RELATIVISTIC EIKONAL APPROXIMATION FOR INELASTIC PROCESSES*

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The eikonal approximation for the contributions to inelastic scattering from arbitrarily crossed ladder graphs is considered. The particle represented by one of the "side-pieces" of the ladders is allowed to change its state and mass at each rung; the other side-piece particle is restricted to have at most one change of state, and that only if its elastic interactions are the same in the two states. When the denominators of the latter side-piece can be linearized the amplitudes reduce to forms essentially identical to those foud in non-relativistic coupled--channel potential theory. This result is independent of the ratio of the energy to the side--piece mass and connects smoothly the non-relativistic and extreme relativistic regimes. Its implications for absorptive models are discussed briefly.

The coupled channel eikonal approximation has been used extensively in high-energy particle and nuclear physics in recent years. It has furnished a derivation of the absorptive model [1–4] for two-body and quasi-two-body exchange reactions, and has also been used to determine the effect of inelastic intermediate states in the high-energy scattering of elementary particles from nuclei [5].

Until recently [6–8] the coupled channel eikonal approximation has been based exclusively on potential theory. This starts from a Hamiltonian

$$H = K+h+V$$

where $K = p^2/2m$ is the kinetic energy, h the internal Hamiltonian, and V the potential which has inelastic as well as elastic matrix elements. A convenient basis for scattering problems is the complete set of states $|\boldsymbol{p}, \mu\rangle$, where

$$[K+h]\,|\boldsymbol{p}, \mu\rangle = \left[\frac{p^2}{2m}+e(\mu)\right]|\boldsymbol{p}, \mu\rangle.$$

The T-matrix is given, formally at least, by the Born series

$$T = \sum_n T^{(n)},$$

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where

$$T^{(n)} = V(G_0 V)^{n-1},$$

with

$$G_0 = (E + i\varepsilon - K - h)^{-1}.$$

For purposes of comparison with a relativistic formula to be obtained below, the explicit form

$$\langle \boldsymbol{p}_f, \mu_f | T^{(n)} | \boldsymbol{p}_i, \mu_i \rangle = \sum_{\{\mu\}} \int \frac{d^3 p_{n-1}}{(2\pi)^3} \cdots \frac{d^3 p_1}{(2\pi)^3} \times$$

$$\times V(\mu_f, \mu_{n-1}; \boldsymbol{p}_f - \boldsymbol{p}_{n-1}) \frac{1}{\dfrac{p_i^2}{2m} + e(\mu_i) + i\varepsilon - \dfrac{p_{n-1}^2}{2m} - e(\mu_{n-1})} \cdots$$

$$\ldots V(\mu_2, \mu_1; \boldsymbol{p}_2 - \boldsymbol{p}_1) \frac{1}{\dfrac{p_i^2}{2m} + e(\mu_i) + i\varepsilon - \dfrac{p_1^2}{2m} - e(\mu_1)} V(\mu_1, \mu_i; \boldsymbol{p}_1 - \boldsymbol{p}_i),$$

where

$$\langle \boldsymbol{p}_2, \mu_2 | V | \boldsymbol{p}_1, \mu_1 \rangle \equiv V(\mu_2, \mu_1; \boldsymbol{p}_2 - \boldsymbol{p}_1),$$

is needed. The different terms in the Born series can be represented by simple diagrams: A particular $n = 3$ term is shown in Fig. 1.

To put some sort of relativistic foundation under this theory (which is, after all, usually used at relativistic energies), some recent work [9–16] on the relativistic eikonal

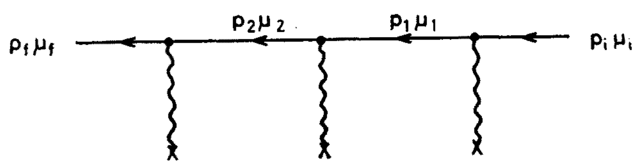

Fig. 1. The $n = 3$ diagram in coupled-channel potential theory

approximation can be used as a guide. Consider the relativistic analogue of Fig. 1 shown in Fig. 2. The top line (the "p-line") is allowed to change its state and mass at each vertex, while the bottom line (the "q-line") is restricted to be almost inert, with at most one change

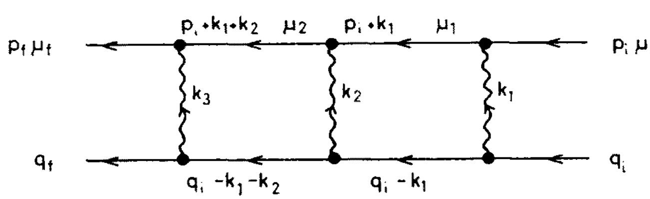

Fig. 2. The $n = 3$ ladder diagram in field theory. The amplitude $A^{(3)}$ also contains the 5 other diagrams obtained by permuting the points of attachment of the rungs to the q-line

of state, and that only if the two states have the same mass and elastic interactions. The restrictions on the q-line are imposed so that the simplifying cancellations [16] which occur in the elastic case can also occur here.

The general term in the relativistic Born series which sums all arbitrarily crossed ladder diagrams is

$$A^{(n)}(\mu_f, \mu_i) = -i^n \sum_{\{\mu\}} \int \frac{d^4k_n}{(2\pi)^4} \cdots \frac{d^4k_1}{(2\pi)^4} (2\pi)^4 \delta^4(p_f - p_i - k_1 - \ldots - k_n) \times$$

$$\times f(\mu_f, \mu_{n-1}; -k_n^2) \ldots f(\mu_1, \mu_i; -k_1^2) \left[(p_i + k_1 + \ldots + k_{n-1})^2 - M(\mu_{n-1})^2 + i\varepsilon\right]^{-1} \ldots$$

$$\ldots \left[(p_i + k_1)^2 - M^2(\mu_1) + i\varepsilon\right]^{-1} \Big\{ \sum_P \left[(q_i - k_{P1} - \ldots - k_{P(n-1)})^2 - \right.$$

$$\left. - M^2 + i\varepsilon\right]^{-1} \ldots \left[(q_i - k_{P1})^2 - M^2 + i\varepsilon\right]^{-1} \Big\}.$$

Here $f(\mu_2, \mu_1; -k^2)$ is the propagator for the exchange lines represented by the rungs of the ladders, and the sum over P indicates the sum over all permutations of the points of attachment of the rungs to the q-line.

This expression for $A^{(n)}$ is now simplified by assuming that the q-line denominators can be linearized, *i. e.* that

$$(q_i - k)^2 - M^2 = -2q \cdot k + (q_f - q_i) \cdot k + k^2 =$$

$$= -2q \cdot k,$$

where $q = \frac{1}{2}(q_f + q_i)$. Clearly this has a chance of being a valid approximation only if $\boldsymbol{q}$ is very large compared to the momentum transfer and the mass of the exchanged particle, and this can be due to either $\boldsymbol{q}$ or M being large, or both. (For the moment the momenta in the center-of-mass system are considered, for it is here where the exchange momenta are likely to be small.) It should be pointed out that in elastic scattering this linearization is not valid for all theories, and where it is valid it does not hold for the individual diagrams but only the sum over all the crossings [16].

Consider now the integrals over the energy components of the k_j when the linearization of the q-line denominators is valid. These have the form

$$I_0 = \int \frac{dk_{n0}}{2\pi} \cdots \frac{dk_{1\,0}}{2\pi} 2\pi\delta(p_{f0} - p_{i0} - k_{1\,0} - \ldots - k_{n0}) \times$$

$$\times F(k_{1\,0}, \ldots, k_{n0}) \Big\{ \sum_P \left[-2q \cdot (k_{P1} + \ldots + k_{P(n-1)}) + i\varepsilon\right]^{-1} \ldots$$

$$\ldots \left[-2q \cdot k_{P1} + i\varepsilon\right]^{-1} \Big\}.$$

Changing the variables of integration to

$$x_j = 2q \cdot k_j = 2q_0 k_{j0} - 2\boldsymbol{q} \cdot \boldsymbol{k}_j$$

this reduces to

$$I_0 = \frac{1}{(4\pi q_0)^{n-1}} \int dx_1 \ldots dx_n F[k_{1\,0}(x_1), \ldots, k_{n0}(x_n)] \times$$

$$\times\, \delta(x_1 + \ldots + x_n) \{ \sum_P [-x_{P1} - \ldots - x_{P(n-1)} + i\varepsilon]^{-1} \ldots$$

$$\ldots [-x_{P1} + i\varepsilon]^{-1} \}.$$

The evaluation of this integral is trivial because of the identity [11]

$$\delta(x_1 + \ldots + x_n) \{ \sum_P [-x_{P1} - \ldots - x_{P(n-1)} + i\varepsilon]^{-1} \ldots$$

$$\ldots [-x_{P1} + i\varepsilon]^{-1} \} = (-2\pi i)^{n-1} \delta(x_1) \ldots \delta(x_n).$$

In the case $n = 2$ this is equivalent of the well-known formula

$$\frac{1}{-x + i\varepsilon} + \frac{1}{x + i\varepsilon} = -2\pi i \delta(x).$$

A proof of the general case begins with the integral representations

$$\frac{1}{-x + i\varepsilon} = \frac{1}{i} \int_{-\infty}^{\infty} d\tau e^{i\tau(-x)} \theta(\tau)$$

and

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau e^{i\tau x}.$$

After a change of variables

$$\delta(x_1 + \ldots + x_n) \sum_P [-x_{P1} - \ldots - x_{P(n-1)} + i\varepsilon]^{-1} \ldots$$

$$\ldots [-x_{P1} + i\varepsilon]^{-1} = \frac{1}{2\pi i^{n-1}} \int dT_1 \ldots dT_n \times$$

$$\times\, e^{-i(T_1 x_1 + \ldots + T_n x_n)} \times$$

$$\times \sum_P \theta(T_{P1} - T_{P2}) \theta(T_{P2} - T_{P3}) \ldots \theta(T_{P(n-1)} - T_{Pn}).$$

But

$$\sum_P \theta(T_{P1} - T_{P2}) \ldots \theta(T_{P(n-1)} - T_{Pn}) = 1,$$

leading directly to the result claimed above. The physics involved in the identity is probably most evident in this last equation. Because we are summing over all permutations each exchanged particle can be emitted from the q-line with equal probability at any time: this probability is unaffected by other emissions occurring along the same line.

With this identity

$$I_0 = \frac{1}{(2iq_0)^{n-1}} F[k_{1\,0}(0), \ldots, k_{n0}(0)]$$

where

$$[x = 2q_0 k_0 - 2\boldsymbol{q} \cdot \boldsymbol{k} = 0] \Rightarrow k_0(0) = -v_q k_z.$$

Here $-\boldsymbol{q}$ defines the z direction, and

$$v_q = |\boldsymbol{q}|/q_0$$

is the center-of-mass velocity parameter. It is important to note that v_q has not been assumed nearly equal to one: The linearization of the q-line denominators may be valid for small v_q if the mass M is large enough compared to the exchanged mass.

When k_0 is replaced by $-v_q k_z$

$$-k^2 \Rightarrow (1-v_q^2)k_z^2 + \boldsymbol{k}_\perp^2 = \boldsymbol{k}'^2$$

and

$$(p_i+k)^2 - M^2(\mu) \Rightarrow -2\boldsymbol{p}_i' \cdot \boldsymbol{k}' - \boldsymbol{k}'^2 + M^2(\mu_i) - M^2(\mu),$$

where

$$\boldsymbol{k}_\perp' = \boldsymbol{k}_\perp, \quad \boldsymbol{p}_\perp' = \boldsymbol{p}_\perp$$

$$k_z' = \sqrt{1-v_q^2}\, k_z, \quad p_z' = \frac{p_z + v_q p_0}{\sqrt{1-v_q^2}}.$$

Note that $\boldsymbol{p}_i'$ is just the incident momentum in the reference frame where $\boldsymbol{q}' = 0$. In the original center-of-mass frame the "potentials" $f[(1-v_q^2)\, k_z^2 + \boldsymbol{k}_\perp^2]$ were Lorentz contracted; by changing the variables of integration from $\boldsymbol{k}$ to $\boldsymbol{k}'$ the potentials again become spherically symmetric in the average rest frame of the q-line particle.

Inserting these results in the original expression for $A^{(n)}$

$$A^{(n)}(\mu_f, \mu_i) = \frac{-i}{(2M)^{n-1}} \sum_{\{\mu\}} \int \frac{d^3k_n'}{(2\pi)^3} \cdots \frac{d^3k_1'}{(2\pi)^3} \times$$

$$\times (2\pi)^3 \delta^3(\boldsymbol{\Delta} - \boldsymbol{k}_1' - \ldots - \boldsymbol{k}_n') f(\mu_f, \mu_{n-1}; \boldsymbol{k}_n'^2) \ldots$$

$$\ldots f(\mu_1, \mu_i; \boldsymbol{k}_1'^2) \times$$

$$\times [-2\boldsymbol{p}_i'(\boldsymbol{k}_1' + \ldots + \boldsymbol{k}_{n-1}') - (\boldsymbol{k}_1' + \ldots + \boldsymbol{k}_{n-1}')^2 + M^2(\mu_i) - M^2(\mu_{n-1}) + i\varepsilon]^{-1} \ldots$$

$$\ldots [-2\boldsymbol{p}_i' \cdot \boldsymbol{k}_1' - \boldsymbol{k}_1'^2 + M^2(\mu_i) - M^2(\mu_1) + i\varepsilon]^{-1},$$

where $\Delta = p'_f - p'_i$ for energies much greater than $M(\mu_f) - M(\mu_i)$. Changing variables of integration to

$$p_1 = p'_i + k'_1$$
$$p_2 = p'_i + k'_1 + k'_2$$
$$\vdots$$

this can be written as

$$\frac{i}{2M} A^{(n)}(\mu_f, \mu_i) = \sum_{\{\mu\}} \int \frac{d^3 p_{n-1}}{(2\pi)^3} \cdots \frac{d^3 p_1}{(2\pi)^3} \times$$

$$\times \left[\frac{1}{2M} f(\mu_f, \mu_{n-1}; (p'_f - p_{n-1})^2)\right] \cdots \left[\frac{1}{2M} f(\mu_1, \mu_i; (p_1 - p'_i)^2)\right] \times$$

$$\times [p_i'^2 + M^2(\mu_i) + i\varepsilon - p_{n-1}^2 - M^2(\mu_{n-1})]^{-1} \cdots$$

$$\cdots [p_i'^2 + M^2(\mu_i) + i\varepsilon - p_1^2 - M^2(\mu_1)]^{-1}.$$

This has precisely the form of the corresponding terms in the potential theory Born series, with

$$2m \langle p'_f \mu_f | T^{(n)} | p'_i \mu_i \rangle \Leftrightarrow \frac{i}{2M} A^{(n)}(\mu_f, \mu_i)$$

$$2mV(\mu_2, \mu_1; k) \Leftrightarrow \frac{1}{2M} f(\mu_2, \mu_1; k^2)$$

$$2me(\mu_2) - 2me(\mu_1) \Leftrightarrow M^2(\mu_2) - M^2(\mu_1),$$

and with the external momenta evaluated in the q-line rest frame [17].

There is thus some reason for hoping that coupled channel potential theory can provide an adequate description of certain high energy processes. It must be remembered, however, that fairly strong restrictions have been placed on the types of graphs considered: only crossed ladders with an "inert" q-line were included.

The eikonal approximation in coupled channel potential theory [5, 18, 19] is discussed most efficiently in terms of the scattering wave functions in coordinate space. The coupled Schroedinger equations have the form

$$\left[-\frac{\nabla^2}{2m} + e(\mu_2)\right] \psi(\mu_2, r) + \sum_{\mu_1} V(\mu_2, \mu_1; r) \psi(\mu_1, r) =$$

$$= \left[\frac{p_i^2}{2m} + e(\mu_i)\right] \psi(\mu_2, r),$$

with the boundary condition

$$\psi(\mu, \boldsymbol{r}) \underset{r\to\infty}{\to} \delta(\mu, \mu_i)e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}} + \text{ outgoing spherical waves.}$$

If the wave functions are written in the form

$$\psi(\mu, \boldsymbol{r}) = \sqrt{\frac{p_i}{p(\mu)}}\, e^{i[\boldsymbol{p}_i\cdot\boldsymbol{r}+(p(\mu)-p_i)z]}\Gamma(\mu, \mu_i; \boldsymbol{b}; z, -\infty)$$

where

$$p(\mu) \approx p_i + \frac{m}{p_i}[e(\mu_i)-e(\mu)]$$

is the asymptotic momentum in the μ-th channel, then dropping terms quadratic in $\boldsymbol{p}-\boldsymbol{p}_i$ in the usual way gives the simple matrix differential equation

$$i\frac{\partial\Gamma}{\partial z} = U\Gamma.$$

Here Γ is the matrix defining the wave function, while U is a modified potential matrix

$$U(\mu_2, \mu_1; \boldsymbol{r}) = \frac{m}{\sqrt{p(\mu_2)p(\mu_1)}}\, e^{-i[p(\mu_2)-p(\mu_1)]z}V(\mu_2, \mu_1; \boldsymbol{r}).$$

The matrix Γ satisfies the boundary condition

$$\Gamma \underset{z\to-\infty}{\to} 1$$

and from its value at $z = +\infty$ the T-matrix can easily be calculated:

$$\langle \boldsymbol{p}_f\mu_f|T|\boldsymbol{p}_i\mu_i\rangle = \sum_{\mu}\int d^3re^{-i\boldsymbol{p}_f\cdot\boldsymbol{r}}V(\mu_f, \mu; \boldsymbol{r})\psi(\mu, \boldsymbol{r}) \approx$$

$$\approx i\sqrt{\frac{p_fp_i}{m^2}}\int d^2be^{-i(\boldsymbol{p}_f-\boldsymbol{p}_i)\cdot\boldsymbol{b}}\times$$

$$\times[\Gamma(\mu_f, \mu_i; \boldsymbol{b}; \infty, -\infty)-\delta(\mu_f, \mu_i)].$$

The problem, then, is to solve the differential equation for Γ. The formal solution, of course, is

$$\Gamma = \mathscr{Z}\exp\left[-i\int_{-\infty}^{z} d\zeta U(\boldsymbol{b}, \zeta)\right],$$

where $\mathscr{Z}$ is the z-ordering operator, but if, as will usually be the case,

$$[U(\boldsymbol{b}, z), U(\boldsymbol{b}, z')] \neq 0$$

this cannot be converted to any simple analytic form which is useful for calculation.

In some cases, however, it is sufficient to expand Γ in powers of the off-diagonal elements of U. This is because in high energy physics the cross-sections for particular inelastic channels are usually small compared to elastic scattering cross-sections. The expansion is essentially identical to that used in time-dependent perturbation theory:

$$\Gamma(\infty, -\infty) = \Gamma_0(\infty, -\infty) - i \int_{-\infty}^{\infty} dz_1 \Gamma_0(\infty, z_1) U_1(z_1) \Gamma_0(z_1, -\infty) +$$

$$+(-i)^2 \int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{z_2} dz_1 \Gamma_0(\infty, z_2) U_1(z_2) \Gamma_0(z_2, z_1) \times$$
$$\times U_1(z_1) \Gamma_0(z_1, -\infty) +$$
$$+ \ldots,$$

where

$$\Gamma_0(\mu_2, \mu_1; \boldsymbol{b}; z_2, z_1) = \delta(\mu_2, \mu_1) \times$$
$$\times \exp\left[-i \frac{m}{p(\mu_1)} \int_{z_1}^{z_2} d\zeta V(\mu_2, \mu_1; \boldsymbol{b}; \zeta)\right]$$

functions as a propagator for the uncoupled channels.

Dropping terms of second and higher order in U_1, the elastic scattering is given by

$$\langle \boldsymbol{p}_f \mu_i | T | \boldsymbol{p}_i \mu_i \rangle = i \frac{p_i}{m} \int d^2 b e^{-i(\boldsymbol{p}_f - \boldsymbol{p}_i)\cdot \boldsymbol{b}} \times$$
$$\times [e^{i\chi(\mu_i, \boldsymbol{b})} - 1],$$

while inelastic scattering is given be the DWBA formula with eikonal-approximation initial and final state wave functions:

$$\langle \boldsymbol{p}_f \mu_f | T | \boldsymbol{p}_i \mu_i \rangle = \int d^3 r e^{-i\boldsymbol{p}_f \cdot \boldsymbol{r} + i\chi^{(-)}(\mu_f, \boldsymbol{r})} \times$$
$$\times V(\mu_f, \mu_i; \boldsymbol{r}) e^{i\boldsymbol{p}_i \cdot \boldsymbol{r} + i\chi^{(+)}(\mu_i, \boldsymbol{r})}.$$

The elastic scattering amplitude involves the full eikonal function

$$\chi(\mu, \boldsymbol{b}) = -\frac{m}{p(\mu)} \int_{-\infty}^{\infty} dz V(\mu, \mu; \boldsymbol{r}),$$

and these functions can be determined from the scattering amplitude by inverting the Fourier transform. The inelastic amplitude, on the other hand, involves the partial eikonal functions [20]

$$\chi^{(\pm)}(\mu, \boldsymbol{r}) = -\frac{m}{p(\mu)} \int_0^{\infty} d\zeta V(\mu, \mu; \boldsymbol{r} \mp \hat{z}\zeta).$$

For central potentials it is possible to obtain these partial eikonal functions directly in terms of the complete eikonal functions, without passing through the intermediate step of obtaining the potential.

The equation for χ in terms of V is an Abel integral equation [21] which can be inverted to give V as an integral transform of χ. If this expression for V is substituted in the integrals defining the partial eikonals a few simple manipulations lead to

$$\chi^{(\pm)}(\mu, r) = \frac{1}{2}\chi(\mu, b) \pm \frac{z}{\pi}\int_r^\infty \frac{b'db'}{\sqrt{b'^2-r^2}} \times$$

$$\times \frac{[\chi(\mu, b')-\chi(\mu, b)]}{(b'^2-b^2)} .$$

The combination

$$\chi^{(-)}(\mu_f, r)+\chi^{(+)}(\mu_i, r) = \chi_a(b)+$$

$$+\frac{z}{\pi}\int_r^\infty \frac{b'db'}{\sqrt{b'^2-r^2}} \frac{[\chi_d(b')-\chi_d(b)]}{(b'^2-b^2)},$$

where

$$\chi_a(b) = \tfrac{1}{2}\,[\chi(\mu_f, b)+\chi(\mu_i, b)]$$

and

$$\chi_d(b) = \chi(\mu_f, b)-\chi(\mu_i, b),$$

is required for the inelastic scattering amplitude.

There has been considerable discussion in the past over exactly how to modify the Born term in the absorptive model [1–4] when the initial and final states have different elastic interactions. (This is, however, often an academic question, since the final state interactions are in many cases not directly measurable). A common prescription has been to multiply the Born amplitude by the geometric mean of the initial and final state elastic S-matrices. This corresponds to taking only the χ_a term above, and this is accurate only if χ_d is small, or if the exchange potential $V(\mu_f, \mu_i; r)$ has a range which is small compared to those of the elastic interactions.

To summarize the results presented here, it has first of all been shown that in the high energy or large mass limit the sum of all arbitrarily crossed inelastic ladder diagrams is equivalent to coupled channel potential theory provided the linearization of the q-line denominators is a good approximation. This result suggests that in some circumstances coupled channel potential theory may provide a useful description of inelastic quasi-two-body processes when appropriate kinematic variables are used. Finally, the coupled channel potential theory eikonal approximation derivation of the absorptive model shows that the usual geometric mean prescription will be inadequate unless the initial and final state interactions are very similar, or unless the exchange interaction is of very short range.

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