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HIGH ENERGY ELON-NUCLEUS COLLISIONS

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The optical formulation of high energy coherent reactions of elementary particles (elons) with nuclei is derived in an heuristic manner, which nevertheless competes in rigor with extant presentations of the subject.

The formalism is applied to elastic scattering on nuclei, including the possibility of collective low-lying nuclear excitations. Coherent production of ϱ and A mesonic states is discussed. Reliability of the nucleus as a measuring tool for elon attenuation is emphasized, and many corrections to the simplest approximation are examined. An approximate treatment of inelastic shadowing is given, and the speculation put forth that at very high energies there might be eigenstates of indefinite multiplicity which suffer anomalously low attenuation in nuclear matter.

The subject of this talk has been treated in many works, in a general theoretical framework borrowed from wave optics, and applied to nuclear scattering most elegantly by Glauber [1]. In the following, we shall specialize to moderately large nuclei, $A \gtrsim 10$, so that the nucleus may be described with some accuracy as an "optical medium". The small-nucleus case has been explored extensively by Glauber and collaborators [2], Czyż and Leśniak [3], Bassel and Wilkin [4], and Harrington [5].

Since the literature is so extensive, we shall derive results in a telegraphic manner, which amounts to providing the audience with a mnemonic device for obtaining relevant formulae with minimum effort. The approach follows the lines of Goldhaber and Joachain [6], but a few new points come up.

Because we work at high energy, we are interested in the nuclear analogue of Fraunhofer scattering: a wave of some elementary particle ("elon") impinges on a nucleus.

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Just on the other side of the nucleus appears a wave of this or another elon. Given the amplitude of this wave on an imaginary screen behind the nucleus, we may calculate the scattering amplitude just as in the Fraunhofer limit of optics, for small angles only. Since high energy scattering is largely confined to small angles, we need not chafe at this restriction of the Fraunhofer formula. Then the whole problem is reduced to the question: How does the incident wave evolve as it passes through the nucleus? All the dynamics are hidden in this question.

We must now begin to make assumptions, some of which will be qualified as the argument develops. The most important assumption is that the elon wave interacts with the nuclear medium as if it were an assembly of free neutrons and protons, whose position and momenta are given by the nuclear wave function. To understand this assumption, we examine some of its implications. First of all, the elon mean free path in nuclear matter is given by the elon-nucleon cross-section and the density distribution of nucleons, including any multinucleon correlations (in practice, correlation effects are usually fairly small, just at the ragged edge of detectability). Secondly, the elon does not encounter "extra particles" in the nucleus. For example, the density of virtual pions is not appreciably higher than that due to the pion clouds of all the individual nucleons, treated as free. Again, one may estimate small corrections coming from many-nucleon interactions, but for processes of interest here, these estimated corrections are negligible. A third, closely related, implication of the free-nucleon hypothesis is the notion that the shape of the nucleon is unaltered by the presen‹ e of other nucleons nearby. Unless such a shape distortion were very large even in the low density surface region of the nucleus, it would be unobservable.

Having decided that the nucleus is not a "black box", but rather a well defined entity, we may begin to compute. Note that our assumption lies at the heart of all "microscopic" theories of nuclear structure and low energy nuclear reactions. However, the analysis of high energy elon-nucleus reaction is much easier than the task of the low energy theorist, because the nucleus may be assumed to sit still while a fast elon "photographs" it. To interpret the elon scattering amplitude as a photograph (actually, more of a hologram), we need not suppose that the nucleus is completely motionless during the time of passage, but only that any pulses which it produces in the nuclear medium travel more slowly than the elon itself.

Although we are most interested in wave propagation, it is helpful to orient ourselves with the simpler problem of a classical beam of particles, having intensity $I(\boldsymbol{r})$, passing in the z direction through a gas of scatterers each with cross-section σ. If the scatterers are uncorrelated, we have the familiar equation

$$\frac{dI(\boldsymbol{r})}{dz} = -\varrho(\boldsymbol{r})\sigma I(\boldsymbol{r}) \tag{1}$$

where $\varrho(\boldsymbol{r})$ is the gas density at $\boldsymbol{r}$.

The lowest order effect of correlations is to give the modified attenuation formula

$$\frac{dI}{dz} = -\varrho\sigma(1-\varrho\sigma R_c)I \tag{2}$$

where R_c is the correlation length, defined positive for attractive correlations, negative for repulsive.

This agrees with intuition; for example, attractive correlations, or clustering, imply that if a beam particle does not strike a particular scatterer, it has a better than random (*i. e.*, no correlations present) chance of avoiding the scatterers clustered with the first one — the attenuation is weakened. The formula (2) depends on the assumption that R_c is small compared to the mean free path of a beam particle in the gas. Otherwise, one expects a similar formula to hold, with an effective R_c, which depends not only on correlations but also on the finite range of the force between beam and target particles. The correlation length is defined (Ref. [6]) in terms of the two particle density distribution by the relations,

$$\varrho^{(2)}(\boldsymbol{r}, \boldsymbol{r}') \equiv \varrho(\boldsymbol{r})\, \delta(\boldsymbol{r}-\boldsymbol{r}') + \\ + \varrho(\boldsymbol{r})\, \varrho(\boldsymbol{r}')\, (1 + C(\boldsymbol{r}, \boldsymbol{r}')) \tag{3}$$

and

$$\varrho(\boldsymbol{r})\, R_c = \int_z^{\infty} dz' \varrho(\boldsymbol{r}')\, C(\boldsymbol{r}, \boldsymbol{r}') \tag{4}$$

with $\boldsymbol{r} = (x, y, z')$ in the integral.

For a uniform medium, ϱ and R_c are constants, but for a finite system such as a nucleus, both quantities depend on position. In saturated nuclear matter, as in the centre of a large nucleus, repulsive short range forces are believed to keep any two nucleons from occupying the same position, implying $R_c \approx -\frac{1}{2}$ or -1 fm. However, near the edge of a nucleus, where the density is lower, there is less knowledge of R_c. It might even become positive, reflecting alpha-particle clustering, but the range $-1 \lesssim R_c \lesssim +1$ seems quite adequate to account for any plausible nuclear correlations [6].

We may adapt the above discussion to wave propagation. First consider the problem of a high energy wave interacting with a potential, for example, a Coulomb potential. Neglecting spin, we write a Klein-Gordon equation

$$\{(E-V)^2 + \nabla^2 - m^2\}\, \psi = 0 \tag{5}$$

and solve in the eikonal approximation, taking $E^2 - m^2 = k^2$ and $\psi \equiv e^{ikz}\varphi$, giving

$$\left\{\frac{\partial}{\partial z} + i(V/v)\right\} \varphi = i(V^2 + \nabla^2)\varphi/2k \tag{6}$$

with $v = k/E$ the phase velocity of the wave. The eikonal approximation consists in neglecting the right-hand side for large k. The resulting equation is identical in form with that obtained in non-relativistic physics. The only difference in the relativistic case is that v must be less than c, hence the effect of the potential is independent of k for k large. In the non-relativistic case the potential becomes infinitely weak for $k \to \infty$.

The equation for ψ in the eikonal approximation is

$$\frac{d\psi}{dz} = i(k - V/v)\psi. \tag{7}$$

When V is produced by the scattering in a gas of uncorrelated particles, the equation is

$$\frac{d\psi}{dz} = (ik - \varrho(\boldsymbol{r})\sigma'/2)\psi \tag{8}$$

with $\sigma' = \sigma(1-i\alpha)$, and α the ratio of real to imaginary part of the forward scattering amplitude of the elon on a gas particle. The convenient σ' notation was introduced by Margolis and collaborators [7]. Note that the intensity $I = |\psi|^2$ obeys precisely the classical eqation (1) for its attenuation. However, in the more general case, including correlations, one must discuss wave propagation instead of intensity attenuation. The role of σ in the classical case is played by $\sigma'/2$ for the wave, giving for correlation effects, instead of Eq. (2),

$$\frac{d\psi}{dz} = \{ik + \varrho\sigma'/2(1-\varrho\sigma' R_c/2)\}\psi \tag{9}$$

so that wave superposition implies that correlation effects on intensity are half as strong as the classical case indicates. For pions in saturated nuclear matter, we have $\varrho \approx (1/8) \times$ $\times$ nucleon $(\text{fm})^{-3}$, $\sigma \approx 2.5\ \text{fm}^2$, and $-0.5 \gtrsim R_c \gtrsim -1$ fm, so the correlation correction is no more than 30%. Since the centre of a nucleus is practically opaque to pions, this correction is impossible to measure. In the more transparent surface region, the correction should not be more than 15%, but the sign is unknown. Having pointed out this effect, we may simplify the writing of formulae by omitting it in the rest of our discussion.

Integrating Eq. (7) we obtain the effect of the nuclear obstacle on the wave at the far side of the nucleus,

$$\psi(\boldsymbol{r}) = e^{ikz}e^{2i\delta(\boldsymbol{b})}$$

$$2i\delta(\boldsymbol{b}) = -\int_{-\infty}^{\infty} dz'\varrho(\boldsymbol{b}, z')\sigma'/2. \tag{10}$$

Of course, we assume $\varrho(\boldsymbol{b}, z)$ has fallen to a negligible value at the point z. There are corrections to this expression, which we shall discuss later. First, let us compute the elastic scattering amplitude, assuming Eq. (10) is exact.

By Huygens' principle, the small angle Fraunhofer scattering is given by adding up the wave amplitudes at different points $\boldsymbol{b}$ on a screen at fixed z just behind the nucleus, with phase factors $e^{\boldsymbol{q}\cdot\boldsymbol{b}}$

$$F(\boldsymbol{q}) = (ik/2\pi)\int d^2\boldsymbol{b}e^{i\boldsymbol{q}\cdot\boldsymbol{b}}[1-e^{2i\delta(\boldsymbol{b})}]. \tag{11}$$

The numerical factor before the integral sign may be fixed by considering the case of very weak interaction, $\sigma\varrho R \ll 1$, where R is the nuclear radius. To lowest order in σ' we have

$$F(0) = f(0)\int d^3r\varrho(\boldsymbol{r}) = Af(0) \tag{12}$$

and, from Eq. (11) plus the optical theorem

$$\sigma_{\text{Nucleus}} = (4\pi/k)\,\text{Im}\,F = A\sigma, \tag{13}$$

where $f(0)$ is the forward scattering amplitude on a single nucleon, and $A = N+Z$ is the nuclear mass number (N neutrons, Z protons). For radius R, $(1-e^{2i\delta(b)})$ is close to unity for $|\boldsymbol{b}| < R$, and falls to zero as $|\boldsymbol{b}|$ increases beyond R. The resultant F represents a diffraction pattern of a black disk with fuzzy edges. The differential cross-section $|F|^2$ has its maximum at $\boldsymbol{q} = 0$ with secondary maxima and minima separated by $\Delta q \approx 1/R$.

What are the corrections to Eq. (11)? Since we are restricting the discussion to small angles and high energies, the Fraunhofer approximation is a good one, so that the corrections must all correspond to modifications of $\delta(\boldsymbol{b})$. The first modification arises if the elon in question is charged. We consider explicitly the long range Coulomb potential Zee'/r where e' is the elon charge. The cheapest way to find the effect of this potential is to make contact with the conventional partial wave expansion, which looks quite similar to Eq. (10)

$$F(\boldsymbol{q}) = \sum_l \frac{(2l+1)}{2ik} P_l(\cos\theta)\,[e^{2i\delta_l}-1]. \tag{14}$$

For small θ it may be shown [6] that Eq. (14) is equivalent to Eq. (11) with the substitution $l+\frac{1}{2} = kb$. To separate the long range Coulomb effects, we write $\delta_l = \Delta_l+\sigma_l$ where σ_l is the Coulomb phase shift. Then we have

$$F(\boldsymbol{q}) = F_{\text{Nuc}}+F_{\text{PC}}$$

where F_{PC} is the well-known [6] scattering amplitude for the pure Coulomb potential of a point charge Ze, and F_{Nuc} is defined by

$$F_{\text{Nuc}} = \sum_l \frac{(2l+1)}{2ik} P_l(\cos\theta) e^{2i\sigma_l}(e^{2i\Delta_l}-1) \tag{15}$$

and Δ_l is the sum of the strictly nuclear force effect, and the difference between the phase shift for a distributed charge (the actual case), and a point charge, $\Delta_l \to \Delta(\boldsymbol{b})$ vanishes outside the nucleus. Inside the nucleus ($b \lesssim R$), it is

$$2i\Delta(\boldsymbol{b}) = -\sigma' \int_{-\infty}^{\infty} dz\,\varrho(\boldsymbol{r})/2 -$$

$$-i \int_{-\infty}^{\infty} dz[V_{\text{dist}}(\boldsymbol{r})-Zee'/r]/v. \tag{16}$$

The integral of $V_{\text{PC}} = Zee'/r$ has a logarithmic singularity at $b = 0$, but this is numerically unimportant.

In our discussion to this point, we assumed only short-range correlations between nucleons. However, many nuclei have bands of low-lying excited states corresponding to collective motions which may be described by a few collective coordinates α_i. For example, these coordinates might be the orientation angles of the long axis of an ellipsoidal nucleus like uranium. Such collective motions are on a time scale of one inverse MeV, or 200 fm/c. Since the elon passes through the nucleus in a time $<2R/c < 15$ fm/c, it is a good approximation to neglect changes in the collective coordinates during the elon passage. Then the scattering amplitude becomes a function of the collective coordinates

$$F = \hat{F}(\boldsymbol{q}, \alpha_i). \tag{17}$$

We may consider two cases for application of this formula. First, strictly elastic scattering; the nucleus recoils in its ground state, giving

$$F(\boldsymbol{q}) = \langle 0 | \hat{F}(\boldsymbol{q}, \alpha_i) | 0 \rangle. \tag{18}$$

If the ground state has $J = 0$ and the α_i describe an orientation axis, then $F(\boldsymbol{q})$ is obtained by averaging $\hat{F}$ over directions of orientation. Obviously, this would tend to blur the maxima in $|F|^2$, though not drastically altering the minima because F may have oscillating signs. The second case occurs if the state of the recoiling nucleus is not fully known; it may be the ground state, or a member of a band of low-lying excited states. Then the differential cross-section is

$$d\sigma/d\Omega = \sum_{n \text{ in band}} |\langle n|\hat{F}|0\rangle|^2 =$$

$$= \langle 0| \ |\hat{F}(\boldsymbol{q}, \alpha_i)|^2 | 0 \rangle. \tag{19}$$

This time the cross-section is averaged over α_i, producing an even greater blurring of diffraction maxima and minima. This is seen dramatically in the 19.3 GeV/c $p-{}^{208}$Pb and $p-{}^{238}$U scattering data of Bellettini *et al.* (Ref. [6]). Spherical lead has a much sharper diffraction pattern than prolate uranium — a fine example of photographing the nucleus. Professor Walecka informs us that elastic electron scattering data of the Stanford group exhibit the milder blurring associated with Eq. (18). Other fairly direct quantitative evidence for nuclear deformation comes from muonic X-rays [8].

There are two more corrections to be considered, which modify the simple theory behind Eq. (10). First, we have assumed exponential attenuation of the elon wave in the nucleus. Actually, the exponential is an approximation appropriate for a gas with an infinite number of particles. This is a difficulty even in the classical theory with which we began. The probability that the beam will pass through an obstacle is the product of the probabilities for evading each and every one of the particles in the obstacle. Only in the limit $A \to \infty$ with finite density ϱ does the product become an exponential. Thinking of the classical case leads one easily to the conclusion (neglecting correlations) that $e^{2i\delta}$ has a correction of order $-i\delta\, e^{2i\delta}/A$. On the edge of a nucleus, one has $-i\delta \approx \frac{1}{2}$ typically or a 2% correction to the amplitude $(1-e^{2i\delta})$ for $A = 10$, and much smaller corrections at the centre. However, for production of an elon b different from the incoming elon a, the "1" term is not present, **and** 5% (in amplitude) or 10% (in cross--section) corrections might occur. For $A = 20$ this would be halved. We shall ignore this exponential approximation error, which is of comparable order to the uncertainties arising from correlations.

The final correction to be discussed at this point might be labelled "edge effects". The point is best seen by applying formula (11) to scattering on a **single** nucleon;

$$f(\boldsymbol{q}) = \frac{ki}{2\pi} \int d^2 b e^{i\boldsymbol{q}\cdot\boldsymbol{b}} [1 - e^{2i\delta_s(\boldsymbol{b})}] \tag{20}$$

where δ_s is the phase shift at impact $\boldsymbol{b}$, for a nucleon target at the origin of coordinates. Taking $f(\boldsymbol{q})$ in a Gaussian form, $f = f_0\, e^{-Bq^2/2}$, one arrives by Glauber's [1] technique at the plausible conclusion that the density distribution $\varrho(\boldsymbol{r})$ should be replaced by an effective density distribution,

$$\varrho_{\mathrm{eff}}(\boldsymbol{r}) = (2\pi B)^{-3/2} \int d^3r' e^{-(\boldsymbol{r}-\boldsymbol{r}')^2/2B} \varrho(\boldsymbol{r}') \tag{21}$$

smeared to take account of the finite range $\sqrt{3B} = 1.1$ fm for hadron-nucleon interactions. In the interior of the nucleus, $\varrho_{\mathrm{eff}}(\boldsymbol{r})$ is almost equal to $\varrho(\boldsymbol{r})$ because ϱ is nearly constant, but at the edge of the nucleus ϱ_{eff} is more diffuse than $\varrho(\boldsymbol{r})$. Note that we still have the same normalization of ϱ,

$$\int d^3r \varrho_{\mathrm{eff}}(\boldsymbol{r}) = \int d^3r \varrho(\boldsymbol{r}) = A. \tag{22}$$

For typical hadron-nucleon interactions the effect of the smearing is to increase the effective half-density radius of a large nucleus by about 0.4 fm. The radius is best determined experimentally by electron scattering, which gives a nuclear charge distribution, and since the root mean square charge radius of an individual nucleon is known to be about $\sqrt{3B/2}$, the charge radius of the nucleus is about 0.2 fm greater than the radius of the density distribution of nucleons. Thus, the net effect is to increase the "hadronic interaction radius" of the nucleus by about 0.2 fm compared to the charge radius [6].

One kind of correlation effect which is unlikely to play much part in elastic scattering, but conceivably could be more important in production reactions, is "channeling". If the nuclear interior has a crystalline structure, there could be special paths between two or more aligned columns of nucleons, with a lower than average "optical density" or opacity all along the path. Because individual nucleons are diffuse objects, this effect could only be big if the spacing between adjacent columns were bigger than 1 fm which is not likely.

Having made a fairly complete survey of the high energy technique for computing elastic scattering on large nuclei, let us turn to the subject of coherent production. This is a class of reactions in which outgoing elon b is different from incoming a, but the struck nucleus recoils as a whole in its ground state. For weak a to b transitions, the nuclear amplitude is

$$\mathscr{A}(\boldsymbol{q}) = \alpha \int d^3r e^{i\boldsymbol{q}\cdot\boldsymbol{r}} e^{i\delta_a(\boldsymbol{r})} e^{i\delta_b(\boldsymbol{r})} \varrho_{\mathrm{eff}}(\boldsymbol{r}) \tag{23}$$

where α is the forward scattering amplitude for $a \to b$ on a single nucleon, and δ_a and δ_b give the complex phase shifts of the incoming and outgoing waves,

$$\delta_a(\boldsymbol{r}) = \int_{-\infty}^{z} dz' \varrho_{\mathrm{eff}}(\boldsymbol{r}') \sigma_a'/2 \tag{24}$$

$$\delta_b(\boldsymbol{r}) = \int_{z}^{\infty} dz' \varrho_{\mathrm{eff}}(\boldsymbol{r}') \sigma_b'/2. \tag{25}$$

Depending on the interaction radius for $a \to a$, $a \to b$ and $b \to b$ on a single nucleon, the effective densities in equations (23), (24) and (25) may differ in their degree of surface diffuseness. As far as I know, the significance of such differences has not been explored in the published literature, but investigations relevant to this point are underway in Cracow [9].

The most nearly complete experimental studies of coherent production reactions have been devoted to photoproduction of vector mesons, especially the ϱ resonance, in a variety of nuclei. For this case, $\sigma_a = \sigma_\gamma = 0$ permits us to simplify the formula if $q_{||}$, the component of $\boldsymbol{q}$ parallel to z, may be neglected.

$$\mathscr{A}(\boldsymbol{q}, \gamma \to \varrho) = \alpha(\gamma \to \varrho) \int d^2 b e^{i\boldsymbol{q}\cdot\boldsymbol{b}} \int_{-\infty}^{\infty} dz \times$$

$$\times \varrho(\boldsymbol{r}) e^{-\int_z^\infty \varrho\sigma_b'/2dz'} = (\alpha/f_\varrho(0)) F_\varrho(\boldsymbol{q}) \tag{26}$$

where $f_\varrho(0)$ is the forward scattering amplitude of a ϱ on a nucleon, and $F_\varrho(\boldsymbol{q})$ is the ϱ-nucleus elastic scattering amplitude. We have the result then, that the amplitudes for ϱ photoproduction and for ϱ elastic scattering bear the same ratio on the nucleus and on a nucleon! This remarkable result, an example of the vector meson dominance hypothesis, holds subject to the condition that $q_{||}$ is negligible. The VMD hypothesis in general states that, for any elementary reaction $\gamma \to \chi$, the amplitude is simply proportional to the corresponding amplitude for $\varrho \to \chi$ with a universal proportionality constant

$$\alpha(\boldsymbol{q}, \gamma \to \chi) = \varepsilon\alpha(\boldsymbol{q}, \varrho \to \chi). \tag{27}$$

Stodolsky has remarked how this hypothesis for elementary reactions automatically leads to the corresponding result for nuclei at high bombarding energies. The amplitude for $\varrho \to \chi$ in a nucleus is the sum of the amplitudes for the ϱ to scatter on any nucleon, multiplied by the amplitude for the ϱ to produce a χ which escapes from the nucleus, after this first interaction (the first interaction could itself be the step $\varrho \to \chi$). The amplitude for the photon to produce χ is the same sum, except that the amplitude on the first nucleon is multiplied by ε, hence VMD for the nucleus. In this derivation we have omitted the other vector mesons, ω and φ, which complicate the practice but not the principle of VMD.

Experimental results for ϱ photoproduction seem to be converging to a consensus that the data require a ϱ-nucleon total cross-section of about 25 mb, like that of the π, and probably also a repulsive (negative) real part of the forward ϱ-nucleon amplitude, again like that of the π. All this is very pleasing for advocates of the quark model, which says ϱ and π should interact the same way at high energies.

The most extensive results aside from photoproduction are those for the reaction $\pi \to A_1$, where the A_1 is a system of mass about 1100 MeV, and width perhaps 400 MeV. The nature of the A_1 (*e. g.*, resonance or not?) is unclear. There is no compelling evidence for its production in any reaction except $\pi \to A_1$. The data from a Freon bubble chamber [10], and from a counter experiment with a range of nuclear targets [11], both indicate $\sigma(A_1N) \lesssim \sigma(\pi N)$. One might look to explain the difficulty by the breakdown of our coherent production formalism, but the success for photoproduction certainly makes this an unappealing last resort, to be adopted only if necessary. Varying the range of one or more of the three $\varrho(\boldsymbol{r})$ which appear in the coherent production amplitude is the only alternative way to escape from the result for $\sigma(A_1N)$. For the Freon data, variation of all three in the same way produced little effect on the conclusions, but a more selective procedure could conceivably alter the results [9]. The counter data have better angular resolution that the heavy liquid bubble chamber, so that they give a strong constraint, especially on the ϱ_{eff} for the $\pi \to A_1$ transition, which does not appear significantly bigger than that for p-nucleus scattering. We want to discuss somewhat more quantitatively than in Ref. [10], why the total coherent production cross-section is relatively insensitive to changes in all the ϱ_{eff}'s at once.

By the Fourier integral theorem, we have

$$\sigma_{coh} = \int dt \frac{d\sigma}{dt} = \int \frac{d^2q}{\pi} |\mathscr{A}(\boldsymbol{q})|^2 = $$

$$= 4\pi \frac{d\sigma^s}{dt}(0) \int d^2b I(\boldsymbol{b}) \tag{28}$$

with

$$I(\boldsymbol{b}) = \left| \int_{-\infty}^{\infty} dz e^{i\delta_a(r)} e^{i\delta_b(r)} \varrho(\boldsymbol{r}) e^{iq_{||}z} \right|^2 \tag{29}$$

and $\frac{d\sigma^s}{dt}(0)$ is the forward A_1 production cross-section on hydrogen. We may write

$$\sigma_{coh} = 4\pi \frac{d\sigma^s}{dt}(0) A \langle e^{i(\delta_a(r) + \delta_b(r) + q_{||}z)} \times$$

$$\times \int dz' \varrho(r') e^{-i(\delta_a{}^*(r') + \delta_b{}^*(r') + q_{||}z')} \rangle \tag{30}$$

where $\langle \quad \rangle$ means "$\boldsymbol{r}$ — averaged over the nuclear density". The bracket obeys the inequality

$$\langle \quad \rangle \lesssim \langle \int_{-\infty}^{\infty} dz' \varrho(\boldsymbol{r}') e^{-\int_{-\infty}^{\infty} dz' \varrho(r') \sigma_m} \rangle \lesssim$$

$$\lesssim 1/(e\, \sigma_m) \tag{31}$$

where σ_m is the minimum of σ_a and σ_b. For light nuclei and pionic cross-sections, the bracket approaches its upper limit. Since a function varies slowly near its maximum, σ_{coh} is insensitive to changes in nuclear parameters, including the uncertainties in smearing of ϱ by the hadron-nucleon interaction. For asymptotically large nuclei, production occurs in a ring of radius R and width $dR \sim \lambda^2/R$ (neglecting nuclear surface thickness, $t \lesssim \lambda/3$), where λ is the pion mean free path near the nuclear surface. This indicates that the total coherent production cross-section should approach a constant as $A \to \infty$. Numerical calculations suggest that this might happen near the end of the periodic table.

Equation (29) for the total coherent cross-section indicates that σ_{coh} is a quite direct measure of the attenuation of the A_1 production by nuclear interactions. What could make that attenuation small even if $\sigma(A_1N)$ were large? There are two obvious possibilities. The first is the so-called "inelastic shadowing" first emphasized by Stodolsky. The idea is best seen in the context of elastic scattering on the nucleus as a whole. Consider, for example (the only one with good data available), a neutron wave incident on a nucleus. As it collides with some target nucleon, the neutron may turn into an excited neutron, or N*, which later deexcites, again joining the neutron wave. Let us treat this problem in a somewhat more abstract way. At a fixed energy, the mass of an N* which can be coherently produced is limited by a longitudinal ($q_{||}$) momentum transfer cutoff. We simplify things by assuming that the $q_{||}$ cutoff restricts us to a finite set of N*'s and setting $q_{||}$ to zero for production of any member of this set. Then we obtain a matrix equation for propagation of any combination of N*'s through the nucleus

$$\frac{d\varphi_i}{dz} = -\left(\frac{\sigma'}{2}\right)_{ij}\varphi_j \tag{32}$$

where j is summed. The diagonal components of the σ' matrix give the attenuation of the i-th component of φ by itself. The off-diagonal components change φ_i to φ_j. Time reversal invariance implies that the matrix is symmetric and may be diagonalized by a complex orthogonal (but not necessarily unitary) matrix. The eigenvectors must all have positive or zero cross-sections, but the imaginary parts of the eigenvalues are unrestricted. Clearly, if some eigenvectors have smaller than nucleon-nucleon cross-sections, then the nuclear attenuation of n may be less than its free cross-section would indicate. If we add one more assumption, that the diagonalizing matrix is unitary, then it is a theorem that the nuclear attenuation must be less than or equal to that inferred from the free nucleon cross-section. That is, inelastic shadowing would make the nucleus more transparent. An indication of such an effect comes from the neutron-nucleus total cross-section measurements of Engler *et al.* [12], who find that between 8 and 21 GeV the cross-sections on **all** nuclei decrease by 3%. This is surprising because large nuclei are almost opaque disks of radius $R \sim A^{1/3}$, so that their cross-sections should change less than those of small targets. Numerical estimates indicate that the effective attenuation cross-section on nucleons in Pb must decrease by $9 \pm 3\%$, while the hydrogen cross-section decreased by only $3 \pm 1\%$. We have a possible inelastic shadowing contribution of about -1.6 mb to the effective attenuation cross-section. In the A_1 case, it is quite conceivable in principle that inelastic shadowing could increase its nuclear production, but for present experimental data the beam energy is such that the A_1 itself is by far the most important state produced, so that this explanation of its copious production fails. At very high energies, there is the amusing possibility of an eigenvector of σ'_{ij} with a very low (near zero) cross-section on nucleons. The eigenvector **might** be a superposition of multipion states. Its low absorption could also enhance its production in nucleon-nucleon collisions, making it a dominant feature of interactions with nucleons **or** nuclei.

The only alternative explanation of the low A_1 attenuation which seems plausible to me is that the A_1 internal structure changes drastically between its formation and its detection as a $\varrho\pi$ state. This will be studied in another publication.

Two topics omitted from this report are included both in Ref. [6] and in the contribution by Professor Bingham to these proceedings. These are incoherent scattering from nuclei, and selection rules for coherent production.

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