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# COHERENT PHOTON REACTIONS AND THE DIPION CONTINUUM

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We discuss the role played by charged pion pairs in mediating high energy, forward Compton scattering, $\varrho^0$ meson photoproduction and $\varrho^0$ meson elastic scattering. With nucleon targets we find deviations from the "$\varrho^0$-photon analogy" theory relating these interactions, primarily in the real parts of the amplitudes. Generalizing to nuclear targets, we take into approximate account the multiple scattering of a pion pair as well as that of the $\varrho^0$. We discuss why the quantities appearing in the conventional, Glauber multiple scattering theory of these reactions require some reinterpretation.

## 1. *Introduction*

It is the purpose of these lectures to discuss in some detail the dynamical role played by charged pion pairs in forward high energy photoproduction of charged pion pairs, photoproduction of $\varrho^0$ mesons *per se* and Compton scattering. The dipion continuum couples directly to both photon and $\varrho^0$ and its importance over other continua arises from the dipion threshold being right in between the photon and $\varrho^0$ mass. Consequently, we take into account that pion pairs participate both "resonantly", as the decay product of a photoproduced $\varrho^0$ meson, and "non-resonantly", as entities independent of $\varrho^0$ dynamics. The full treatment of these effects will prove relevant to popular $\varrho^0$ dominance theories of the isovector part of the photon coupling with hadrons.

First of all we will find that the inclusion of these effects leads to a refinement of the most naive view of $\varrho^0$ dominance, the "$\varrho^0$-photon analogy". The "$\varrho^0$-photon analogy" states that the photon in all isovector, hadronic interactions is mediated simply by a $\varrho^0$ meson. This means that, when viewed as a function of photon mass, an isovector photon amplitude displays the "free" $\varrho^0$ meson propagator. Thus, there would exist a universal $\gamma-\varrho^0$ coupling $g_{\gamma\varrho}$, which multiplicatively relates $\varrho^0$ meson and isovector photon amplitudes. In the relevant examples that we shall discuss (suppressing some spin and isospin dependence), if $f_{\varrho\to\varrho}(0)$ is the amplitude for forward $\varrho^0$ meson elastic scattering from a nucleon, $g_{\gamma\varrho}f_{\varrho\to\varrho}(0)$ and $g^2_{\gamma\varrho}f_{\varrho\to\varrho}(0)$ would then be the amplitudes for forward $\varrho^0$ photoproduction and forward, isovector Compton scattering respectively. A number of recent experiments

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indicate that, while giving a rough approximation, these relations are not exactly true. These experiments range from detailed comparison of $\varrho^0$ photoproduction with Compton scattering [1] to measurements of photon absorption by complex nuclei [2].

In these lectures we take into account that the photon can be mediated by the dipion continuum as well as the $\varrho^0$. In doing so we will adopt a more sophisticated view of $\varrho^0$ meson dominance. We still picture the photon's isovector interaction as a direct conversion to a $\varrho^0$ meson *via* a universal $\gamma-\varrho^0$ coupling, but we now allow the $\varrho^0$ meson to itself convert into other things — like pion pairs — before interacting. This is $\varrho^0$ dominance in the field theoretic sense, sometimes referred to as the field-current identity [3], and we discuss what perhaps is the simplest, non-trivial digression from the naive view.

In addition we shall also discuss some new, dynamical features introduced by a combination of "non-resonant" pion pairs and the $\varrho^0$'s instability. We shall find that the nonresonant background not only interferes with pion pairs from $\varrho^0$ decay, but also may re-combine and form a final $\varrho^0$ or photon. Because the mass of the $\varrho^0$ is above the dipion threshold, new singularities then appear in $\varrho^0$ meson production and scattering amplitudes which give rise to large and "anomalous" refractive pieces.

Generalizing to nuclei, we will find that the multiple scattering of mediating pion pairs will make some reinterpretation of the terms in a "traditional" Glauber [4] multiple scattering series necessary. While the required changes should be small at presently available laboratory energies, the effect could have future significance. Finally, considering forward Compton scattering from large nuclei, we give some reasons why the conversion of the amplitude from being $\propto A$ to being $\propto A^{2/3}$ can still take place, but that the transition energy may be shifted up from that predicted from estimates employing the $\varrho^0$-photon analogy (see Ref. [6]).

In these talks I will assume that at least some passing acquaintance exists with the traditional approaches to this subject. This includes some notion of the multiple scatternig picture of Glauber, as applied to $\varrho^0$ photoproduction from nuclei, *e.g.* the paper by Drell and Trefil [5], and as applied to Compton scattering *e.g.* many papers [6].

In Section 2 we outline a simple $\varrho^0$-dominance picture of the coupling of photons, $\varrho^0$'s and charged pion pairs. In Section 3 we apply these ideas to the forward, high energy reactions $\gamma+N\to\varrho^0+N$ and $\gamma+N\to\pi^++\pi^-+N$. Section 4 explores our more "sophisticated" $\varrho^0$-dominance theory, relating forward, high energy $\gamma+N\to\gamma+N$, $\gamma+N\to\varrho^0+N$ and $\varrho^0+N\to\varrho^0+N$. In Section 5 we attempt to generalize the discussion of Section 3 to nuclear targets but flounder in complexities which are only crudely and approximately removed by the "spatial" considerations of Section 6, which offer as well a crude, qualitative solution to our dilemma associated with the energy dependence of forward Compton scattering from large nuclei.

## 2. *The $\varrho^0$ meson, pion pairs and the $\gamma-\varrho^0$ coupling*

We begin by introducing the $\varrho^0$ meson as a spin, $I$ spin $=1$, $T_3=0$ resonance in $\pi^+-\pi^-$ elastic scattering. We view this scattering field theoretically as the dipion system coupling directly (with a strength, $f_\varrho$) to an unstable particle, the $\varrho^0$ (Fig. 1).

We are going to neglect any other interfering contribution to $\pi^+ - \pi^-$ scattering and neglect any form factor in the $\varrho^0 - \pi^+\pi^-$ coupling. Briefly discussing the $\varrho^0$ meson propagator, it is easy to see that the part of the propagator $\propto K_\mu K_\nu$ does not contribute because we

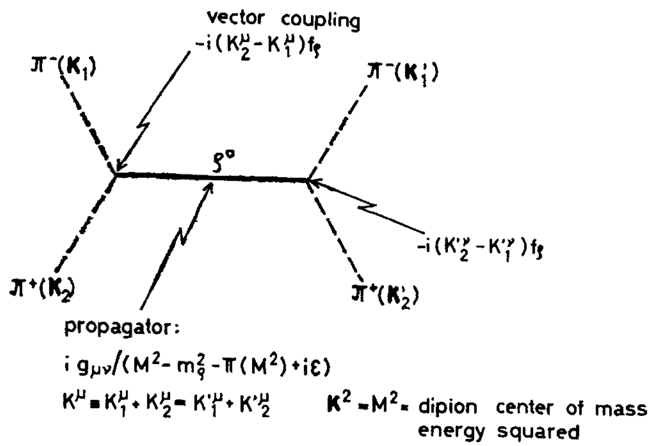



Fig. 1

couple the $\varrho^0$ to conserved currents. $\pi(M^2)$ is the part of the $\varrho^0$'s "vacuum polarization" $\propto g_{\mu\nu}$ and is given by evaluating all possible "bubble diagrams" as illustrated in Fig. 2.

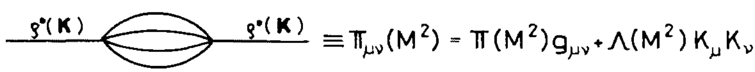



Fig. 2

Because we will be interested in $M$'s above the dipion threshold, $\pi(M^2)$ will take on an absorptive or imaginary part. We estimate Im $\pi(M^2)$ by evaluating the absorptive part of the simple bubble diagram (Fig. 3).

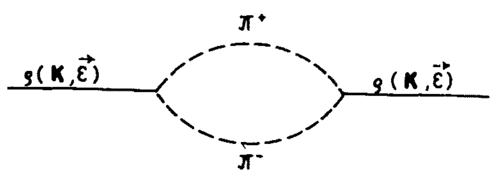



Fig. 3

Writting $\mathrm{Im}\,\pi(M^2) \equiv -m_\varrho \Gamma(M^2)$, we find:

$$\Gamma(M^2) = \frac{m_\varrho}{M}\left(\frac{(M^2/4) - m_\pi^2}{(m_\varrho^2/4) - m_\pi^2}\right)^{3/2} \Gamma(m_\varrho^2),$$

$$\Gamma(m_\varrho^2) = \frac{2}{3}(f_\varrho^2/4\pi)\frac{((m_\varrho^2/4) - m_\pi^2)^{3/2}}{m_\varrho^2}. \qquad (2.1)$$

We note that $\Gamma(m_\varrho^2)$ is just the "decay rate" of the $\varrho^0$ meson as calculated from the simple Feynman diagram shown in Fig. 4.

Understanding Re $\Pi(M^2)$ is a lot harder — but after renormalization, as $M^2 \to m_\varrho^2$ Re $\Pi(M^2)$ must $\to 0$ at least as fast as $(M^2 - m_\varrho^2)^2$ as any other behaviour would just alter

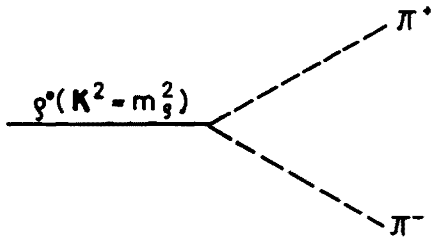



Fig. 4

the phenomenological values of $m_\varrho$ and $f_\varrho$. For lack of other inspiration [7] we will assume Re $\Pi(M^2) \approx$ constant over our region of interest, *i.e.* Re $\Pi(M^2) \approx 0$.

So, now we have an approximate expression for $p$ wave $\pi^+ - \pi^-$ scattering in terms of two free parameters $m_\varrho$ and $f_\varrho$. The propagator now has an approximate Breit-Wigner form:

$$ig_{\mu\nu}/(M^2 - m_\varrho^2 + i\Gamma(M^2)m_\varrho).$$

Although it is really inconsistent with our $M$ dependent expression for $\Gamma(M^2)$, for eventual use in a Glauber multiple scattering series, when we wish to "isolate" the contribution from the $\varrho^0$ meson as a particle — in the sense of an integration over $M^2$ — we will just assume the $\varrho^0$ propagator has a simple pole at $M^2 = m_\varrho^2 - i\Gamma(m_\varrho^2)m_\varrho$.

We introduce the photon into this picture by looking at the charged pion's electromagnetic form factor for photon masses ($M^2$) in the $\varrho^0$ resonance region. The photon couples to charged pions in the spin, $I$ spin 1, $T_3 = 0$ mode and the form factor should

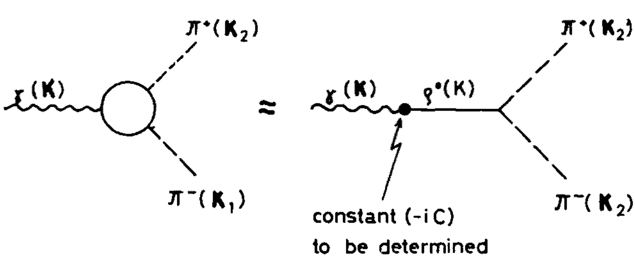



Fig. 5

thus display the $\varrho^0$ meson propagator. In the $\varrho^0$ region, as a function of $M^2$, assuming the $\varrho^0$ propagator is the most rapidly varying contribution, we can decompose the $\gamma - \pi^+\pi^-$ coupling as shown in Fig. 5.

The form factor $F_\pi(M^2)$ (normalized so $F_\pi(0) = e$) is then $\approx -Cf_\varrho/(M^2 - m_\varrho^2 + i\Gamma(M^2)m_\varrho)$ in the $\varrho^0$ region. We note that if $\varrho^0$ dominance is true the constant, $C$, is a universal number

$(= m_\varrho^2 g_{\gamma\varrho})$, and subject to all our approximations thus far, should be given by $em_\varrho^2/f_\varrho$ $(g_{\gamma\varrho} = e/f_\varrho)$. One analysis [8] of the Orsay colliding beam reaction: $e^+ + e^- \to (\gamma) \to \pi^+ + \pi^-$ with $f_\varrho$, $m_\varrho$ and $C$ left as free parameters yields: $m_\varrho = 776 \pm 6$ MeV; $\Gamma(m_\varrho^2) = 127 \pm 12$MeV $(f_\varrho^2/4\pi = 2.48 \pm 0.24)$, and $C = (em_\varrho^2/f_\varrho)(1.18 \pm 0.09)$ with the probability of a worse fit: 55%. Thus we have a simple picture of the coupling of $\varrho^0$'s, pion pairs and photons which does "reasonable justice" to experiment.

### 3. *The high energy reactions*: $\gamma + N \to \pi^+ + \pi^- + N$ *and* $\gamma + N \to \varrho^0 + N$

Neglecting the $\varrho^0$ for the time being, we sketch the Drell-Söding [9] theory of the high energy photoproduction of "non-resonant" pion pairs. For simplicity we will work in the lab frame and neglect nucleon recoil and any difference between neutrons and

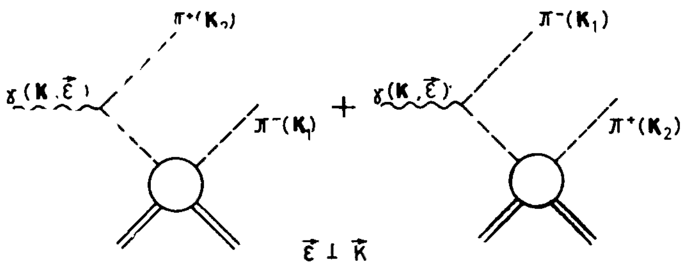



Fig. 6

protons. Pictorially, we view the incident photon as spontaneously converting into a pion pair one of which eventually elastically scatters from the nucleon target with sufficient longitudinal momentum transfer to "kick" the virtual dipion system onto its mass shell. The relevant Feynman diagrams are shown in Fig. 6. We calculate their contribution for a high energy, forward going dipion system of invariant mass $M$. We will assume that the off-mass-shell pion scatters like a forward going on-mass-shell pion (the momentum transfer to the nucleon is small $\approx M^2/2K$) and neglect differences between the $\pi^+$ and $\pi^-$. In this case both diagrams contribute equally and the Drell-Söding contribution $\vec{\varepsilon} \cdot \vec{M}_{\text{DS}}$, is

$$\vec{\varepsilon} \cdot \vec{M}_{\text{DS}} \propto e \frac{2f_{\pi N}(0)}{M^2} \vec{\varepsilon} \cdot (\vec{K}_2 - \vec{K}_1). \tag{3.1}$$

$f_{\pi N}(0)$ is the forward pion-nucleon scattering amplitude, written as $(iK/4\pi)\sigma_{\pi N}(1 - i\alpha_\pi)$ where $\sigma_{\pi N} \approx 26$ mb, $\alpha_\pi \approx -0.15$ and $K$ is the photon energy. An important feature of the Drell-Söding amplitude is that the pion pairs it produces are $p$ wave in their centre of mass — just like those from $\varrho^0$ decay. (This amazing fact is a result of cancellations of angular factors in the off-mass-shell propagator and the off-mass -shell scattering "blob".) It is not at all obvious that the Drell-Söding mechanism is the only source of non-resonant pion pairs. One is just motivated by its plausability — but later on we shall introduce some refinements. Related to this question is the awkward fact that this theory is not gauge invariant — a point we shall ignore.

We now bring the $\varrho^0$ into the picture by assuming that pion pairs can arise from the decay of a photoproduced $\varrho^0$. The work above indicates that one way a $\varrho^0$ might be photoproduced is by a recombination of Drell-Söding pion pairs as illustrated in Fig. 7.

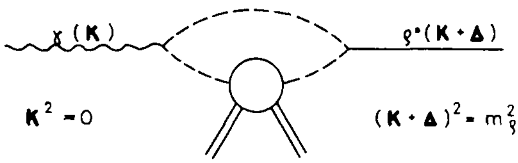



Fig. 7

It is not very realistic to assume that this is the only way $\varrho^0$'s may be photoproduced [10, 11], and to these amplitudes we add "remaining mechanisms" as shown in Fig. 8

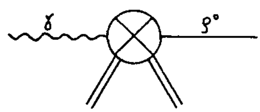



Fig. 8

However, the amplitude of Fig. 7 has an interesting feature in that, because $m_\varrho > 2m_\pi$, the pion pairs adjoining the $\varrho^0$ can be on their mass shells. This on-mass-shell contribution can be calculated as the "absorptive contribution" of the pion loop integral. The "arithmetic" involved is rather easy because Drell-Söding pion pairs are $p$ wave. We term this contribution "anomalous" $\varrho^0$ photoproduction [12] and will later see that its presence implies some breakdown of the $\varrho^0$-photon analogy. We find for forward production (using $g_{\gamma\varrho} = e/f_\varrho$)

$$f_{\gamma\to\varrho}(0)/_{\text{anomalous}} \approx i g_{\gamma\varrho} f_{\pi N}(0)\, \frac{2\Gamma(m_\varrho^2)}{m_\varrho} \,.$$

(We have suppressed all notation for spin. For forward production we assume polarizations are all transverse and that spin is conserved.) Since $f_{\pi N}(0)$ is principally absorptive or "imaginary", it is easy to see that anomalous $\varrho^0$ photoproduction is principally refractive or "real".

If we do view the $\varrho^0$ through a decay into lepton pairs, anomalous $\varrho^0$ production should manifest itself as a larger than "normal" refractive piece in $\varrho^0$ photoproduction. However, if we view the photoproduced $\varrho^0$ through its principal decay mode of pion pairs, the amplitude for anomalous $\varrho^0$ production and subsequent decay is naturally viewed as a "final state interaction" of the Drell-Söding background, and its inclusion multiplies that background by a factor $e^{i\delta}\cos\delta$ (where $\delta$ is the $p$ wave $\pi^+-\pi^-$ phase shift associated with the $\varrho^0$ resonance) which $\to 0$ as the system resonates.

We now "lump" the remaining contribution from Fig. 7 and the whole contribution of Fig. 8 together into a term that we call "normal" $\varrho^0$ photoproduction. If we make a rough estimate by assuming $f_{\gamma\to\varrho}(0)/_{\text{normal}}$ is primarily absorptive and $\approx g_{\gamma\varrho} f_{\pi N}(0)$(which is a naive $\varrho^0$

dominance, quark model type of prediction) and look at all the terms contributing to the reaction: $\gamma+N\to\pi^++\pi^-+N$, we get results as shown (schematically) in Fig. 9.

The illustrated "distortion" of the $\varrho^0$ mass spectrum is a well known experimental feature of this reaction [13] and the mechanisms we have discussed provide a natural way of understanding it.

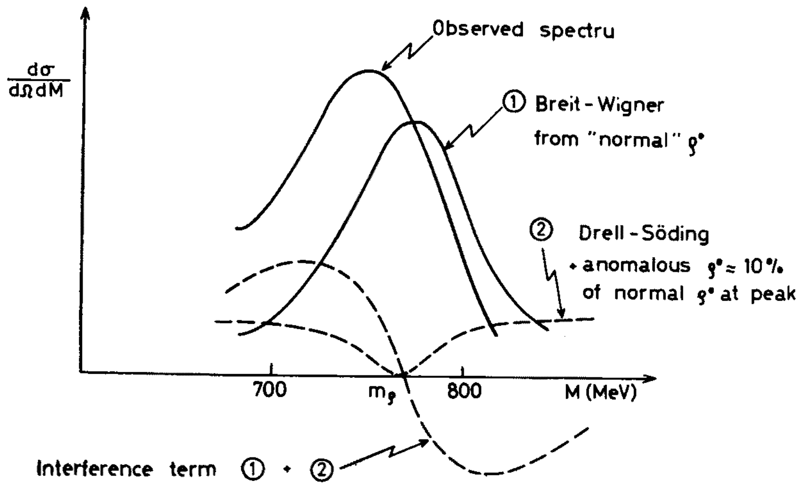



Fig. 9

*4. $\varrho^0$ meson dominance and the reactions $\gamma+N\to\gamma+N$, $\gamma+N\to\varrho^0+N$* and *$\varrho^0+N\to\varrho^0+N$*

$f_{\varrho\to\varrho}(0)$ is the amplitude for forward $\varrho^0$-nucleon elastic scattering (again we suppress spin notation and assume polarizations are all transverse and conserved). If we view this amplitude as a function of the masses of the initial and final $\varrho^0$($m_i^2$ and $m_f^2$ respectively), $\varrho^0$ dominance says that the amplitude for forward $\varrho^0$ photoproduction is $g_{\gamma\varrho}f_{\varrho\to\varrho}(0)/m_i^2=0$ and that the isovector part of forward Compton scattering is $g_{\gamma\varrho}^2 f_{\varrho\to\varrho}(0)/m_i^2,\ m_f^2=0$.

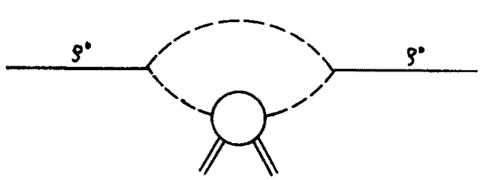



Fig. 10

In addition to these hypotheses the "$\varrho^0$-photon analogy" theory states that such extrapolations in $m_i^2$ and $m_f^2$ from 0 to $m_\varrho^2$ produce no changes. However, our work in the last section indicates that $f_{\varrho\to\varrho}(0)$ contains contributions from the amplitude shown in Fig. 10, where the $\varrho^0$ is "mediated" by pion pairs.

Now, in extrapolating from $m_i^2$ and/or $m_f^2=0$ to $m_\varrho^2$ we reach and go well beyond the dipion threshold; so it is not unreasonable to expect significant "mass dependence" in these amplitudes and thereby predict deviations from the "$\varrho^0$-photon analogy".

To proceed, for simplicity we will assume all other amplitudes contributing to $f_{\varrho\to\varrho}(0)$, do extrapolate smoothly and obey, in essence, the prescription of the $\varrho^0$-photon analogy. A heuristic way of presenting this hypothesis is to say that the $\varrho^0$ can elastically scatter in two ways; either mediated by pion pairs as shown in Fig. 10 or as a "bare" $\varrho^0$, as shown in Fig. 11. whose scattering shows no structure as a function of $m_i^2$ or $m_f^2$.

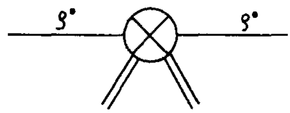



Fig. 11

Studying the amplitudes of Fig. 10, a significant mass dependence can be isolated by looking at on-mass-shell contributions to the pion loop integral [12]. These contributions will give rise to "anomalous" contributions in $\varrho^0$ photoproduction and $\varrho^0$ elastic scattering but not in Compton scattering (because of a threshold effect). Quoting our previous work, the anomalous contribution to $\varrho^0$ photoproduction is

$$f_{\gamma\to\varrho}(0)/_{\text{anomalous}} \approx i g_{\gamma\varrho} f_{\pi N}(0)\, \frac{2\Gamma(m_\varrho^2)}{m_\varrho}\,. \tag{4.1}$$

In the case of $\varrho^0$ elastic scattering either the pions coupled to the initial or final $\varrho^0$ can be on-mass-shell, and the anomalous contribution turns out to be:

$$f_{\varrho\to\varrho}(0)/_{\text{anomalous}} \approx i f_{\pi N}(0)\, \frac{2.4\Gamma(m_\varrho^2)}{m_\varrho}\,. \tag{4.2}$$

To put these amplitudes in perspective, if we assume that the rest of the amplitude $f_{\varrho\to\varrho}(0)$ is principally absorptive and $\approx f_{\pi N}(0)$, then the anomalous contributions give rise to refractive, real parts which are 30–40% of the imaginary parts. The word "anomalous" is used to describe these real parts because they are not given by the "usual" dispersion relations, are large, and do not fall off with energy.

The presence of these effects in themselves has nothing to do with $\varrho^0$ dominance. Noting the characteristic factor of the width over the mass, we should regard the anomalous real parts as dynamical effects indigenous to unstable particles.

We now look at the high energy forward $\varrho^0$-nucleon elastic scattering amplitude in its "entirety", as a function of $m_i^2$ and $m_f^2$. We are interested only in the singularity represented by the dipion continuum and write the whole contribution of the loop integral as an unsubtracted dispersion integral over mediating dipion masses, using the "on-mass--shell" amplitudes to give the contribution across the "cut" which begins at $m_i^2, m_f^2 = 4m_\pi^2$. We write the contribution from the "bare" $\varrho^0$ of Fig. 11 as $\hat{f}_{\varrho\to\varrho}(0)$.

$$f_{\varrho\to\varrho}(0)/_{m^2_i, m^2_f} \approx$$

$$\approx \hat{f}_{\varrho\to\varrho}(0) - 2 f_{\pi N}(0) \int\limits_{4m^2_\pi}^{\infty} d(M^2)\left(\frac{i}{m_i^2 - M^2 + i\varepsilon}\right)\frac{m_\varrho \Gamma(M^2)}{\pi}\left(\frac{i}{m_f^2 - M^2 + i\varepsilon}\right) \tag{4.3}$$

Formally, we can interpret the dipion contribution to expression (4.3) as the $\varrho^0$ converting directly to a "pseudo particle" of mass $M$ with a coupling $-i(m_\varrho \Gamma(M^2)/\pi)^{1/2}$ and this "pseudo particle" scattering from the nucleon with an amplitude $2f_{\pi N}(0)$.

An interesting digression at this point would be to compare $\varrho^0$ meson scattering as mediated by pion pairs with, say, deuteron scattering as mediated by its constituent nucleons. One difference is that in deuteron scattering one is interested in the "$t$" dependence, not $m_i^2$ or $m_f^2$ dependence. Although we will not pursue the matter, some $t$ dependence arises from the pion loop integral of Fig. 10 for non-forward scattering. In this case $p$ wave pion pairs of mass, $M$, can be scattered into pion pairs of different partial waves and masses. This effect is not taken into account by expression (4.3) which applies only to forward scattering.

Unfortunately, the integral over $M^2$ in (4.3) diverges, the integrand $\propto 1/M^2$ for large $M^2$. However, if we are interested in only the differences [11] among Compton scattering, $\varrho^0$ photoproduction, and $\varrho^0$ elastic scattering, finite results can be achieved. We recognize that $\hat{f}_{\varrho\to\varrho}(0)$ is an "unphysical" quantity and insert $g_{\gamma\varrho}^2 f(0)$, as the phenomenological scattering amplitude for forward isovector Compton scattering. Now, we can write the differences (aside from the factors of $g_{\gamma\varrho}$) between this amplitude and the others as once subtracted integrals over $M^2$ which do converge. The arithmetic is straightforward, and we write our results as

$$f_{\gamma\to\varrho}(0) = g_{\gamma\varrho} f(0)[1+\eta_1+i\alpha_1],$$

$$f_{\varrho\to\varrho}(0) = f(0)\,[1+\eta_2+i\alpha_2].$$

For definiteness, assuming $f(0) \approx f_{\pi N}(0)$, $\alpha_1$ and $\alpha_2$ are just the "anomalous" contributions calculated before and are of order 30% and 40% respectively. $\eta_1$ and $\eta_2$ are the off-mass--shell contribution from Fig. 10 and in both cases are of order $-10\%$.

To summarize, we are led to expect deviations from the photon analogy of order 30 to 40% in the real parts and of order 10% in the imaginary parts of these amplitudes. We should not take the detailed calculations too seriously, but I think they should be regarded as indications of real, physical effects, and we should pay attention to their order of magnitude and direction. These ideas are somewhat encouraged by the experimental deviations which are observed [1].

In the next section we discuss the consequences of these ideas for reactions employing nuclear targets.

## 5. *The reactions* $\gamma+A\to\pi^++\pi^-+A$ *and* $\gamma+A\to\varrho^0+A$

We ignore the $\varrho^0$ resonance for the time being and discuss the theory of high energy, forward photoproduction of "nonresonant" pion pairs from complex nuclei. In order that our basic ideas be more applicable to nuclear targets, we introduce a refinement into the Drell-Söding theory. In essence, when the photon spontaneously converts into pion pairs, we will allow for the possibility of both pions scattering from the target.

To see how this possibility might be important we consider the high energy, forward production of symmetrical pion pairs of mass, $M$, by a photon of energy $K$. Now, $M^2/2K$

is the longitudinal momentum transfer to the target required to produce the pion pairs. The "uncertainty principle" then asserts that the incident photon converts into the pion pair at a distance in front of the target characteristically $\lesssim 2K/M^2$. However, the pions' "trajectories" diverge from each other at an angle $\theta \approx 2\sqrt{M^2 - 4m_\pi^2}/K$. Thus, the characteristic distance separating the two pions as they strike the target is $\lesssim 4\sqrt{M^2-4m_\pi^2}/M^2$ and is at most $\approx 1.4$ fermis and goes as $4/M$ for large $M$. We conclude that definitely for large nuclei, and possibly even for nucleons, contributing amplitudes where both pions strike the target (see Fig. 12) should be important.

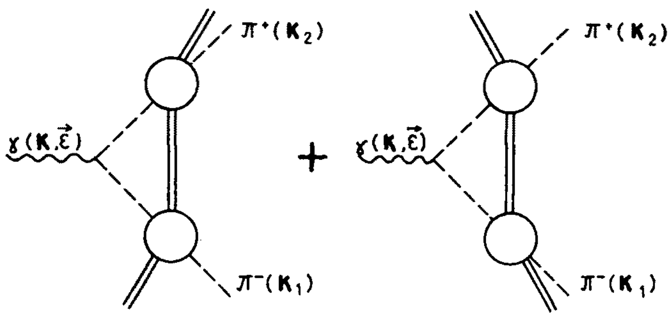



Fig. 12

Referring elsewhere [14] for a detailed analysis, we will content ourselves with some relevant results:

1. To a good approximation the effect of the amplitudes shown Fig. 12 is to reduce somewhat the amplitudes calculated with the Drell-Söding theory. For nucleon targets a crude estimate gives this reduction factor as $\approx 3/4$ (this result does not change any of our previous, qualitative results in any important way). The pion pairs are still predominantly $p$ wave in their centre-of-mass.

2. It turns out that not only are the pions' trajectories close together when striking the target, but to a good approximation we can neglect differences in their impact parameters altogether! A corollary (below) of this result applies to nuclei as targets.

3. In complex nuclei we can consider the photoproduced dipion state to propagate and multiply scatter like a "pseudo particle" with a forward scattering amplitude from

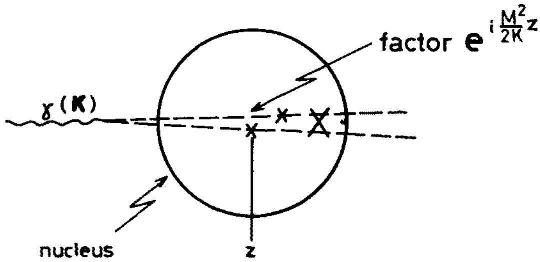



Fig. 13

a nucleon $f_P(0) \approx 2 \times \frac{4\pi i}{K} \sigma_{\pi N}(1 - i\alpha_\pi) \times$ (reduction factor) where $K$ is the photon's energy. Note that, contrary to some speculations, these results indicate it might be quite difficult to distinguish resonant from non-resonant particles by the way they multiply scatter in complex nuclei.

Making the approximations of the optical model, we write the production amplitude for pion pairs of mass $M$ by a photon of energy $K$ as $\vec{\varepsilon} \cdot \vec{M}$ where

$$\vec{\varepsilon} \cdot \vec{M} \propto e \frac{f_P(0)}{M^2} \vec{\varepsilon} \cdot (\vec{K}_2 - \vec{K}_1) \times N_{\text{eff}}^{\pi\pi}(M^2/2K),$$

$$N_{\text{eff}}^{\pi\pi}(M^2/2K) \approx \int d^2\vec{b}\, dz\, e^{\frac{iM^2}{2K}z} n(\vec{b}, z) \exp\left[\frac{2\pi i}{K} f_P(0) \int_z^\infty dz_1 n(\vec{b}, z_1)\right], \tag{5.1}$$

$z$ and $\vec{b}$ represent directions $\parallel$ and $\perp$ to $\vec{K}$ respectively and $n(\vec{b}, z)$ is the one particle nuclear number density. The phase factor $e^{i\frac{M^2}{2K}z}$ arises at the nucleon (position $\vec{b}$, $z$) where the pion pair first scatters and is "kicked" onto its mass shell and is due to the difference of $M^2/2K$ between the wave vectors of the initial photon and the pion pair. The factor $\exp\left[\frac{2\pi i}{K} f_P(0) \int_z^\infty dz_1 n(b, z_1)\right]$ gives the optical model "absorption" of that pion pair from the point $z$ on out. We illustrate these ideas pictorially in Fig. 13 where an "×" represents an interaction with a single nucleon.

We now consider pion pairs from the decay of a photoproduced $\varrho^0$. Our previous work says that one way we can photoproduce a $\varrho^0$ is to let the pion pairs of Fig. 13 combine together, as illustrated in Fig. 14.

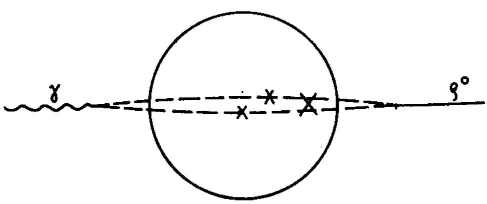



Fig. 14

If we look at the part of the amplitude of Fig. 14 where the pions adjoining the $\varrho^0$ are on-mass-shell, we get an "anomalous" amplitude for $\varrho^0$ photoproduction relevant to the nuclear problem. We find

$$f^A_{\gamma\to\varrho}(0)/_{\text{anomalous}} \approx i g_{\gamma\varrho} f_P(0) \frac{\Gamma(m_\varrho^2)}{m_\varrho} \times N_{\text{eff}}^{\pi\pi}(m_\varrho^2/2K). \tag{5.2}$$

As before, if we view the $\varrho^0$, say, through decay into lepton pairs the anomalous term should manifest itself as a larger than "normal" refractive piece in $\varrho^0$ photoproduction [14]. On the other hand, if we look at it through its principal decay mode of pion pairs it combines naturally with the nonresonant background so as to multiply that background by a factor $e^{i\delta}\cos\delta$. We would expect these amplitudes to combine with other "normal" amplitudes, arising from $\varrho^0$ photoproduction, in such a way that the dipion mass spectrum exhibits a "breakdown" of contributions similar to that shown in Fig. 9 [15].

Now, to discover those "other" mechanisms: one thing we have neglected is the fact that the $\varrho^0$ can multiply scatter in a nucleus. The "traditional" theory of Drell and Trefil takes this into account by assuming that the $\varrho^0$ is photoproduced on one nucleon with an amplitude $f_{\gamma\to\varrho}(0)$ and then is subject to rescattering by other nucleons with an amplitude $f_{\varrho\to\varrho}(0)$. Making the approximations of the optical model, the Drell-Trefil theory yields

$$f^A_{\gamma\to\varrho}(0) = f_{\gamma\to\varrho}(0)\times N^{\varrho}_{\text{eff}}(m^2_\varrho/2K),$$

$$N^{\varrho}_{\text{eff}}(m^2_\varrho/2K) \approx \int d^2\vec{b}\,dz\,e^{\frac{im^2_\varrho}{2K}z}\,n(\vec{b},z)\exp\left[\frac{2\pi i}{K}f_{\varrho\to\varrho}(0)\int_z^\infty dz_1 n(\vec{b},z_1)\right]. \tag{5.3}$$

The interpretation of the factors in $N^{\varrho}_{\text{eff}}$ are identical to those in the interpretation of $N^{\pi\pi}_{\text{eff}}$. We illustrate the Drell-Trefil contribution in Fig. 15. This mechanism was originally propos-

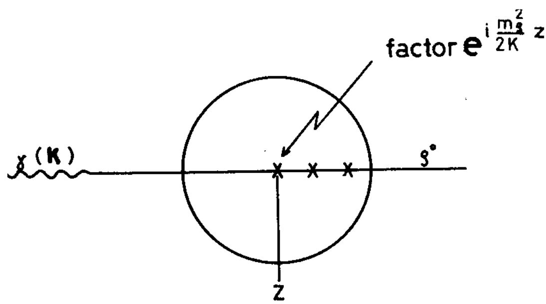



Fig. 15

ed with an eye toward measuring $f_{\gamma\to\varrho}(0)$ and $f_{\varrho\to\varrho}(0)$ independently, but we shall find that these quantities in the expression for $N^{\varrho}_{\text{eff}}$ require some reinterpretation.

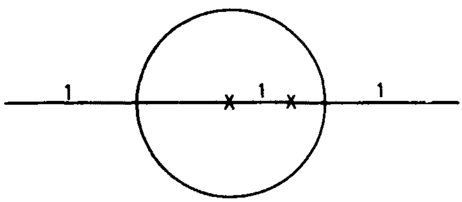
Just as the amplitudes of Fig. 14 neglected the multiple scattering of the $\varrho^0$, so do the Drell-Trefil amplitudes of Fig. 15 neglect the multiple scattering of the mediating dipion state. A true picture must allow for the multiple scattering of both the $\varrho^0$ and dipion states. In fact one can easily imagine a horrible complexity of "mixed" $\varrho^0$ meson-dipion multiple scattering as legitimate contributions to $\varrho^0$ photoproduction. However, at least quantitatively, we can make some sense of these things by looking at some of the properties of our two body scatterings in coordinate space.

### 6. *Two body amplitudes in coordinate space and multiple scattering in nuclei*

If a high energy (energy $K$) particle 1 of mass $M_1$ elastically scatters, mediated by particle 2 of mass $M_2$, then the longitudinal distance required by the scattering is of order $|2K/(M_1^2-M_2^2)|$. This result is required by the uncertainty principle and is just the inverse of the longitudinal momentum transfer required to convert from one particle to the other, while conserving energy. For a very nice full treatment of this subject we refer to some recent lectures by Gribov [16].

Now, coherent hadronic scatterings from complex nuclei characteristically take place near the downstream surface of the nucleus in a "shell" a few fermis thick. Characteristically, any contribution from a hadron scattering anywhere else is negligible because the final hadron is absorbed. The thickness of this shell, then, is determined by the mean free path, $\lambda$, of the hadron in the nucleus (according to the optical model $\lambda \approx 2/(\sigma_{tot}\langle n\rangle)$). For simplicity, we will assume that $\lambda$ is the same for all complex nuclei and relevant hadrons, and we will take $\lambda \approx 3$ fermis (it value for pi mesons).

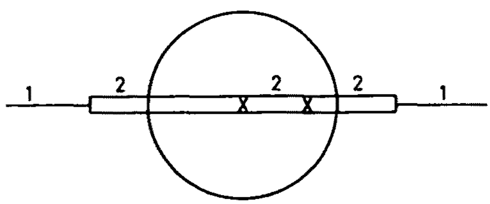
The relationship between the "$\lambda$" of particle 2 and the distance required to complete the scattering is quite important. We consider two extreme cases. With little effort [17] it is possible to show that, if $|2K/(M_1^2 - M_2^2)| \ll \lambda$, when particle 1 scatters from a nucleus it is particle 1, itself, which scatters multiply as shown in Fig. 16.




$|2K/(M_1^2 - M_2^2)| \ll \lambda$

Fig. 16

On the other hand, if $|2K/(M_1^2 - M_2^2)| \gg \lambda$, it is the mediating particle 2 which multiply scatters from the nucleus as a whole as shown in Fig. 17.




$|2K/(M_1^2 - M_2^2)| \gg \lambda$

Fig. 17

It is important to note that in these extreme limits complicated, "mixed" multiple scattering can be neglected.

This behaviour suggests some qualitative features of the problem of $\varrho^0$ photoproduction from complex nuclei where both the $\varrho^0$ and mediating pion pairs are free to multiply scatter. If we consider photon energies high enough so that $2K/m_\varrho^2 \gg \lambda$, we can neglect the difference between the mass of the photon and the $\varrho^0$. We now view the role of the mediating pion pairs in a nucleus in the following "crude" manner. Defining a "cut off mass", $M_c$, $M_c^2 \equiv 2K/\lambda$, we will assume all mediating pairs with mass $> M_c$ participate only in the two body interactions of photons and $\varrho^0$ mesons, and those with mass $< M_c$ multiply scatter from the nucleus as a whole.

Thus the relevant two body amplitudes, used in the Drell-Trefil theory, crudely speaking, receive contributions from mediating pion pairs of mass $> M_c$. So, for use in expression (5.3) (making use of expression (4.3)) we would substitute something like:

$$f_{\gamma\to\varrho}(0) \to f^{F}_{\gamma\to\varrho}(0)+g_{\gamma\varrho}f_P(0)\int_{4m_\pi^2}^{M_c^2} d(M^2)\left(\frac{i}{-M^2+i\varepsilon}\right)\frac{m_\varrho\Gamma(M^2)}{\pi}\left(\frac{i}{m_\varrho^2-M^2+i\varepsilon}\right)$$

and

$$f_{\varrho\to\varrho}(0) \to f_{\varrho\to\varrho}(0)+f_P(0)\int_{4m_\pi^2}^{M_c^2} d(M^2)\left(\frac{i}{m_\varrho^2-M^2+i\varepsilon}\right)\frac{m_\varrho\Gamma(M^2)}{\pi}\left(\frac{i}{m_\varrho^2-M^2+i\varepsilon}\right). \quad (6.1)$$

One amusing effect of this substitution is that the "anomalous" contributions to the two body amplitudes (which involve $M = m_\varrho$) are all cancelled away. [18]). (This cancellation actually turns out to be exact). However, the effect is less dramatic for the "normal" contributions which at $K = 20$ GeV just experience reductions of order 10%.

With our approximations the "complete" amplitude for $\varrho^0$ photoproduction is given by the Drell-Trefil term, (Eq. (5.3)) with the above modifications (6.1) plus a "dipion" contribution from the mechanism of Fig. 14, where intermediate pion pairs have mass $< M_c$ and can be written as:

$$f^A_{\gamma\to\varrho}(0)/_{\text{dipion}} \approx -f_P(0)N^{\pi\pi}_{\text{eff}}(0)\int_{4m_\pi^2}^{M_c^2} d(M^2)\left(\frac{i}{-M^2+i\varepsilon}\right)\frac{m_\varrho\Gamma(M^2)}{\pi}\left(\frac{i}{m_\varrho^2-M^2+i\varepsilon}\right). \quad (6.2)$$

The effect of the "dipion" term is to give an "anomalous" contribution (Eq. (5.2)) and at $K = 20$ GeV increase the "normal" $\varrho^0$ production amplitude by about 10%.

Thus at present laboratory energies it would seem that a Drell-Trefil theory, neglecting two body anomalous terms, plus the "anomalous" contribution (5.2) should be a good approximation. However, at "ultra" high energies this may not be so, since the dipion contribution (6.2) increases logarithmically with energy relative to the Drell-Trefil term. This increase is associated with the divergence of the integral in (Eq. (4.3)), and we would expect our ideas to break down "somewhere". It should be pointed out, though, that our substitutions (6.1) make "sense" for $M_c$ up to around 25 GeV or photon energies up to around 4000 GeV! In addition, as energy increases, other continua may become important (as $M_c > M_{\text{threshold}}$). Such contributions will tend, as well as contribution (6.2), to increase $\varrho^0$ production amplitudes.

Leaving the question of $\varrho^0$ photoproduction, we now discuss forward Compton scattering from complex nuclei. If we make the "traditional" assumption that the $\varrho^0$ is the only hadronic state mediating the proton (thus neglecting for simplicity contribution from other vector mesons), then according to our previous picture, we can regard the photon as particle 1 and the $\varrho^0$ meson as particle 2. At low energies, when $2K/m_\varrho^2 \ll \lambda$ the amplitude should $\propto A$, since the photon, itself, multiply scatters quite weakly. However, at high energies, where $2K/m_\varrho^2 \gg \lambda$ this amplitude should $\propto A^{2/3}$, since the $\varrho^0$ meson multiply scatters quite strongly. $K \approx 5$ GeV, when $2K/m_\varrho^2 = \lambda$, represents the "transition energy" between these extremes. Such a transition, which would be quite noticeable in large nuclei, seems not to appear as predicted [2]. Perhaps one reason for this is our neglect of pion pairs as mediators of the two body reactions.

Detailed analyses [6], allowing for the multiple scattering of only the $\varrho^0$ predict, where $2K/m_\varrho^2 \gg \lambda$, for the forward Compton scattering amplitude:

$$f^A_{\gamma\to\gamma}(0) = A\left[f_{\gamma\to\gamma}(0) - \frac{(f_{\gamma\to\varrho}(0))^2}{f_{\varrho\to\varrho}(0)}\right] + \text{surface terms} \propto A^{2/3}. \tag{6.3}$$

So, if the $\varrho^0$-photon analogy is violated, a term $\propto A$ persists to all energies!

Our crude prescription for discussing mediating pion pairs modifies the traditional analysis in several ways. Mediating pion pairs of mass $< M_c$ will make a direct contribution to the amplitude $\propto A^{2/3}$. Only mediating pion pairs of mass $M > M_c$ effectively contribute to the two body amplitudes in Eq. (6.3). Now, with our $\varrho^0$ dominance hypothesis the only differences among $f_{\gamma\to\gamma}(0)$, $f_{\gamma\to\varrho}(0)$ and $f_{\varrho\to\varrho}(0)$ are due to mediating pion pairs, and, as can be seen by studying Eq. (4.3), as $K$ and hence $M_c \to \infty$, the differences among these "effective" amplitudes $\to 0$. Thus as $K \to \infty$, the term $\propto A$ in forward Compton scattering "eventually" $\to 0$. This result should be regarded as somewhat surprising since pion pairs of arbitrarily high mass mediate the two body Compton scattering; *i. e.* the integral over $M^2$ in Eq. (4.3) diverges. However, our $\varrho^0$ dominance hypothesis couples all of these pion pairs to a $\varrho^0$ propagator which, as far as the photon is concerned, effectively "elongates" the short range contributions from high dipion masses.

Without a doubt these results "yearn" for a less crude theory which would make reasonable predictions to compare with experiment.

## 7. *Conclusions*

The conclusions that one can draw from these considerations are mixed. In some instances, when dealing with highly unstable particles, the effect of a mediating continuum gives rise to new dynamical effects which may be interesting in themselves to study. On the other hand, our considerations can be seen as limiting somewhat the feasibility of using nuclear targets to obtain two body physics. However, this is no disaster and being able to estimate "theoretical error bars" is a step forward in credibility. Finally, on the subject of $\varrho^0$ dominance, it should be emphasized that an "exact" $\varrho^0$-photon analogy is not a reasonable theoretical proposition and small experimental violations thus have no "catastrophic" theoretical moment. Their importance notwithstanding, the photon is more than a superposition of vector mesons.

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[7] We could just evaluate the real part of the "bubble diagram" of Fig. 3 with two subtractions. But the real part can receive contributions from much higher mass intermediate states than can the absorptive part; so this assumption would probably be too naive.

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[10] We part company at this point with the work of G. Kramer, J. L. Uretsky [*Phys. Rev.*, **181**, 1918 (1969)] and more recently G. Kramer, H. R. Quinn [*Nuclear Phys.*, **B27**, 77 (1971)] who view all $\varrho^0$ photo-production as a type of final state interaction of Drell-Söding pion pairs. However, their procedure seems quite different from our "Feynman amplitude" approach, and their results seem glaringly inconsistent with ours.

[11] We refer at this point to the views expressed by D. R. Yennie in *Hadronic Interactions Electrons and Photons*, Academic Press, London, 1971, p. 321.

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[13] I think the strongest experimental evidence to date in favour of these considerations in the reaction $\gamma+p\rightarrow\pi^++\pi^-+p$ is given by H. H. Bingham *et al.*, *Phys. Rev. Letters*, **24**, 955 (1970).

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[17] In the first extreme case multiple scattering of particle 2 is suppressed by phase factors, characteristically exp $[i((M_1^2-M_2^2)/2K)\lambda]$, arising from the longitudinal momentum transfer necessary for conversion between particles 1 and 2. In the second extreme case each multiple scattering of particle 1 is "cancelled out" by an equal and opposite contribution from a multiple scattering of particle 2.

[18] Contray to my own speculation in Ref. [12].