambda^2 + m_i^2}{\Lambda^2 + \omega_p^2}. \tag{4}$$

The masses of the mesons in the s-channel are renormalized by the interactions. Since all the vector meson couplings are fixed by the choice of one coupling constant, g, and the renormalized masses are required to match the experimental masses and widths of the mesons, the free parameters in the model are essentially the cutoffs. Their values are adjusted to fit the data, but all fall within a physically reasonable range, *e.g.*, 1.6–4.5 GeV.

One feature that is automatically included in the model is channel coupling. Channel coupling of $\pi\pi$ to $K\bar{K}$ and $\pi\eta$ to $K\bar{K}$ via K^* exchange turns out to be crucial to an understanding the structure of the s-waves.

3. The Jülich model: Results

With all the elements in place, what remains is to observe and evaluate the results. What one observes with only vector meson exchange is a fairly good quantitative fit, up to about 1 Gev, to the phases in all partial waves which there is either a known *vector meson* resonance or no known resonance at all. In the former case the result is certainly to be expected: the resonances are part of the model. The exceptional case is the $I = 0,\ J = 2$

phase. In that case there is a known resonance — the f_2 — which t-channel exchange interactions cannot reproduce. A good fit to that amplitude requires that the f_2 be included as an element of the model. It should be pointed out, however, that the low energy part of the $I = 0,\ J = 2$ amplitude is well-reproduced by vector meson exchange; the effect of the f_2 is manifest only at energies approaching 1 GeV. Furthermore, the sharp rise in the $I = 0,\ J = 0$ phase near the $K\bar{K}$ threshold is well-reproduced and is absent without channel coupling.

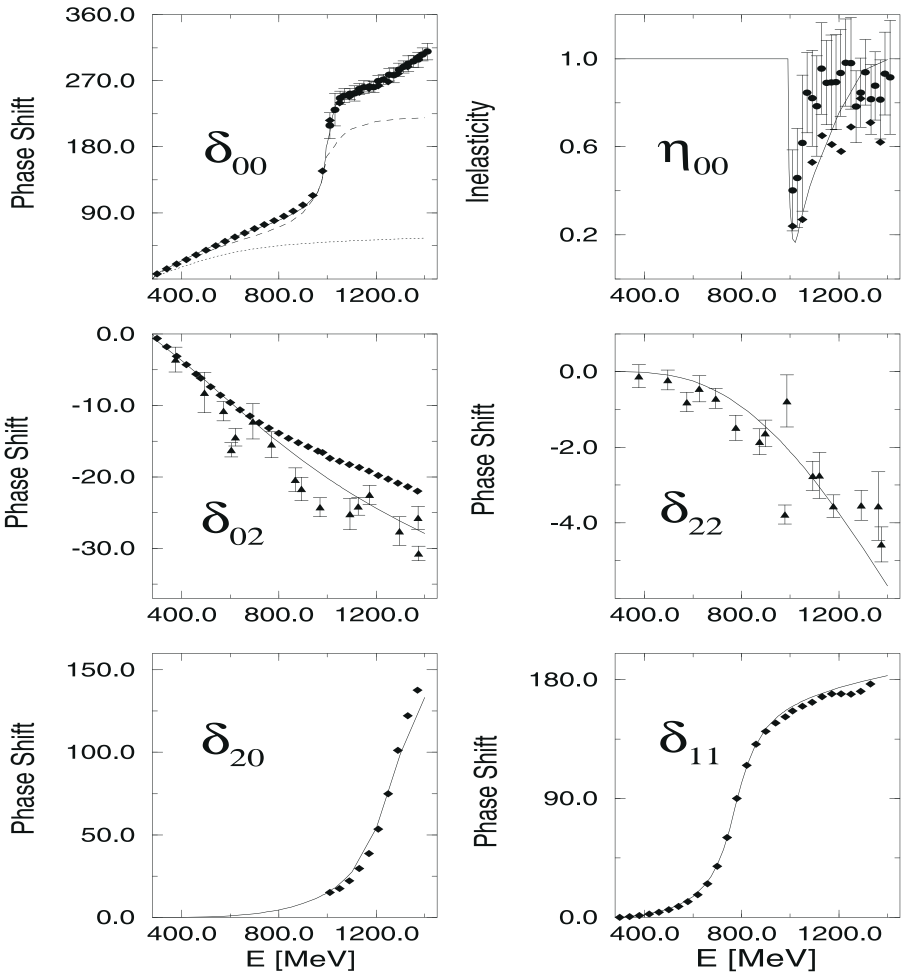

Fig. 2. Scattering parameters in the JI $\pi\pi$ channels for the Jülich Model (full line). The dotted curve in the upper left panel shows the results for vector meson exchange with no channel coupling; the dashed curve adds channel coupling with no ε meson contribution.

To achieve a good fit to the $I = 0,\ J = 0$ phase at energies above 1 GeV requires the introduction of a scalar-isoscalar meson, labeled ε, with a bare mass of about 1.3 GeV, derivatively coupled to the $\pi\pi$ system:

$$\mathcal{L}_{\pi\pi\varepsilon} = \frac{g_\varepsilon}{m_\varepsilon}(\partial_\mu \pi \cdot \partial^\mu \pi)\varepsilon. \tag{5}$$

(A similar description of the $I = \frac{1}{2},\ J = 0$ phase in the πK sector is achieved by including an SU(3) octet partner of the ε — the κ — by similar coupling, in analogy with Eq.(2).)

The end result for the $\pi\pi$ sector is shown in Fig. 2, in which we observe a quantitativley acceptable description of all phases from threshold to well beyond 1 GeV. As stated above, one of the advantages of this dynamical approach is the possibility of separating dynamical effects from true $q\bar{q}$ states. The f_0 in this model is one of the former, and is the result of a $K\bar{K}$ quasi-bound state coupled weakly to the $\pi\pi$ channel. In the $\pi\eta$ channel (not shown) the a_0 appears as an enhanced threshold effect due to strong coupling to the $K\bar{K}$ channel.

4. Further considerations

While the success of the model in reproducing the meson-meson scattering data is clear, it is only a first step towards the ultimate goal of a model with broad applicability. One should now pause and ask what approximations have been made, what symmetries have been compromised, and what effects these might have on further applications.

In principle, the amplitudes should be unitary, Lorentz invariant, crossing symmetric, and chirally symmetric in the limit of vanishing pion mass. Unitarity is ensured by the use of a scattering equation, and the kernel of the equation is Lorentz invariant. While the terms in the potential are crossing symmetric, the use of form factors and iteration of the potential in the scattering equation, which effectively sums ladder, triangle, and bubble diagrams, produces an amplitude which violates crossing symmetry. However, the quality of the fit to the data means that the amplitudes satisfy the Roy equations [7], so that crossing symmetry is restored, in some sense, after the fact. Chiral symmetry, however, is not. Direct calculation reveals that the scattering lengths, which should vanish as the pion mass goes to zero, instead diverge. The question then is whether the breaking of approximate chiral symmetry has any important consequences.

Extension of the original model to the problem of pion interactions in nuclear matter reveal that it does [8]. When modifications of the pion propagator in the nuclear medium are taken into account, the $\pi\pi$ s-wave interaction in the medium is strongly enhanced at low energy due to the coupling of the pion to nucleon-hole and Δ-hole states in the sub-threshold region, which is now physically accessible. The effect grows with the density of the medium and produces $\pi\pi$ bound states that eventually drop below zero total energy at only slightly above nuclear matter density. Such an instability is clearly unacceptable and demands a remedy.

Further exploration of the model revealed that the increased attraction in the sub-threshold region is directly linked to the divergence of the scattering lengths in the chiral limit. If the amplitude is forced to obey the constraint that the scattering length vanish in the chiral limit, or even that the potential satisfy approximate chiral symmetry [9], then the $\pi\pi$ pairing instability noted above remains, but is shifted to somewhat higher density: about two times nuclear matter density.

5. Improvements: Pion interactions in nuclear matter

The modification of the basic model to achieve the desired behavior of the amplitude is done in two steps. First, the $\rho\pi\pi$ interaction lagrangian derived from Eq.(2) is replaced by that of the gauged non-linear sigma model [10]:

$$\mathcal{L}_{\rm gnls} = \frac{1}{4\mathrm{f}_\pi^2}\boldsymbol{\pi}^2(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) - \frac{m_\pi^2}{8\mathrm{f}_\pi^2}(\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2 + g(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\cdot\boldsymbol{\rho}^\mu - \frac{g^2}{2m_\rho^2}(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})^2. \quad (6)$$

This adds repulsive contact terms to the pure meson exchange terms of Fig. 1. Second, the vertex functions in the potential are modified so that all terms in the potential are of the form [9]

$$V_i(k,q;s) = \lambda_i(k,s)U_i(k,q,s)\lambda_i(q,s), \quad (7)$$

where $\lambda(k,s) \sim m_\pi$ as s, $m_\pi \to 0$. This modification ensures the vanishing of the scattering lengths in the chiral limit, although the potential now formally violates Lorentz invariance. The resulting model gives approximately the same quality fit to the scattering data.

The improvements in the behavior of the model after modification are seen in Fig. 3, where three measures of comparison are displayed. The direct comparison of the imaginary part of the s-wave amplitude M_{00} between the original model (BbS) and the "chirally improved" model [9, 11] shows the suppression of low-energy strength in the latter. In order to examine the effect of this change on the two-pion exchange contribution to the in-medium nucleon-nucleon potential [11], the resulting spectral function η_{00}, which can be interpreted as the distributed coupling constant squared as a function of mass squared for the exchange of a scalar-isoscalar meson is shown, along with the resulting isoscalar central potential, supplemented by a common repulsive ω exchange contribution. The changes are dramatic. There are still quantitive difficulties for the saturation properties of nuclear matter, but the disaster of the pion pair instability at near-nuclear matter density is suppressed, if not altogether eliminated.

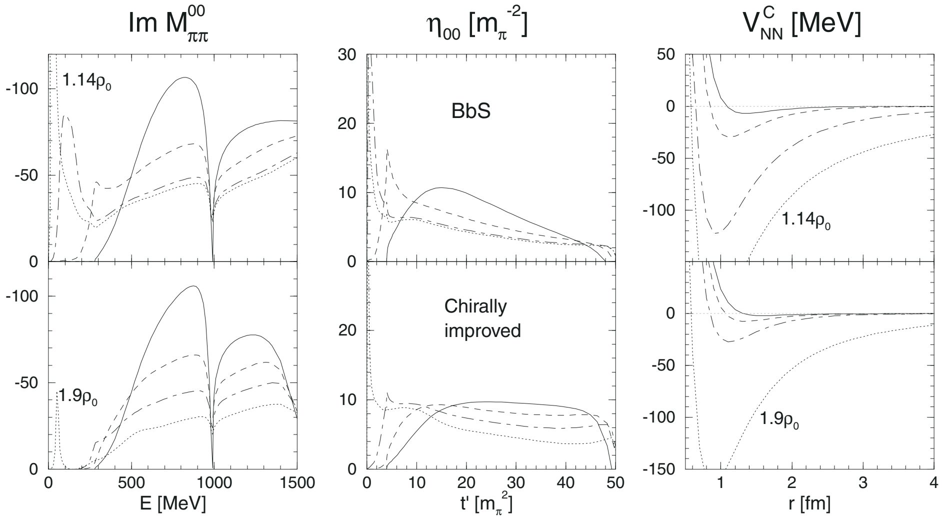

Fig. 3. Results of the Jülich Model in nuclear matter. The left panels show the imaginary part of the $(JI = 00)$ $\pi\pi$ amplitude, the center panels the $N\bar{N}$ spectral function, and the right panels the corresponding central NN potential supplemented with ω exchange, at various nuclear matter densities. The full lines are for free space ($\rho = 0$), the dashed lines are for $\rho = \frac{1}{2}\rho_0$, the dashed-dotted lines are for $\rho = \rho_0$, and the dotted lines are for the densities indicated. The upper panels show the results for the original Jülich Model, the lower panels for the model which enforces the chiral scattering length constraints according to Eqs.(6) and (7).

6. Conclusions

The dynamical model for meson-meson interactions described here provides a good description of on-shell meson-meson scattering. At higher energies it should be an adequate model for such applications as final state interactions. When extended to the nuclear medium, its shortcomings due to its violation of approximate chiral symmetry become apparent. Modifications of the potential which enforce a chiral scattering length constraint improve the behavior of the model in nuclear matter without significantly altering the good fit to the scattering data, and demonstrate the importance of chiral symmetry for the interactions of nucleons in nuclei.

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